Revista Mexicana de Física 19 (1970) 339-366

AN EXTENSION OF A SCHEMATIC THEORY OF NUCLEAR REACTIONS TO INCLUDE ISOSPIN AND ISOBARIC ANALOGUE RESONANCES*

P. A. Mello and M. Moshinsky

Instituto de Física, Universidad Nacional de México

(Recibido: julio 31, 1970)

ABSTRACT: In a previous paper one of the authors discussed a schematic theory of nuclear reactions in which both the colliding particles and the compound system are described as point particles, the main advantage being that the time-dependent analysis of the problem is considerably simplified. In the present paper we extend the schematic theory to include the isotopic spin formalism in the specific example of a two-channel problem involving isobaric analogue resonances.

I. INTRODUCTION

In a previous paper, one of the authors[1] discussed a schematic theory of nuclear reactions, in which both the colliding particles and the compound system are described as point particles; the so-called internal region (in the R'-matrix language) has radius zero but still retains all of the characteristics of a compound system. The idea here for the internal region is similar to

*Work supported by the Comisión Nacional de Energía Nuclear, México.

that of a classical system of n masses interconnected by springs. This system has a set of n normal modes, that are analogous to the levels of our zero-radius compound nucleus. On the other hand, we can consider a semi-infinite string along the x-axis extending from $x = 0$ to $x = \infty$, which can be set into vibration; the amplitude of the vibration at a position x at time t is analogous in the nuclear reaction problem to the wave function of the two separated fragments, when the distance between them at time t is r. In the classical system, the really interesting problem arises when we tie the end of the string to the system of springs at $x = 0$; this is analogous, in the nuclear problem, to allowing for a probability different from zero for the transition between the system being in the form of two separated fragments and in the form of a compound system. This is achieved by appropriate boundary conditions at $r = 0$. The classical model allows a complete time-dependent description and similarly, in the quantum mechanical case, the main advantage of shrinking the colliding particles and the compound system to point particles is that in many instances the time dependence can again be analized[2].

In a previous paper[3] which will be referred to as I from now on, a classical model was constructed for the two-channel problem[4] involving isobaric analogue resonances and its complete time dependence was analized in a subsequent paper[5].

In the present paper we extend the schematic theory of nuclear reactions to the two-channel problem mentioned above. In this problem one considers a proton $|p>$ incident on a target $|C>$

$$|p> \equiv \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$

$$|C> \equiv \left|T_0, T_0\right\rangle, \qquad (1.1)$$

the target having isospin T_0 and projection $(N - Z)/2 = T_0$. In the compound system one has two possibilities $T_{\gtrless} = T_0 \pm 1/2$ for the isospin. The states with $T_<$ correspond to the ground and low-lying excited states of the compound nucleus as their isospin and projection are the same. The states $T_>$, as their projection is $T_0 - 1/2$, are isobaric analogues of the ground or low excited states of the nucleus with Z protons and $N + 1$ neutrons. The first $T_>$ state, from medium heavy nuclei on, will appear in the region of energy in which the $T_<$ states are already quite dense. The $T_>$ state will then interact with this "sea" of $T_<$ states, giving rise to a modulating effect on their widths which is observed in high resolution experiments.

If we want to construct states with isospin $T_>$ or $T_<$, we make use of Clebsch-Gordan coefficients

$$|T_{\gtrless}, T_0 - \tfrac{1}{2}> = \left\langle T_0 \tfrac{1}{2} T_0 - \tfrac{1}{2} \middle| T_{\gtrless} (T_0 - \tfrac{1}{2}) \right\rangle |pC> +$$

$$+ \left\langle T_0 \tfrac{1}{2} (T_0 - 1) \tfrac{1}{2} \middle| T_{\gtrless} (T_0 - \tfrac{1}{2}) \right\rangle |nA \quad . \quad (1.2)$$

We employed the notation

$$|n> \equiv \left|\tfrac{1}{2} \tfrac{1}{2}\right>$$

$$|A> \equiv |T_0 (T_0 - 1)> \alpha T_- |C> \qquad (1.3)$$

to indicate the isospin part of the neutron wave function and the isobaric analogue of the target, respectively. The simplest model we can make, that gives rise to $T_>$ and $T_<$ states is then one that couples the $|pC>$ channel of (1.1) with the $|nA>$ channel of (1.3). A strong coupling between these two channels was observed[6] in the charge exchange (p, n) reaction, leading to the isobaric analogue of the target.

