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EUCLIDEAN EXPANSION OF THE SCATTERING AMPLITUDE IN NUCLEAR PHYSICS

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ABSTRACT

It is shown that there is a close relation between the usual energy-averaged scattering amplitude for elastic nuclear collisions and the Euclidean expansion of the non-relativistic scattering amplitude. This expansion is the non-relativistic analogue of the crossed-channel Regge analysis used in relativistic particle physics.

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RESUMEN

Se muestra que hay una relación íntima entre la amplitud de dispersión, promediada sobre la energía en los métodos habituales, para colisiones elásticas nucleares y el desarrollo euclidiano de la amplitud de dispersión. Este desarrollo es la analogía no-relativista del análisis tipo Regge en canales cruzados, el cual se emplea en la física relativista de partículas elementales.

INTRODUCTION

1. Since Feshbach, Porter and Weisskopf's[1] classical work, an ever increasing interest has been devoted to the analysis of the averaged scattering amplitude in low-energy nuclear physics. In such an analysis, the cross sections obtained from experimental data taken with poor resolution in energy or the energy-averaged cross sections correspond to the scattering of a wave packet sharply defined in time. This cross section is described by an optical potential determined as some kind of average over the nucleons of the target, since the sharp packet stays a very short time near the nucleus[2]. For better resolution in energy the so-called intermediate structure does appear and we may say that we are seeing the structure produced by scattering processes with a longer time delay. This structure has been explained in terms of the so-called doorway states[3]. In a simple model[4], the intermediate structure can be identified with the coherent excitation of nucleon-hole pairs in the target. For an even better energy resolution a finer structure appears. This fine structure may be due to more complicated processes with longer time delays. In the limit of very good resolution one will have the very narrow compound-nucleus resonances, corresponding to very long time delays.

2. A natural frame for the discussion of the averaged scattering amplitude has been given in a previous paper[5]; this frame is the Euclidean representation of the non-relativistic scattering amplitude, written as an expansion over the parameter ν characterising the exchanged objects. Each term in the expansion transforms

irreducibly under the Galilei group.

For non-forward scattering the little group is E(2) and therefore one may consider the exchange of E(2) poles. Such poles are labeled by the number ν which is the non-relativistic analogue of the parameter α which labels the O(2,1) poles in the Regge-Joos representation[6]. For forward scattering the little group is E(3), and one may exchange E(3) poles, labeled by a parameter μ which corresponds to the σ of Toller[7] labeling the O(3,1) Lorentz poles.

The purpose of this note is to show that the analysis of the non-relativistic amplitude by taking finer and finer averages is equivalent to taking into account singularities in the μ and ν planes which are farther and farther away from the origin. Such a feature suggests a parametrization of the gross averaged amplitude by one or a few effective singularities near the origin of the μ and ν planes. This parametrization, which is the non-relativistic analogue of the Regge-pole analysis used with success in high-energy particle physics, might be a more convenient description than the usual optical-model analysis of the averaged amplitude.

3. Let us consider the forward scattering of spinless particles. For forward scattering the little group is E(3), which is the non-relativistic limit of the homogeneous Lorentz group. The generators of E(3) are

$$L_{\alpha\beta} = - i \left(k_\alpha \frac{\partial}{\partial k_\beta} - k_\beta \frac{\partial}{\partial k_\alpha}\right) \tag{1}$$

$$L_{\alpha 0} = i \frac{\partial}{\partial k_\alpha} \equiv \xi_\alpha \qquad \alpha, \beta = 1,2,3 \tag{2}$$

There are two invariant operators,

$$\xi_1^2 + \xi_2^2 + \xi_3^2 = - \nabla_k^2 \tag{3}$$

and

$$\epsilon_{\alpha\beta\gamma} L_{\alpha\beta} L_{\gamma 0} \tag{4}$$

but (4) vanishes identically for forward scattering of spinless particles.

A complete set of labels for the vectors of an irreducible representation of E(3) is provided by the eigenvalues of L^2 and L_{12}.

The simultaneous eigenfunctions of (3), L^2 and L_{12} will be denoted by $f_{\mu lm}(k)$ or $f_{\mu lm}(k, \theta, \phi)$,

$$(\xi_1^2 + \xi_2^2 + \xi_3^2) f_{\mu lm} = \mu^2 f_{\mu lm}, \quad 0 \leqslant \mu < \infty \tag{5}$$

$$L^2 f_{\mu lm} = l(l+1) f_{\mu lm}, \quad l = 0, 1, 2, \ldots \tag{6}$$

$$L_{12} f_{\mu lm} = m f_{\mu lm}, \quad m = -l, \ldots, +l \tag{7}$$

In the case we are considering, the expansion of the scattering amplitude in terms of such eigenfunctions gives us

$$A(k) = A_D + \int_0^\infty \mu^2 a(\mu) \frac{\sin \mu k}{\mu k} d\mu \tag{8}$$

where $A(k)$ is the scattering amplitude and k is the wave number. The term A_D gives the contribution of the discrete representation of E(3) and may be included formally in the integral if we allow $a(\mu)$ to be a distribution. Therefore we may write

