Vol. XVIII, 4 REVISTA MEXICANA DE FISICA 1969

"FREE AND EFFECTIVE N-N INTERACTIONS AND THE STRUCTURE OF LIGHT NUCLEI"*

Susana Ramírez, E. Yépez
Facultad de Ciencias, Universidad Nacional de México
and
M. de Llano
Instituto de Física, Universidad Nacional de México

(Recibido: 13 de junio 1969)

RESUMEN

Se estudian las diferencias entre interacciones nucleón-nucleón "libres" y "efectivizadas" a través de sus resultados sobre las energías de amarre de ^3H, ^4He, ^{16}O así como el espectro energético del ^6Li, suponiendo siempre el espacio de configuraciones más simple de acuerdo al modelo de capas de oscilador armónico: $(Os)^3$, $(Os)^4$, $(Os)^4\,(Op)^{12}$ y $(Os)^4\,(Op)^2$, respectivamente. Se emplearon potenciales N-N realistas con carozo repulsivo blando así como elementos de matriz asociados a diversos tipos de interacciones efectivas. Resulta clara-

*Work supported by Comisión Nacional de Energía Nuclear (México) and Dirección General del Profesorado, U.N.A.M.

mente patente la necesidad de "renormalizar" una interacción N-N libre cuando se trabaje en un problema secular correspondiente a un espacio de configuraciones restringido.

ABSTRACT

The differences between "free" and "effectivized" nucleon-nucleon interactions are studied through their effects on the binding energies of ^{3}H, ^{4}He, ^{16}O, as well as the energy spectrum of ^{6}Li, by assuming the simplest oscillator shell model configuration for each: $(Os)^3$, $(Os)^4$, $(Os)^4\,(Op)^{12}$ and $(Os)^4\,(Op)^2$, respectively. Soft-cored, realistic N-N potentials are used, as well as effective interaction matrix elements of various kinds. The need to "renormaliz " a free N-N interaction whenever one does a restricted configuration space secular problem is clearly manifested.

I. INTRODUCTION

Since the pionnering work of Brueckner and others dating from 1954 or so, much attention has been given to the problem of deducing, from a given interaction between two isolated nucleons, an (effective) interaction appropriate for treatment in a limited-configuration-space secular problem (e.g., a shell model calculation). The advent of soft-cored "realistic" *N-N* interactions, fitting the empirical data associated with two-nucleon systems, has permitted obtaining *finite* values for the relevant shell-model matrix elements and, consequently, enabling one to carry trough shell model calculations with these potentials. Thus, it becomes possible to compare the results, on nuclear structure properties, produced by the "free" versus that of an "effective" interaction which has been deduced from the former by one method or another; hence, one may evaluate the relative necessity of deriving such effective interactions.

II. FREE AND EFFECTIVE N-N INTERACTIONS

The most general form of the nucleon-nucleon interaction potential consistent with the usual invariance symmetries associated with non-relativistic strong-interactions can be written[1] as the well-known sum of central, tensor, spin-orbit and quadratic spin-orbit parts. In the last decade[2] a great deal of effort has been put into obtaining "realistic" potentials; that is: an interaction potential function of the above mentioned form with radial functions *parameterized* to fit both N-N scattering experiments for $0 \lesssim E_{lab} \lesssim 350$ MeV *and* the ground state properties of the bound n-p (deuteron) system. Among the best-known "realistic" potentials are the Hamada-Johnston and Yale forms whose radial forms were assumed, "a priori", to be independent of p^2 and l^2, an effect supposedly simulated by the introduction of an infinitely repulsive (hard) core for $r \lesssim 0.5$ F. The former potential has 28, the latter 52, non-zero adjustable parameters. (Recently, one-boson-exchange meson-theoretic N-N potentials with as few as $\simeq 5$ adjustable parameters and which fit scattering data remarkably well have been derived[3]). Our interest here will focus only on two "realistic" potentials whose radial forms are a superposition of attractive and repulsive gaussian functions, with *finite* "soft" cores, which turn out to be extremely convenient to handle when dealing with harmonic oscillator orbitals.

The *Tamagaki*[4] (G3RS-1) potential has 56 parameters and has radial functions of gaussian form (see Figs. 1 to 4):

$$v_\lambda(r) = \sum_{i=1}^{3} V_{\lambda i}^0 \, e^{-(r/\eta_{\lambda i})^2} \qquad (\lambda = c, t, lS, W, ll), \tag{1}$$

with $V_{\lambda i}^0$, $\eta_{\lambda i}$ the strengths and ranges adjusted to fit the N-N phase shift data of Arndt & MacGregor (1966). No mention is made of any attempt to fit the bound-state deuteron properties (a calculation which is in progress[15]). On the whole, it appears that the fit to the *scattering* data is at least as good as that accomplished by the Hamada-Johnston potential.

The *Eikemeier & Hackenbroich*[5] (or *Tübingen*) potential consists also of

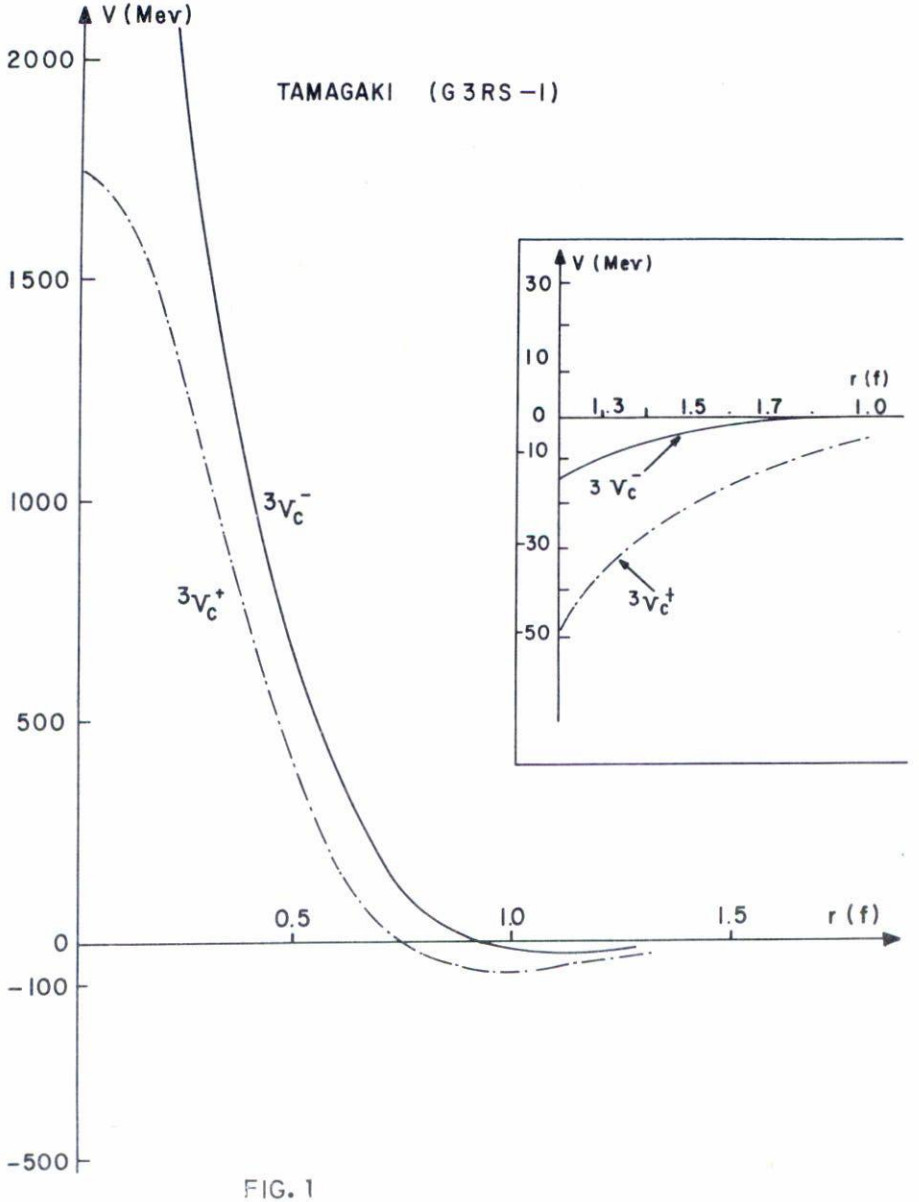
V (Mev)
2000
1500
1000
500
0
-100
-500
TAMAGAKI (G 3RS -I)
$^3V_c^-$
$^3V_c^+$
0.5
1.0
1.5
r (f)
V (Mev)
30
10
0
-10
-30
-50
r (f)
1.3
1.5
1.7
1.0
3 V_c^-
$^3V_c^+$

