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ON THE SCREENING OF IMPURITIES IN HOT-ELECTRON TRANSPORT PROBLEMS

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ABSTRACT

A transport approximation of the screening parameter in hot-electron problems is considered.

RESUMEN

Se considera una aproximación de transporte para el parámetro de pantalla en problemas de electrones calientes.

The effect of ionized impurities in the transport properties of a semiconductor is frequently calculated using the following scattering potential [1]

$$\phi(r) = \frac{Ze}{\epsilon} \frac{e^{-Kr}}{r} \tag{1}$$

where Ze is the impurity charge, ϵ the dielectric constant and K is the inverse of the screening length

$$K^2 = \frac{4\pi e^2 n}{\epsilon k T_L}$$

where n is the effective carrier density (only electron density considered here) and T_L is the (near) equilibrium temperature. This result is obtained by solving the equilibrium Boltzmann equation coupled with the Poisson equation, thereby correlating the electron density and the potential.

If one is concerned with hot-electron transport problems, the distribution of electrons is far from equilibrium and the same approach couples the equations:

$$\frac{\mathbf{p}}{m} \cdot \nabla_r f + e(\mathbf{F} - \nabla\phi) \cdot \nabla_p f = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}} \tag{2a}$$

$$\epsilon \nabla^2 \phi = -4\pi e n(\mathbf{r}) + 4\pi e \left[z \sum_i \delta(\mathbf{r} - \mathbf{r}_i) + n_0\right] \tag{2b}$$

where $\mathbf{p}$, m, e are the momentum, effective mass and charge of the electron, $\mathbf{F}$ is the applied field, n_0 is the density of the positive ground to balance the electron charge and $\mathbf{r}_i$ is the position of the charged impurity. The right-hand side of (2a) represents the now non-vanishing collision rate and the two equations are coupled through the relation

$$n(\mathbf{r}) = \int f(\mathbf{r}, \mathbf{p})\, d^3 p \tag{3}$$

The set of equations (2) are in fact more general than those needed in the study of the effects of impurities in transport phenomena, because in the latter the impurity generally acts in the form of a perturbation (to appear only in the collision term). A self-consistent solution to the problem can be obtained when interelectronic collisions are dominant. In this case the electron distribution can be approximated by [2] (parabolic energy bands):

$$f(\mathbf{r}, \mathbf{p}) = \frac{h^3 n(r)}{2(2m\pi kT_e)^{3/2}} \exp\left\{- \frac{|\mathbf{p}-\mathbf{p}_0|^2}{2mkT_e}\right\} \tag{4}$$

where $\mathbf{p}_0$ is the drift momentum and T_e is the temperature of the electrons. Considering momentum conservation and using (2a) and (4) we obtain

$$RT_e \nabla n + en\nabla\phi = e\mathbf{F}n + \int \mathbf{p} \left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}} d^3 p \tag{5}$$

where the assumption has been made that in the transport régime no other parameter except $n(r)$ is space-dependent in the distribution function. To make further progress we have to eliminate the correlation in eq. (5) between the terms that have significant variation near the impurity and those which represent the average (over space) in the transport region. This separation can be performed formally by assuming that each side of (5) vanishes independently; this is possible because the left-hand side refers to the impurity region and the right-hand side to the transport region (far from impurity). Hence our self-consistent treatment proposes the following separation:

Transport region

Simultaneous equations in $\mathbf{p}_0$ and T_e:

$$e\mathbf{F}n = -\int \mathbf{p} \left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}} d^3p \tag{6a}$$

$$\frac{ne}{m} \mathbf{F} \cdot \mathbf{p}_0 = -\int E(\mathbf{p}) \left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}} d^3p \tag{6b}$$

Eq.(6b) follows from the energy-conservation relation [2]. In these equations n is taken as the average (space-independent) electron density and $E(\mathbf{p})$ is the electron energy.

The effect of the impurity appears only in the collision (perturbation) term and in fact has an effect only in (6a), because the presence of the impurity in the transport regions is assumed to cause a change only in momentum and not in energy. These equations are the usual hot-electron set [4].

Impurity region

$$kT_e \nabla n + en\nabla\phi = 0 \tag{7a}$$

$$\epsilon\nabla^2\phi = -4\pi e\, n(\mathbf{r}) + 4\pi e\, [Z\delta(\mathbf{r}) + n_0] \tag{7b}$$

Here ϕ is now the potential of a single scatterer. The set of equations (7) are identical to those leading to the usual linearization approximation (1), but now in the screening length the equilibrium temperature T_L is replaced by the electron temperature T_e. This is the only effect due to the new distribution on the scattering by impurities in our "transport-régime" approximation that does not consider in detail the region near the impurity.

Our result is valid independently of the value of the ratio $\mathbf{p}_0^2/m2kT_e$ and hence whether or not the "drift term" in the average energy [3]

$$E = \frac{\mathbf{p}_0^2}{2m} + \frac{3}{2} kT_e$$

is negligible. The result leads to a modification of the screening with field in situations where the field is considered in a better approximation than the linear, because the departure of T_e from T_L is at least of second order in the field strength (see eqs. (3.7), (3.16) and (3.27) of ref. [4]). The substitution of T_e for T_L as the only modification of the screening in non-linear conditions has been used previously on intuitive grounds [3].

Also, in setting (7a) we have not made use of the form of the impurity charge distribution; hence the result should be valid for the screening in other scattering mechanisms as well, in particular for piezo-acoustic scattering where the screening is of importance.

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