Notice that first of all we have made the simplifying assumption that the ground state of the target and its analog have good isospin, the only difference between them being the "Coulomb displacement energy" Δ. If the incident proton is then assumed to move in an average single particle Coulomb potential, rigorously speaking the compound system cannot have good isospin, because the proton in the $p + C$ channel and the neutron in the $n + A$ channel are treated differently. However it has been shown by Robson that one can choose the radius of the internal region (in R-matrix language) such that the isospin impurity arising from the extra proton approximately cancels as far as the compound states are concerned. This leads to the second simplifying assumption, that isospin is a good quantum number for the compound states defined in the internal region (i.e. we neglect the so called "internal mixing"). Therefore, in our schematic theory we shall consider that our point-particle compound system, when isolated completely from the external world, has the possibility of existing in states with *good* isospin. In the external region ($r \neq 0$ in our schematic theory) such an approximation would be impossible

even in the absence of a Coulomb force on the incoming proton, because the neutron feels a potential barrier equal to the $Q(=\Delta)$ of the (p, n) reaction and the proton does not. As an example, we show qualitatively in Fig. 1 the potential V_p felt by a proton in the field of the target and that felt by a neutron (V_n) in the field of the analogue of the target. Both potentials have been drawn identically in the internal region, T being therefore a good quantum number for states defined for $r < a$ only. If we suppress the Coulomb potential felt by the proton outside the target, we get the potential of Fig. 2. The isospin impurity produced by $V_p \neq V_n$ for $r > a$ is known as "external mixing" and gives rise to effects such as the Robson's enhancement factor[3,4], that make themselves evident when we allow for an interaction between the compound nucleus and the external world.

In the next section we consider the extension of the schematic theory to the two-channel problem described above, for the simple case in which the Coulomb interaction on the incoming proton has been turned off, leaving for Section III the discussion of this interaction.

II. A SCHEMATIC THEORY FOR ISOBARIC ANALOGUE STATES WITHOUT COULOMB POTENTIAL

As mentioned in the Introduction we shall study in this section the problem of isobaric analogue resonances in the case where only two channels are considered: the proton channel and the neutron channel, leaving behind a residual nucleus in a state which is the isobaric analogue of the target in its ground state. This problem has been studied by Robson[4] in the framework of the R-matrix theory and our purpose will be to rederive his equations from the standpoint of the schematic theory of nuclear reactions[1]. In Wigner's R-matrix[7] formalism, configuration space is divided into two regions: the internal region, corresponding to all nucleons being close together to form a compound nucleus, and the external region corresponding to two pieces of the compound nucleus being separated by a distance larger than the sum of the radii of these two pieces.

In the schematic theory of nuclear reactions described in the Introduction, one would say that we can find the system either in the form of two separated particles or in the form of a compound system, considered as elementary particles. We describe the first case by means of the vector Ψ in Fock's space

$$\Psi \equiv \begin{bmatrix} \psi_p(r,t) \mid pC > \\ \\ \psi_n(r,t) \mid nA > \end{bmatrix}, \qquad (2.1a)$$

where $\psi_p(r,t)$ and $\psi_n(r,t)$ are the wave functions in relative coordinates for the proton and neutron channels, respectively; $\mid pC >$ and $\mid nA >$ have the same meaning as in (1.1) and (1.3). We describe the situation in which the system is in the form of a compound nucelus by means of the vector Φ

$$\Phi \equiv \begin{bmatrix} \phi_\lambda(t) \mid T_> > \\ \\ \phi_\mu(t) \mid T_< > \end{bmatrix}, \qquad (2.1b)$$

where we have indicated explicitly the possibility of having one compound state with isospin $T_> = T_0 + 1/2$ (the isobaric analogue of a low lying state in the nC system) and one with isospin $T_< = T_0 - 1/2$. Explicitly:

$$\mid T_> > = \frac{\mid pC > + \sqrt{2T_0} \mid nA >}{\sqrt{2T_0 + 1}}$$

$$\mid T_< > = \frac{\sqrt{2T_0} \mid pC > - \mid nA >}{\sqrt{2T_0 + 1}}. \qquad (2.1c)$$

If one thinks of the system as being described by an optical model hamiltonian plus Lane's potential, one has, in fact, a two channel problem and for the $T_<$ state of the compound nucleus one has a broad giant resonance centered several MeV below the analogue state. In the true many body problem, on the other hand, one would have many $T_<$ states (instead of a broad giant resonance) around the analogue, which give rise to the fine structure. This description could be included in the definition of Φ, by adding many states ϕ_μ and the analysis that follows would be almost identical.