$$kA(k) = \int_0^\infty \mu a(\mu) \sin \mu k \, d\mu \tag{9}$$

hence

$$\mu a(\mu) = \frac{2}{\pi} \int_0^\infty k A(k) \sin \mu k \, dk \tag{10}$$

In order to take into account only a certain region of the μ plane we introduce a symmetric cut-off function $f(\mu)$ and

$$\mathfrak{Z}(k) = \frac{1}{\pi} \int_0^\infty f(\mu) \cos k\mu \, d\mu$$

If

$$kA(\mathfrak{Z}, k) \equiv \int_0^\infty f(\mu)\, \mu a(\mu) \sin \mu k \, d\mu \tag{11}$$

and recalling that A is a function of k^2, we can define $A(k)$ for negative values of k as

$$A(k) \equiv A(-k)$$

Then the integrals can be extended to $-\infty$

$$kA(\mathfrak{Z}, k) = \int_{-\infty}^{+\infty} \mathfrak{Z}(k-\tau)\, A(\tau)\, \tau \, d\tau \tag{12}$$

Therefore, the amplitude averaged with a weight function $\mathfrak{Z}$ with a width k_0 corresponds to taking into account a region of the μ plane up to a distance of the origin of the order $1/k_0$.

If we are interested in the gross features of the scattering amplitude, as is the case in the optical-model analysis, it is enough to consider the region of the μ plane near the origin.

4. Non-forward scattering. Let us consider now the non-forward elastic scattering when momentum is exchanged. The relevant group is the little group E(2), which leaves invariant the momentum transfer vector q.

The E(2) generators, which act in a two-dimensional plane perpendicular to q, are

$$L_{12} = - i \left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1} \right) \equiv - i \frac{\partial}{\partial \phi} \tag{13}$$

$$L_{i0} = i \frac{\partial}{\partial k_i} \equiv \xi_i , \quad i = 1, 2 . \tag{14}$$

The only invariant operator is

$$\xi_1^2 + \xi_2^2 = - \nabla^2_{k_\perp}$$

$$- \nabla^2_{k_\perp} = - k_\perp^{-1} \frac{\partial}{\partial k_\perp} k_\perp \frac{\partial}{\partial k_\perp} - k_\perp^{-2} \frac{\partial^2}{\partial \phi^2} \tag{15}$$

where

$$k_\perp^2 = k_1^2 + k_2^2 , \qquad \phi = \tan^{-1} \frac{k_2}{k_1}$$

A complete set of labels for the vectors of an irreducible representation of E(2) is provided by the eigenvalues of L_{12} .

The simultaneous eigenfunctions of (13) and (15) are denoted* by $h_{\nu m}(k_1 \text{ y } k_2)$

*The physical meaning of μ and ν has been discussed in ref. 5; these parameters can be related to the time delay in the group velocity of cylindrical packets[7,8].

$$(\xi_1^2 + \xi_2^2)\, b_{\nu m} = \nu^2 b_{\nu m}, \quad 0 \leq \nu^2 < \infty$$

$$L_{12}\, b_{\nu m} = m b'_{\nu m}, \quad m = 0, \pm 1, \pm 2, \ldots \qquad (16)$$

If we consider the scattering of spinless particles, the expansion of the amplitude in these eigenfunctions gives us

$$A(k_\perp, q) = A_D(q) + \int_0^\infty \nu a(\nu, q)\, J_0(\nu k_\perp)\, d\nu$$

where q is the momentum transfer and $p = \sqrt{k_\perp^2 + \frac{q^2}{4}}$ (17)

is the wave number.

Again the term $A_D(q)$ is the contribution of the discrete representation and may be included formally in the integrand if one allows $a(\nu, q)$ to be a distribution; we will then have

$$A(k_\perp, q) = \int_0^\infty \nu a(\nu, q)\, J_0(\nu k_\perp)\, d\nu \qquad (18)$$

$$a(\nu, q) = \int_0^\infty k_\perp A(k_\perp, q)\, J_0(\nu k_\perp)\, dk_\perp \qquad (19)$$

As in the preceding case, if one is interested in those terms of the scattering amplitude which have a weak energy dependence, it is enough to take into account only the region of the ν plane near the origin. If we parametrize such a region with one or a few effective singularities, the parameters will depend on the momentum transfer q and therefore one may study the motions of the singularities in the ν plane as functions of q.

5. It has been shown in the previous paragraphs that the concept of Regge-pole exchange, trajectories, etc., have a well-defined analogue in non-relativistic scattering and that this decomposition constitutes a natural frame for the type of analysis by finer and finer averaging used in nuclear physics. In particular, if one is interested only in the rough features of the scattering amplitude, it is enough to consider the singularities in the region of the ν plane near the origin, and by parametrizing this region with one or a few effective poles or branch points one would have a description of the averaged amplitude complementary to the optical model and perhaps more convenient in many cases. The application of this type of parametrization to the analysis of nucleon-nucleus collisions will be considered elsewhere.

However, in order to say more about the dynamical mechanism, it is necessary to know the analytical structure in a larger region of the ν plane. If, as seems to happen in the nuclear case, the different dynamical mechanisms have different characteristic times, this will show up as a clustering of the singularities in the μ and ν planes. Therefore the study of the analytical structure of the scattering amplitude as a function of μ and ν as well as the study of the restrictions that more or less detailed dynamical assumptions impose on such analytical structure seem to be interesting problems worth exploring.

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