FIG. 1

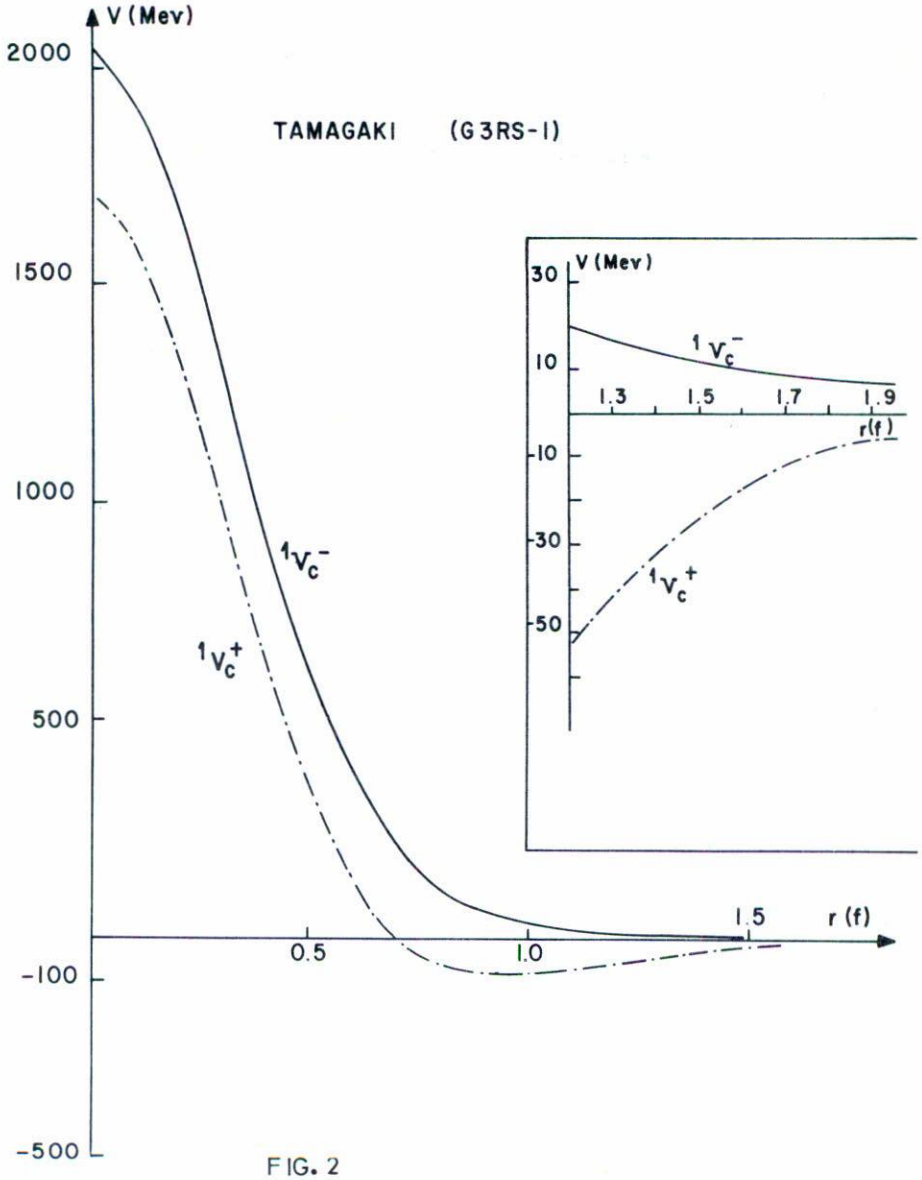
V (Mev)
2000
1500
1000
500
-100
-500
TAMAGAKI (G3RS-I)
$^1V_c^-$
$^1V_c^+$
0.5
1.0
1.5
r (f)
30
10
-10
-30
-50
1.3
1.5
1.7
1.9
r(f)