In the absence of a Coulomb potential in the external region, Φ satisfies the following equation, for $r \neq 0$

$$i\hbar \frac{\partial \Psi}{\partial t} = - \frac{\hbar^2}{2\mu} \nabla^2 \Psi + m c^2 \Psi \qquad (2.2)$$

where μ is the reduced mass of the proton and the target, which is almost identical to the reduced mass in the channel nA; m is a two by two matrix

$$m = \begin{pmatrix} m_p + m_C & 0 \\ 0 & m_n + m_A \end{pmatrix}$$

containing the mass of each channel when the two fragments are separated. The Q of the (p, n) reaction is therefore given by

$$Q = [(m_p + m_C) - (m_n + m_A)] c^2 < 0$$

In a formalism in which the internal region was of finite size, eliminating the Coulomb potential in the external region would correspond to considering for the proton and the neutron the potentials of Fig. 2 instead ot those of Fig. 1. The Q of the (p,n) reaction is sufficient to produce isospin mixing, as explained in the Introduction.

If there were no coupling between the vectors Ψ and Φ of (2.1a) and (2.1b), Φ would satisfy the equation

$$i\hbar \frac{\partial \Phi(t)}{\partial t} = M c^2 \Phi \ , \qquad (2.3)$$

where $\mathbf{M}$ is a 2 x 2 matrix

$$\mathbf{M} = \begin{pmatrix} M_\lambda & 0 \\ 0 & M_\mu \end{pmatrix} \ , \qquad (2.4)$$

M_λ and M_μ being the masses of the $T_>$ and $T_<$ compound states respectively. The splitting between these two states could be thought of as due to Lane's potential $V_1 \, t \cdot \mathbf{T}$, which has different eigenvalues for the two value of isospin.

The problem of interest arises of course when there is a coupling

between Ψ and Φ, in which case Φ will no longer satisfy Eq. (2.3); in order to see what kind of an alteration one should introduce into Eq. (2.3), we ask that the probability of finding the system in the form of two separated fragments, plus the probability of finding it as a compound nucleus be conserved in time, i.e.

$$\frac{d}{dt}\left[\int \Psi^{+}\Psi d\tau + \Phi^{+}\Phi\right] = 0 \qquad (2.5a)$$

or more generally, to have conservation of probability even for superposition of states, we ask that the following scalar product be conserved in time:

$$\frac{d}{dt}\left[\int \Psi'^{+}\Psi d\tau + \Phi'^{+}\Phi\right] = 0 \qquad (2.5b)$$

where Ψ' and Φ' are any other wave functions. By the scalar product $\Psi'^{+}\Psi$ (and similarly for $\Phi'^{+}\Phi$), we imply a scalar product in isospin space also, i.e.

$$\Psi'^{+}\Psi = [\psi_p'^{*}(r,t) < pC|\ ,\ \psi_n'^{*}(r,t) < nA|]\begin{bmatrix} \psi_p(r,t)|\,pC> \\ \psi_n(r,t)|\,nA> \end{bmatrix} =$$

$$= \psi_p'^{*}(r,t)\,\psi_p(r,t) + \psi_n'^{*}(r,t)\,\psi_n(r,t)\,. \qquad (2.6)$$

If we take the hermitian conjugate of (2.2) for a vector Ψ'

$$-i\hbar\frac{\partial\Psi'^{+}}{\partial t} = -\frac{\hbar^2}{2\mu}\nabla^2\Psi'^{+} + \Psi'(mc^2)\ , \qquad (2.7)$$

take the scalar product of (2.2) with Ψ'^{+} and of (2.7) with Ψ, substract and integrate, we get

$$i\hbar\frac{d}{dt}\int \Psi'^{+}\Psi d\tau = -\frac{\hbar^2}{2\mu}\int [\Psi'^{+}\nabla^2\Psi - (\nabla^2\Psi'^{+})\Psi]\,d\tau\,. \qquad (2.8)$$