FIG. 2

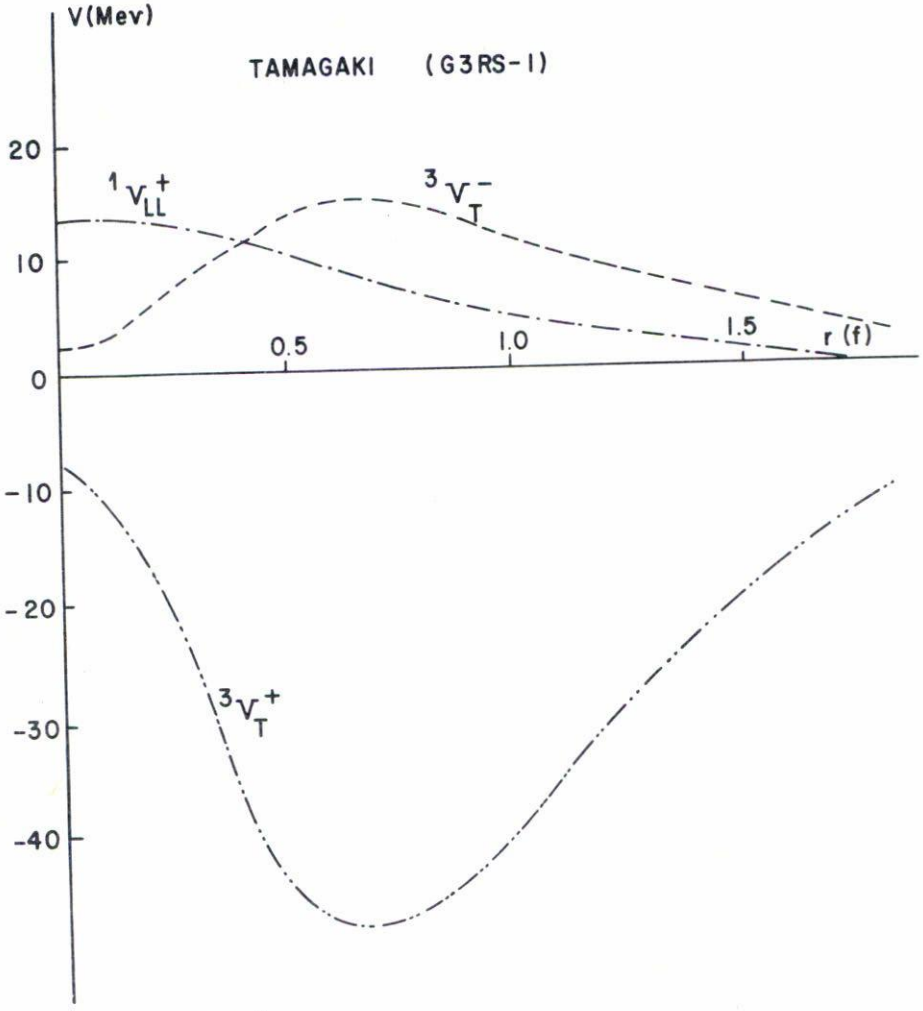

FIG. 1

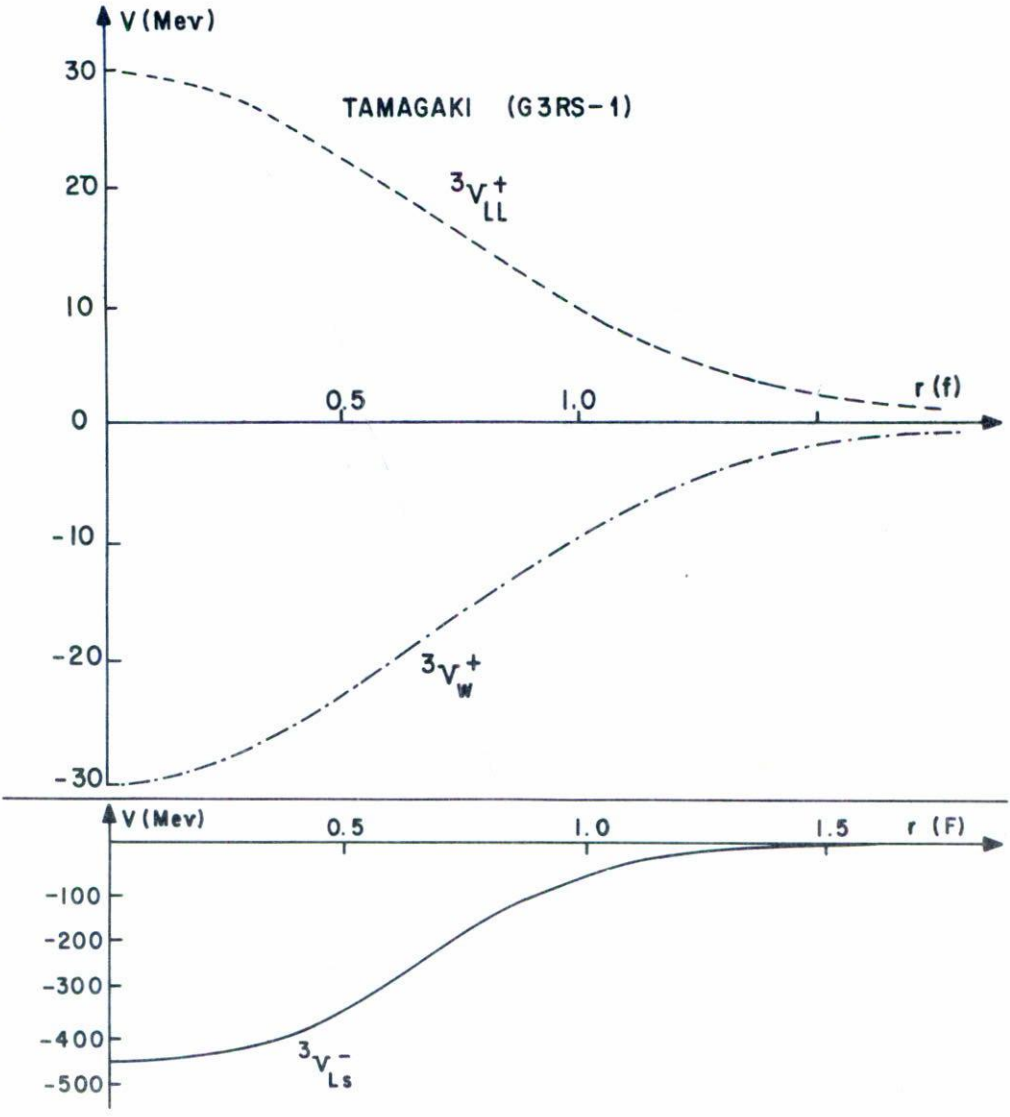
V (Mev)
30
20
10
0
-10
-20
-30
TAMAGAKI (G3RS-1)
$^3V_{LL}^+$
0.5
1.0
r (f)
$^3V_W^+$
V (Mev)
0.5
1.0
1.5
r (F)
-100
-200
-300
-400
-500
$^3V_{Ls}^-$

FIG. 4

gaussians and includes a soft core but is even simpler in that it is *purely central* and to date has been reported only for the even-l states (cf. Fig. 5). It has a total of 12 parameters which were adjusted to the Yale (1962) phase shifts, the low-energy effective-range theory data compiled in 1957 by Hulthén & Sugawara, *and also* the deuteron binding energy.

Regarding the so-called "effective" interactions we adopt also only two. The one given by *Volkov*[6] (number 5 in his notation) (cf. Fig. 6), with 5 parameters adjusted to fit binding energies and rms radii of *both* ^{4}He and ^{16}O, assuming a spherical Slater determinant as a model state.

Elliott and co-workers[7] have recently deduced nuclear matrix elements directly from N-N phase shift data of the Yale group (1968). The matrix elements turned out to be strikingly similar numerically to those obtained by Grillot & McManus[8] who actually solved the Bethe-Goldstone equation in relati· coordinate. It has been argued[9] that the Elliott method for constructing nuclear matrix elements from phase shift data contains the essential ingredients of a Brueckner-Bethe-Goldstone (reaction matrix) effective interaction, namely: short-ranged correlations treated exactly (non-perturbatively) and "healing" of the perturbed to unperturbed function at long ranges. The Elliott matrix elements shall thus be taken in what follows as those of an *effective interaction*. For comparison, matrix elements which *do not*[9] involve "healing" have also been used in the ^{16}O binding energy calculation; these were deduced[16] directly from phase shift data employing a technique due to Koltun[17].