It will be convenient to change from Ψ, as defined in (2.1) to contain the proton and neutron channels, to $\overline{\Psi}$ which will contain the $|T_>\rangle$ and $|T_<\rangle$ channels

$$\overline{\Psi} \equiv \begin{bmatrix} \psi_> (r,t) \,|\, T_> \rangle \\ \psi_< (r,t) \,|\, T_< \rangle \end{bmatrix} \tag{2.9}$$

where

$$\begin{bmatrix} \psi_> \\ \psi_< \end{bmatrix} = O \begin{bmatrix} \psi_p \\ \psi_n \end{bmatrix} , \quad O \equiv \begin{bmatrix} \frac{1}{\sqrt{2T_0+1}} & \sqrt{\frac{2T_0}{2T_0+1}} \\ \sqrt{\frac{2T_0}{2T_0+1}} & -\frac{1}{\sqrt{2T_0+1}} \end{bmatrix} \tag{2.10}$$

Then one clearly has the relation

$$\Psi'^{+}\Psi = \overline{\Psi}'^{+}\overline{\Psi}$$

and (2.8) can be written as

$$i\hbar \frac{d}{dt}\int \overline{\Psi}'^{+}\overline{\Psi} d\tau = -\frac{\hbar^2}{2\mu}\int [\overline{\Psi}'^{+}\nabla^2\overline{\Psi} - (\nabla^2\overline{\Psi}'^{+})\overline{\Psi}]\, dV =$$

$$= -\frac{\hbar^2}{2\mu}\oint [\overline{\Psi}'^{+}\nabla\overline{\Psi} - (\nabla\overline{\Psi}'^{+})\overline{\Psi}] \cdot dS \tag{2.11}$$

where we have applied Green's theorem, and the integration is performed on a closed surface consisting of the sphere at infinity plus a small sphere of radius a centered at the origin (we shall eventually let $a \to 0$). If our wave function consists of a packet which goes to zero at infinity, we shall only have the contribution from the small sphere. Changing the direction of dS so that it will point outwards and choosing s-waves, we have

$$i\hbar \frac{d}{dt}\int \bar{\Psi}'^{+}\Psi d\tau = \frac{2\pi\hbar^2}{\mu}\left[(r\psi'_{>})^{*}\frac{\partial(r\psi_{>})}{\partial r} - (r\psi_{>})\frac{\partial(r\psi'_{>})^{*}}{\partial r} + \right.$$
$$\left. + (r\psi'_{<})^{*}\frac{\partial(r\psi_{<})}{\partial r} - (r\psi_{<})\frac{\partial(r\psi'_{<})^{*}}{\partial r}\right]_{r=0}, \tag{2.12}$$

so that the conservation of total probability (5) now reads:

$$\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi'_{>})^{*}}{\partial r}\right]_{r=0}(r\psi_{>})_{r=0} - \left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_{>})}{\partial r}\right]_{r=0}(r\psi'_{>})_{r=0} +$$
$$+ \left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi'_{<})^{*}}{\partial r}\right]_{r=0}(r\psi_{<})_{r=0} - \left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_{<})}{\partial r}\right]_{r=0}(r\psi'_{<})_{r=0} +$$
$$+ \phi'^{*}_{\lambda}\left(-i\hbar\frac{\partial\phi_{\lambda}}{\partial t} + M_{\lambda}c^2\phi_{\lambda}\right) - \phi_{\lambda}\left(-i\hbar\frac{\partial\phi'_{\lambda}}{\partial t} + M_{\lambda}c^2\phi'_{\lambda}\right)^{*} +$$
$$+ \phi'^{*}_{\mu}\left(-i\hbar\frac{\partial\phi_{\mu}}{\partial t} + M_{\mu}c^2\phi_{\mu}\right) - \phi_{\mu}\left(-i\hbar\frac{\partial\phi'_{\mu}}{\partial t} + M_{\mu}c^2\phi'_{\mu}\right)^{*} = 0. \tag{2.13}$$