III. THE SHELL MODEL PROBLEM

Using both the *free* and *effective* interactions described above we wish to calculate energies within a shell model framework, that is: restricting the many-body function to the most natural model subset of single-particle configurations.

a) ***Doubly-Closed Shell.*** It is assumed that the ground state (model) wave function of a doubly-cl·sed shell nucleus (like ^{4}He, ^{16}O, ^{40}Ca, ...) to be the single Slater determinant Φ_0 composed of the lowest-lying single-particle harmonic

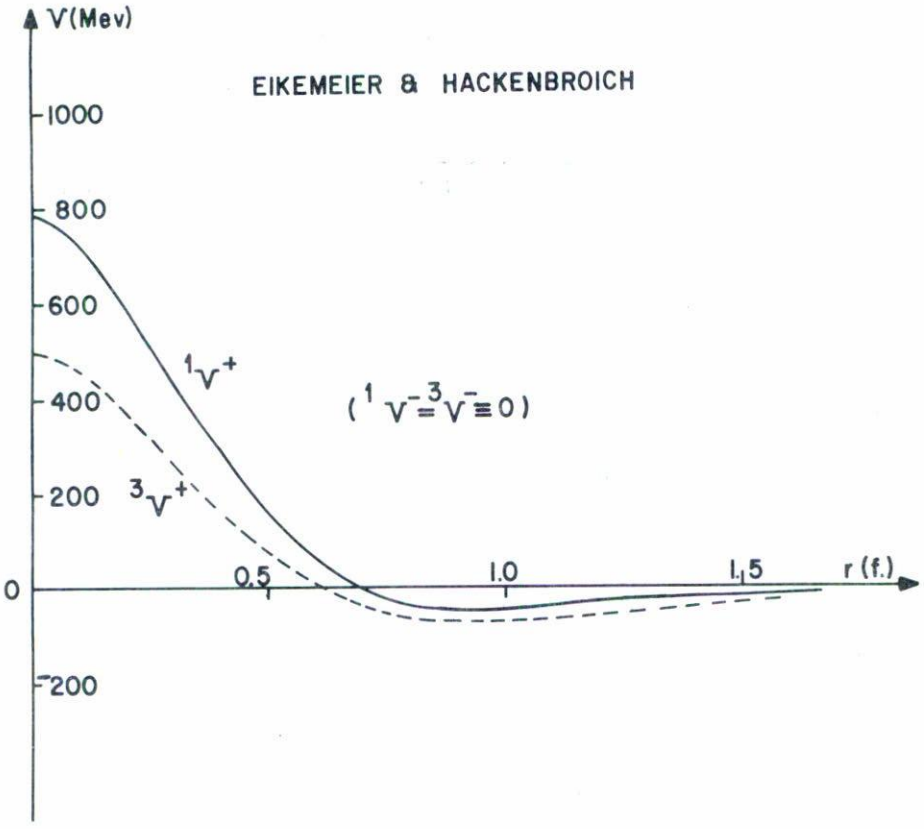
V(Mev)
EIKEMEIER & HACKENBROICH
1000
800
600
400
200
0
-200
$^1V^+$
$^3V^+$
$(^1V^- = {}^3V^- \equiv 0)$
0.5
1.0
1.5
r (f.)

FIG. 5

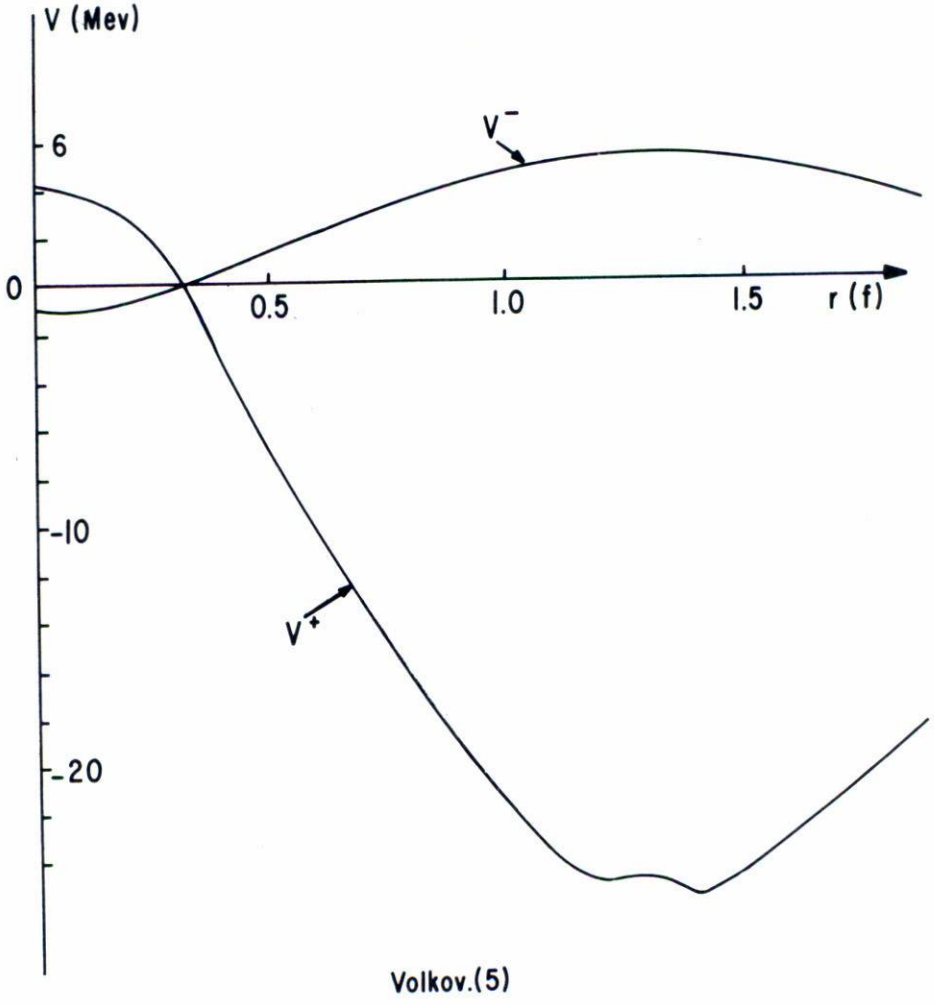

FIG. 6

oscillator orbitals. The hamiltonian of the A-particle system is taken as

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} - \frac{1}{2Am} \left(\sum_{i=1}^{A} p_i \right)^2 + \sum_{i<j}^{A} v_{ij} \qquad (2)$$

$$= T_{\text{int}} + \sum_{i<j}^{A} v_{ij}$$

which is both galilean and translational invariant, and in which v_{ij} will be either a *free* or an *effective* N–N interaction. The expectation value of (2) is then[10]

$$\langle \Phi_0 | H | \Phi_0 \rangle = \frac{1}{2} \sum_{\alpha} \left(2n_\alpha + l_\alpha + \frac{3}{2} \right) \hbar\omega - \frac{3}{4} \hbar\omega$$

$$+ \frac{1}{2} \sum_{\alpha,\beta} \{ \langle \alpha\beta | v_{12} | \alpha\beta \rangle - \langle \alpha\beta | v_{12} | \beta\alpha \rangle \}, \qquad (3)$$

in which the sums are only over *occupied* states.