This bilinear form is of the type

$$\sum_{i=1}^{4}\left(y_i^{*}x_{n+i} - x_i y_{n+i}^{*}\right) = 0 \tag{2.14}$$

and a sufficient condition[1] for (2.14) to vanish identically is that the x_{n+i}'s be a linear combination of the x_i's

$$x_{n+i} = \sum_{j=1}^{4} C_{ij}x_j \tag{2.15}$$

where $\mathbf{C} = \| C_{ij} \|$ is a constant Hermitian matrix. A similar condition holds for the y's. We therefore have

$$(r\psi_>)_{r=0} = C_{>>}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_>)}{\partial r}\right]_{r=0} + C_{><}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_<)}{\partial r}\right]_{r=0} + C_{>\lambda}\phi_\lambda + C_{>\mu}\phi_\mu$$

$$(r\psi_<)_{r=0} = C_{<>}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_>)}{\partial r}\right]_{r=0} + C_{<<}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_<)}{\partial r}\right]_{r=0} + C_{<\lambda}\phi_\lambda + C_{<\mu}\phi_\mu$$

$$-i\hbar\frac{\partial\phi_\lambda}{\partial t} + M_\lambda c^2\phi_\lambda = C_{\lambda>}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_>)}{\partial r}\right]_{r=0} + C_{\lambda<}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_<)}{\partial r}\right]_{r=0} + C_{\lambda\lambda}\phi_\lambda + C_{\lambda\mu}\phi_\mu$$

$$-i\hbar\frac{\partial\phi_\mu}{\partial t} + M_\mu c^2\phi_\mu = C_{\mu>}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_>)}{\partial r}\right]_{r=0} + C_{\mu<}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_<)}{\partial r}\right]_{r=0} + C_{\mu\lambda}\phi_\lambda + C_{\mu\mu}\phi_\mu \,. \tag{2.16}$$

The last two equations are the modifications of (2.3) we were looking for, when Φ is coupled to Ψ.

Assuming that there is no coupling between the wave functions of the compound particles themselves, but only between them and the two channels $T_>$ and $T_<$, we have $C_{\lambda\lambda} = C_{\mu\mu} = C_{\lambda\mu} = C_{\mu\lambda} = 0$. At $r = 0$, ϕ_λ, being a $T_>$ state, will only be coupled to the $T_>$ channel and ϕ_μ to the $T_<$ channel $\therefore C_{\lambda<} = C_{\mu>} = 0$. At $r = 0$, the $T_>$ channel will not be coupled to the $T_<$ so that $C_{><} = C_{<>} = 0$.

Finally we shall assume that the $T_>$ and $T_<$ channels do not couple directly but only through the compound particle, i.e. $C_{>>} = C_{<<} = 0$. This will make the background $\boldsymbol{R}$ matrix[2] vanish, without changing any of the essential features of the problem.

The equations (2.16) simplify in an obvious manner. We shall now proceed to show that these simplified equations are, for the case of s-waves, which is the only one we shall discuss in this paper, entirely analogous to equations (2.3) of Ref. 3 for the classical model. For this purpose we define appropriately renormalized states of the compound nucleus by

$$\phi_\lambda(t) \equiv \frac{\bar{\phi}_\lambda(t)}{C_{>\lambda}} \quad , \qquad \phi_\mu(t) = \frac{\bar{\phi}_\mu(t)}{C_{<\mu}} \tag{2.17}$$

and then introduce a phase factor $\exp\left[-i\hbar^{-1}(m_p + m_C)\,c^2\,t\right]$ in all wave functions, i.e.

$$\psi_p(r,t) \to \psi_p(r,t)\, e^{-\frac{i}{\hbar}(m_p + m_C)\,c^2 t} \quad , \text{ etc.} \tag{2.18}$$

The wave equations for our problem become then

$$-i\hbar\,\frac{\partial(r\psi_p)}{\partial t} = \frac{\hbar^2}{2\mu}\,\frac{\partial^2(r\psi_p)}{\partial r^2} \quad , \quad -i\hbar\,\frac{\partial(r\psi_n)}{\partial t} = \frac{\hbar^2}{2\mu}\,\frac{\partial^2(r\psi_n)}{\partial r^2} - |Q|(r\psi_n) \tag{2.19a, b}$$

$$-i\hbar\,\frac{\partial\bar{\phi}_\lambda}{\partial t} + \left[M_\lambda - (m_p + m_C)\right]c^2\bar{\phi}_\lambda = C^2_{\lambda>}\left[\frac{2\pi\hbar^2}{\mu}\,\frac{\partial(r\psi_>)}{\partial r}\right]_{r=0} \tag{2.19c}$$