The expectation value of the potential energy for a doubly-magic nucleus described by a single Slater determinant becomes

$$\langle \Phi_0 | \sum_{i<j}^{A} v_{ij} | \Phi_0 \rangle = \sum_{nlSj} (2T+1)(2j+1) \langle nlSj | v_{12} | nlSj \rangle \cdot$$

$$\cdot \sum_{n_1 l_1 n_2 l_2 LN\mathfrak{L}} \frac{(2L+1)}{(2l+1)} | \langle nlN\mathfrak{L}, L | n_1 l_2 n_2 l_2, L \rangle |^2 \cdot$$

(4)

Evaluation of the two-body elements $\langle nlSj | v_{12} | n'l'Sj \rangle$ for a general N–N potential v_{12} of the Tamagaki kind is quite direct.

For $A = 4$ (^{4}He), eqs. (3) and (4) give

$$\langle H \rangle = \frac{9}{4}\hbar\omega + 3\,[\langle 0, {}^3S_1 | v_{12} | 0, {}^3S_1 \rangle + \langle 0, {}^1S_0 | v_{12} | 0, {}^1S_0 \rangle] \qquad (5)$$

and it is readily seen that for $A = 3\,({}^3\mathrm{H})$ one has

$$\langle H \rangle = \frac{3}{2}\,[\hbar\omega + \langle 0, {}^3S_1 | v_{12} | 0, {}^3S_1 \rangle + \langle 0, {}^1S_0 | v_{12} | 0, {}^1S_0 \rangle]\,. \qquad (6)$$

Also, for $A = 16\,({}^{16}\mathrm{O})$ there results $[v(n, {}^{2S+1}l_j) \equiv \langle nlSj | v_{12} | nlSj \rangle]$:

$$\langle H \rangle = \frac{69}{4}\hbar\omega + 21\,[v(0, {}^1S_0) + v(0, {}^3S_1)] +$$

$$+ \frac{3}{2}\,[v(1, {}^1S_0) + v(1, {}^3S_1)] + 6\,[v(0, {}^1P_1) + v(0, {}^3P_0)] +$$

$$+ 18\,v(0, {}^3P_1) + 30\,v(0, {}^3P_2) + \frac{15}{2}\,v(0, {}^1D_2) + \frac{3}{2}\,v(0, {}^3D_1) +$$

$$+ \frac{5}{2}\,v(0, {}^3D_2) + \frac{7}{2}\,v(0, {}^3D_3)\,. \qquad (7)$$

b) *The Open-Shell Nucleus* ^6Li. It will be assumed that the ground and excited positive-parity states of ^{6}Li are linear combinations of configurations consisting of an inert ^{4}He core plus two valence nucleons restricted in the Op harmonic oscillator shell. The secular problem is then

$$\det \Big|\, {}_a\langle n_1 l_1 j_1, n_2 l_2 j_2; JT | v_{12} | n_3 l_3 j_3, n_4 l_4 j_4; JT \rangle_a +$$

$$+ \{\Delta E(1 + \delta_{j_3^{\frac{1}{2}}})(1 + \delta_{j_4^{\frac{1}{2}}}) - E_{JT}\}\, \delta_{n_1 l_1 j_1, n_3 l_3 j_3}\, \delta_{n_2 l_2 j_2, n_4 l_4 j_4} \Big| = 0 \qquad (8)$$

where $\Delta E \equiv E_{p\,1/2} - E_{p\,3/2}$ is the single-particle spin-orbit energy splitting taken from experiment as $\Delta E \simeq 4$ MeV, and E_{JT} will be the energy levels with the lowest value corresponding to the contribution to the total binding of ^{6}Li coming from the n-p valence pair. This binding energy is deducible also from experiment[12] through the relation

$$B(np) = B(^6\text{Li}) - B(^4\text{He}) - [B(^5\text{Li}) - B(^4\text{He})]$$

$$- [B(^5\text{He}) - B(^4\text{He})]$$

$$= 6.585 \text{ MeV}.$$

IV. RESULTS

The calculated and experimental binding energies of ^{3}H and ^{4}He (Coulomb repulsion subtracted) are shown in Figs. 7 and 8. Experimental energies were taken from ref. 12; "experimental" $\hbar\omega$ values are deduced from the empirical rms radii found in ref. 18, through the readily-obtained relation

$$r_{rms} = \sqrt{(2n + l + \frac{3}{2}) \frac{\hbar}{m\omega}}\ , \qquad (9)$$

where (n, l) refer to the harmonic oscillator orbit of the "most exterior" nucleon.

The Tübingen potential is seen to give the most binding in both cases (at roughly the "right" $\hbar\omega$) but it is certain that addition of the missing odd-l parts to this potential will *reduce* this binding. The Tamagaki-1 potential grossly underbinds in both cases, as might be expected because of the very repulsive soft core ($\gtrsim 3$ times that of the Tübingen soft cores which are probably *too small* judging from the bad phase-shift fits at higher energies). Both the Elliot[7] matrix elements, deduced from the Yale (1968) phase shifts, and the Grillot-McManus[19] matrix elements obtained by solving[8] the Bethe-Goldstone equation with the

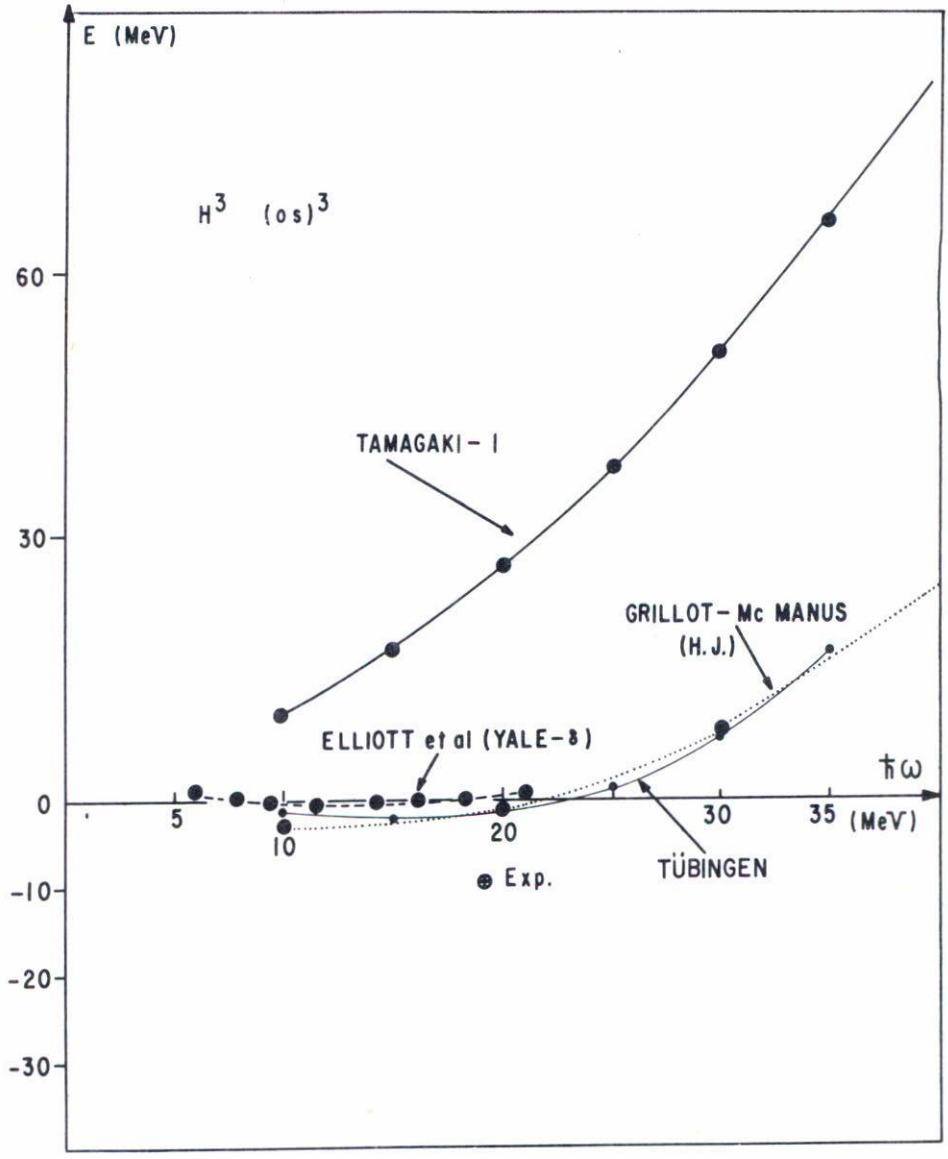
E (MeV)
H^3 $(os)^3$
60
30
0
-10
-20
-30
TAMAGAKI - I
GRILLOT - Mc MANUS
(H.J.)
ELLIOTT et al (YALE-8)
$\hbar\omega$
5
10
20
30
35 (MeV)
Exp.
TÜBINGEN

FIG. 7

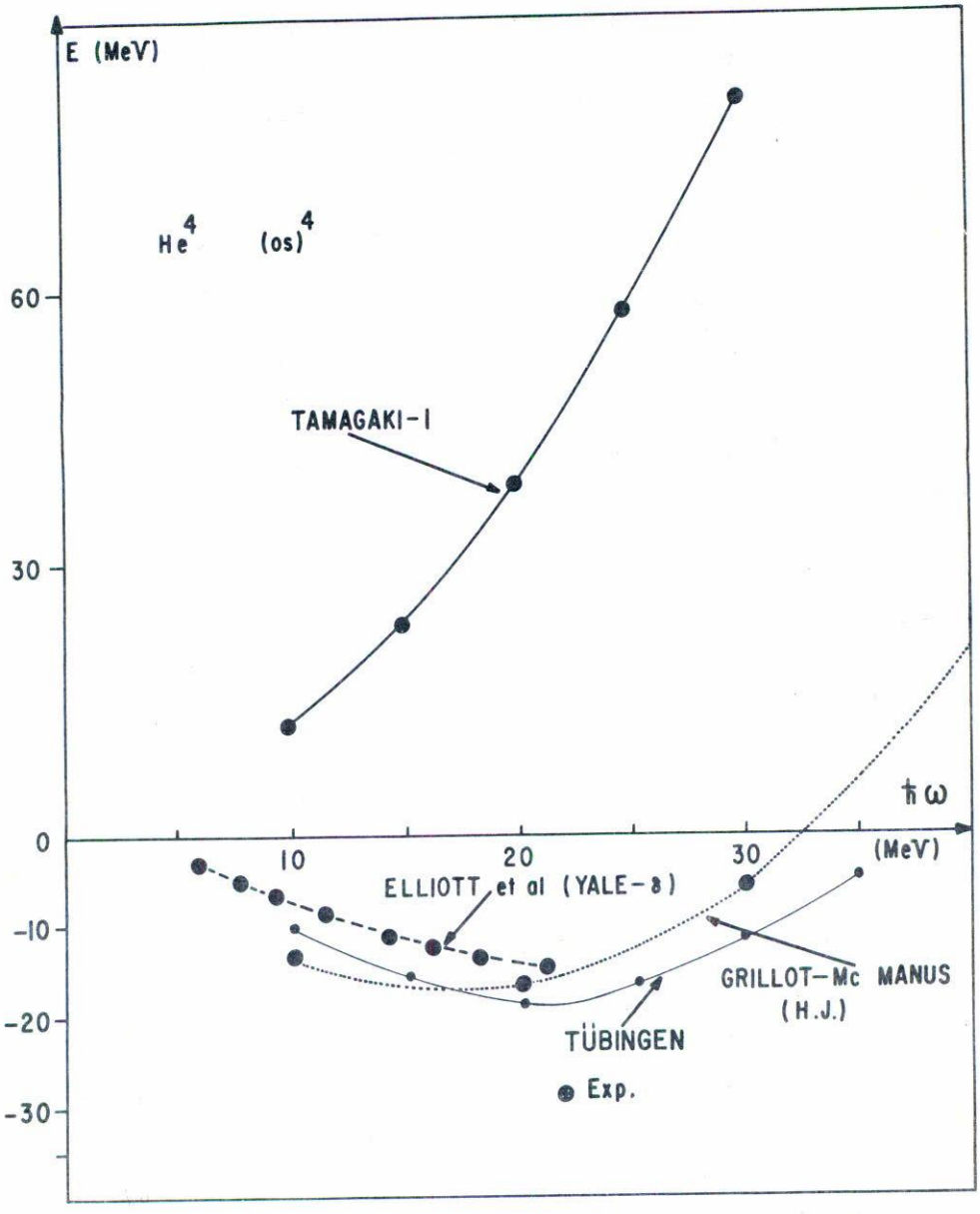
E (MeV)
He^4 $(os)^4$
60
30
0
-10
-20
-30
TAMAGAKI-I
10
20
30
$\hbar\omega$
(MeV)
ELLIOTT et al (YALE-8)
GRILLOT-Mc MANUS
(H.J.)
TÜBINGEN
Exp.

FIG. 8

Hamada-Johnson potential, give *very similar results*. All results, using *free* "realistic" and *effective* interactions, *underestimate the binding*.

The corresponding results for ^{16}O (Fig. 9) are very analogous to those for $A = 3$ and 4. The almost exact agreement of the Tübingen potential is again misleading for the reasons given above. We have also calculated the binding energy using the t-matrix elements associated with the Schrödinger equation, in relative coordinate, of two nucleons in a common oscillator well interacting with each other through a Yale interaction. These have been deduced directly from the Yale (1968) phase shifts. Comparisons[16] with those obtained by *integrating* the Schrödinger with the *explicit* potential function show that, insofar as the diagonal elements go, the two methods are essentially equivalent. These matrix elements will differ from the Bethe-Goldstone ones in the *absence* of the Pauli exclusion principle, found in the latter, which prevents virtually scattered particles from going to states already occupied by other particles. *Absence* of the Pauli effect thus gives *too much* binding (as is to be expected as one is then *overcounting* contributions to the matrix elements) and *inclusion* of the Pauli effect results in *too little* binding The difference between the two "extremes" is very striking indeed (cf. Ref. 9 for a more detailed discussion).

In Fig. 10 is shown the experimental[22] spectrum of ^{6}Li (extreme right) compared to the results of various calculations, all in the $(Os)^4 (Op)^2$ configuration model space, using either a free "realistic" or an "effective" interaction acting between the two Op valence particles. Again, the Tübingen potential (E-H) gives the best overall agreement as regards level sequence and spacings. The Volkov-5 force does not reproduce the ground state! The Elliott matrix elements correctly predict the sequence of the first few levels but the spectrum is too compressed. The spectrum marked "modelistic" is shown only for comparison; it was obtained[20] by adjusting four parameters in a model interaction consisting of pairing, quadrupole, long-ranged tensor and single-body spin-orbit forces. Finally, the Tamagaki (G3RS-1) potential gives the most unrealistic results: the actual ground state $J^{\pi}T = 1^{+}0$, together with the whole spectrum, is pushed too high. It thus appears that the more realistic an interaction (in the sense of fitting reasonably well isolated two-body scattering) the clearer the need either

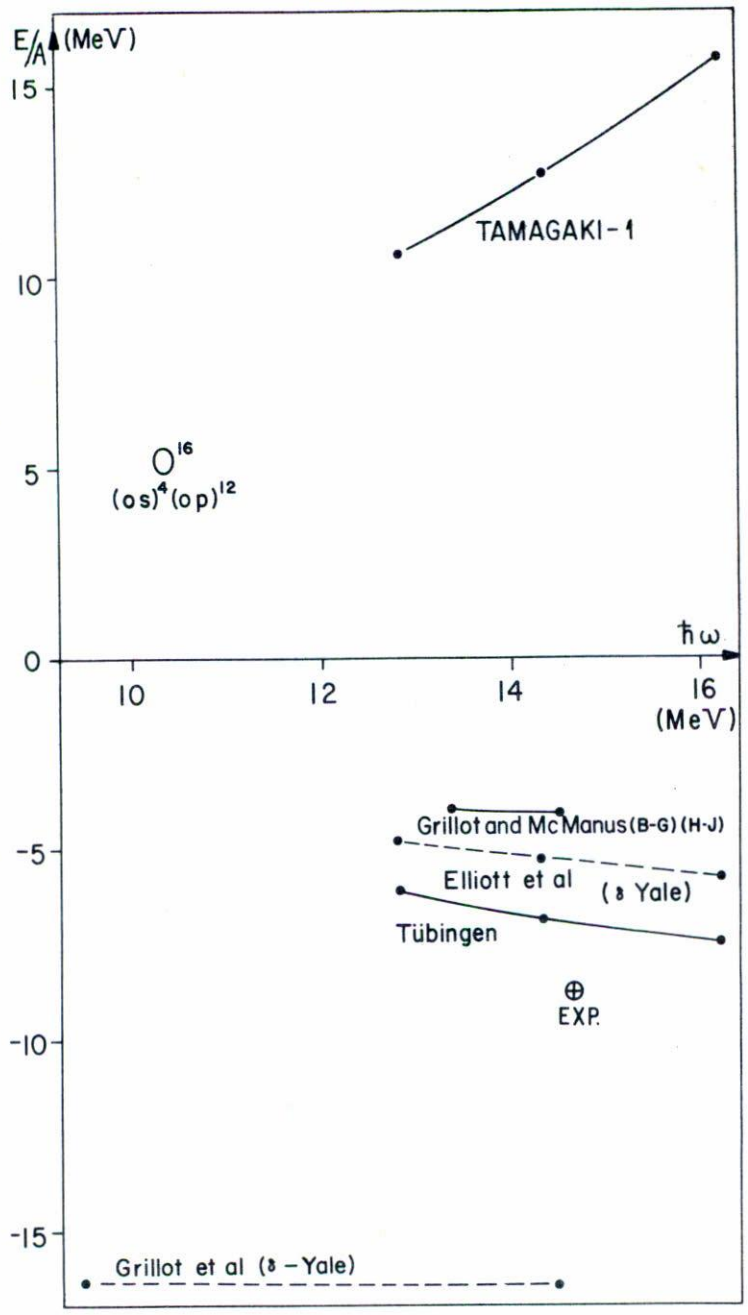
E/A (MeV)
15
10
5
0
-5
-10
-15
TAMAGAKI-1
O16
(os)4(op)12
ħω
10
12
14
16
(MeV)
Grillot and McManus (B-G) (H-J)
Elliott et al (δ Yale)
Tübingen
EXP.
Grillot et al (δ-Yale)

FIG. 9

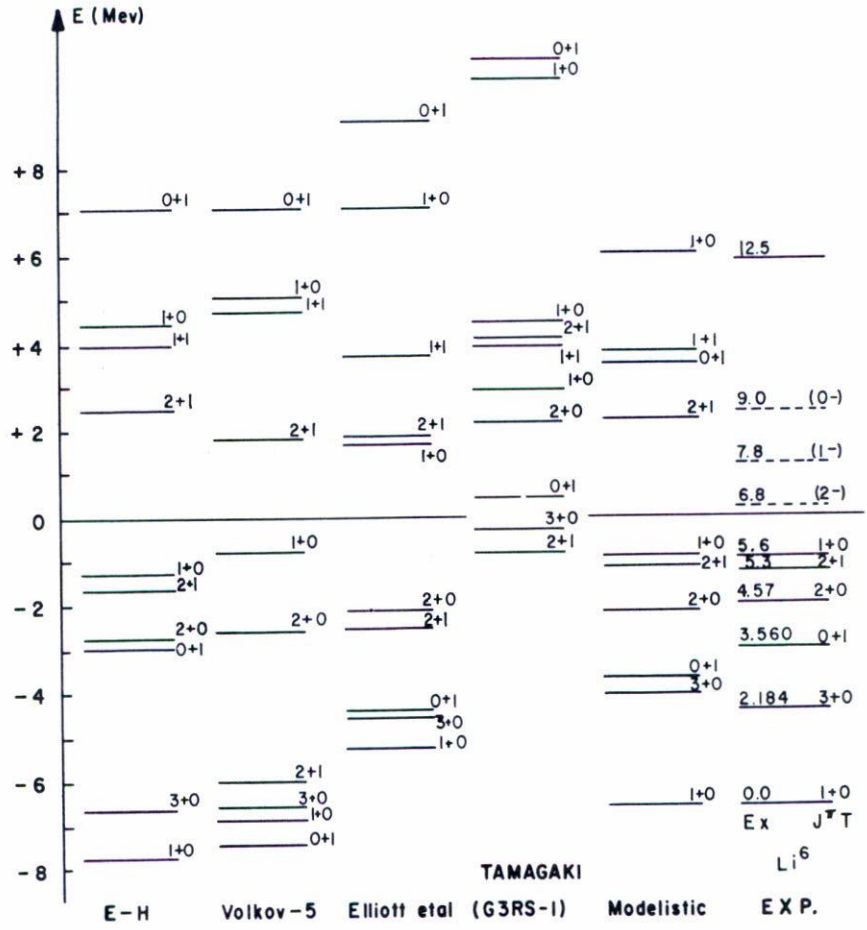
E (Mev)
+8
+6
+4
+2
0
-2
-4
-6
-8
E-H
Volkov-5
Elliott etal (G3RS-1)
TAMAGAKI
Modelistic
EXP.
Ex
J^π T
Li^6
12.5
9.0 (0-)
7.8 (1-)
6.8 (2-)
5.6
5.3
4.57
3.560
2.184
0.0

FIG. 10

of considerably enlarging the model space beyond the simple $(0s)^4\ (0p)^2$ configuration space or, "effectivizing" the interaction in some consistent way. Either of these refinements will build into the two-body relative matrix elements short-ranged correlations by "pushing out" the two-body wave function *away* from the core region of the potential. This would obviously weigh the repulsive parts of the free N-N potential less drastically.

In Fig. 11 are shown the oscillator radial functions (squared and multiplied by r^2) used in evaluating the (diagonal) matrix elements $(nl|v_\gamma(r)|nl)$. A comparison of this figure with, say, Figs. 1 through 4 which show the radial parts of the Tamagaki potential, indicates that lower l-values (predominantly the s-waves) are producing *too much repulsion* in both binding and excited spectra calculations.

V. CONCLUSIONS

Our main conclusion is that it is meaningless to do a nuclear structure calculation with a *free* "realistic" N-N interaction within a *limited* shell-model configuration space. The need to "renormalize" such an interaction seems quite dramatically evident from both binding and excited spectra results.

A basic ingredient in this "renormalization" is the Pauli exclusion principle kernel of the Bethe-Goldstone integral equation for the correlated (or "perturbed") two-body wave function. This kernel essentially prevents the (virtual) scattering of two particles to states otherwise occupied by *other* particles and is responsible for the well-known "healing" of the perturbed to unperturbed wave function[8,21]. The absence of these properties have the drastic result of overbinding a nucleus like ^{16}O to as much as *twice* the experimental value.

ACKNOWLEDGEMENTS

We are grateful to E. Ley-Koo for helpful discussions and to D.V. Grillot for kindly providing us with unpublished matrix elements. Also, one of us (E.Y.) appreciates the assistance of J.L. del Río in some calculations.

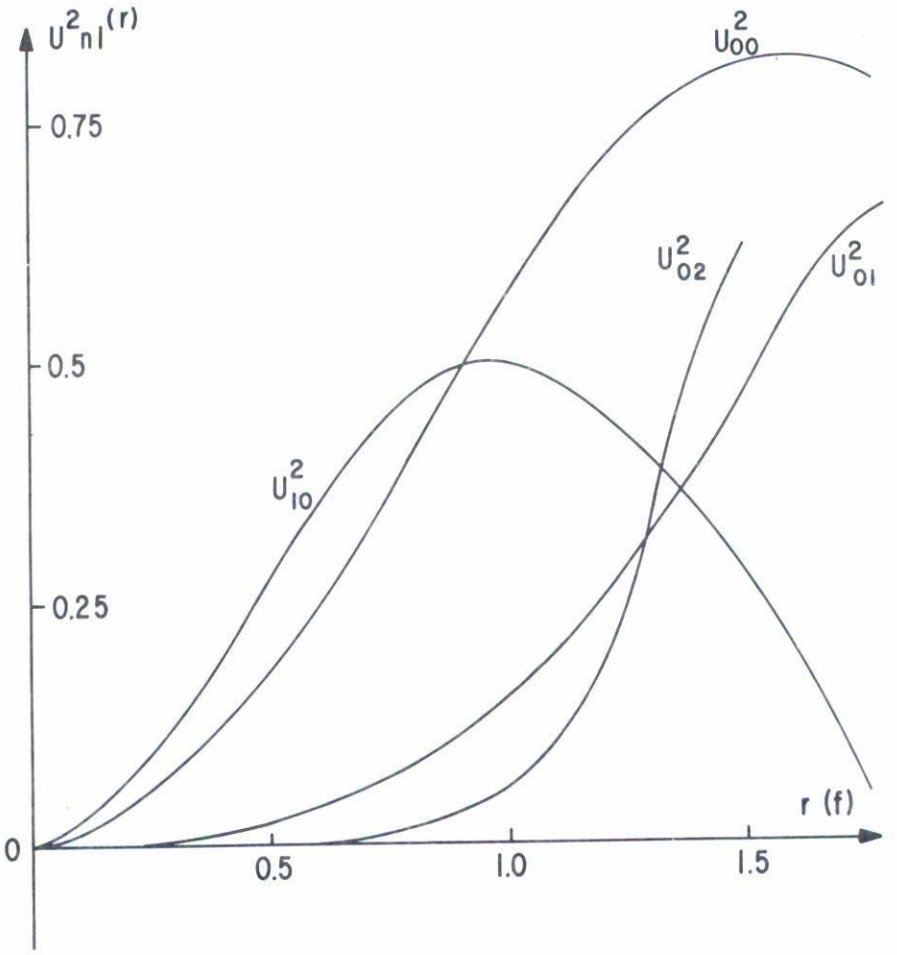

FIG. 11

REFERENCES

1. J. Okubo & R. Marshak, Ann of Phys. (N.Y.) 4 (1958) 166. For details, cf. Canuto, V. and M. de Llano (unpublished notes on N-N Interactions).
2. "Gainesville Conference on the N-N Interaction". Revs. Mod. Phys. 39 (1967) and also, Supplement, Prog. Theor. Phys. 39 (1967) as well as 42 (1968), and references therein.
3. Green, Sawada & Saxon, *The Nuclear Independent-Particle Model* (Academic Press, 1968) and references in the last section of this book.
4. Tamagaki, R., Prog. Theor. Phys. 39 (1968) 91.
5. Eikemeier, H. and H. Hackenbroich, Z.f. Phys. 195 (1966) 412.
6. Volkov, A.B., Nucl. Phys. 74 (1965) 33.
7. Elliott, J.P., A.D. Jackson, H.A. Mavromatis, E.A. Sanderson & B. Singh, Nucl. Phys. A121 (1968) 241.
8. Grillot, D. and H. McManus, Nucl. Phys. A113 (1968) 161.
9. E. Ley-Koo & M. de Llano (to be published).
10. Condon, E.U. & G.H. Shortley, *The Theory of Atomic Spectra* (Cambridge, 1959) p. 171.
11. Mavromatis, H.A., E.A. Sanderson & B. Singh, "Tables of Coefficients for the Expansion of Two-particle jj-coupled Matrix elements in terms of Relative Matrix Elements". (unpublished).
12. König et al, Nucl. Phys. 31 (1962) 18.
13. Brody, T.A. & M. Moshinsky, *Tables of Transformation Brackets* (Gordon & Breach, 1967).
14. Tamagaki, R. & W. Watari, Supplement, Prog. Theor. Phys. 39 (1967).
15. del Río, J.L., (private communication).
16. Grillot, D.V., E. Ley-Koo, M. de Llano and H. McManus, Nucl. Phys. (to be published).
17. Koltun, D.S., Phys. Rev. Lett. 19 (1967) 910.
18. Elton, L.R.B., *Nuclear Sizes* (Oxford, 1961).
19. Grillot, D.V., (private communication).
20. E. Ley-Koo et al, Nucl. Phys. A107 (1968) 35.

21. Day, B., Rev. Mod. Phys. 39 (1967) 719.
22. Ajzenberg-Selove, F. et al, Phys. Rev. 139 (1965) B 592.