$$-i\hbar\,\frac{\partial\bar{\phi}_\mu}{\partial t} + \left[M_\mu - (m_p + m_C)\right]c^2\bar{\phi}_\mu = C^2_{\mu<}\left[\frac{2\pi\hbar^2}{\mu}\,\frac{\partial(r\psi_<)}{\partial r}\right]_{r=0} \tag{2.19d}$$

$$(r\psi_>)_{r=0} = \bar{\phi}_\lambda \quad , \qquad (r\psi_<)_{r=0} = \bar{\phi}_\mu \quad , \tag{2.19e, f}$$

together with the linear transformation (2.10) that relates Ψ to $\bar{\Psi}$.

The equations (2.19) are then entirely equivalent to (2.3) of Ref. 3 if we consider the relations of Table 1.

For stationary states, i.e. if

$$\psi_p(r,t) = \psi_p(r)\, e^{-\frac{iE}{\hbar}t} \tag{2.20}$$

where E is now the kinetic energy, equations (2.19) reduce to the form

$$\left(\frac{d^2}{dr^2} + k_p^2\right)(r\psi_p) = 0 \qquad \left(\frac{d^2}{dr^2} + k_n^2\right)(r\psi_n) = 0 \tag{2.21a,b}$$

$$\{[M_\lambda - (m_p + m_C)c^2] - E\}\bar{\phi}_\lambda \equiv (E_\lambda - E)\bar{\phi}_\lambda = C^2_{\lambda>}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_>)}{\partial r}\right]_{r=0} \tag{2.21c}$$

$$\{[M_\mu - (m_p + m_C)c^2] - E\}\bar{\phi}_\mu \equiv (E_\mu - E)\bar{\phi}_\mu = C^2_{\mu<}\left[\frac{2\pi\hbar^2}{\mu}\frac{\partial(r\psi_<)}{\partial r}\right]_{r=0} \tag{2.21d}$$

$$\bar{\phi}_\lambda = (r\psi_>)_{r=0}, \qquad \bar{\phi}_\mu = (r\psi_<)_{r=0}, \tag{2.21e,f}$$

where we have

$$k_p = \left[\frac{2\mu E}{\hbar^2}\right]^{1/2}, \quad k_n = \left[\frac{2\mu}{\hbar^2}(E - |Q|)\right]^{1/2} \tag{2.22}$$

which, in turn, are identical to equations (3.2) of Ref. 3 if we use the relations of Table 1 which include $\omega^2 \to E$.

From (2.21) it follows that

$$\begin{bmatrix} r\psi_> \\ r\psi_< \end{bmatrix}_{r=0} = \boldsymbol{D} \begin{bmatrix} \frac{\partial r\psi_>}{\partial r} \\ \frac{\partial r\psi_<}{\partial r} \end{bmatrix}_{r=0}, \tag{2.23a}$$

$$\boldsymbol{D} = \begin{bmatrix} \dfrac{\dfrac{2\pi\hbar^2}{\mu} C_{\lambda>}^2}{E_\lambda - E} & 0 \\ 0 & \dfrac{\dfrac{2\pi\hbar^2}{\mu} C_{\mu<}^2}{E_\mu - E} \end{bmatrix} \tag{2.23b}$$

In the case we have not one, but many resonant states in the $T_<$ channel, the lower term of the diagonal matrix $\boldsymbol{D}$ becomes

$$R(E) = \sum_\mu \frac{\gamma_\mu^2}{E_\mu - E} \tag{2.24}$$

as in (3.16) for the classical problem[3].

The $\boldsymbol{R}$-matrix for the nuclear reaction problem is then

$$\begin{bmatrix} r\psi_p \\ r\psi_n \end{bmatrix}_{r=0} = \boldsymbol{R} \begin{bmatrix} \dfrac{\partial r\psi_p}{\partial r} \\ \dfrac{\partial r\psi_n}{\partial r} \end{bmatrix}_{r=0}, \quad \boldsymbol{R} = \tilde{\boldsymbol{O}}\boldsymbol{D}\boldsymbol{O} \tag{2.25a, b}$$

with $\boldsymbol{O}$ being given by (2.10). Carrying this out explicitly we see that

$$R_{cc'} = \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E} + \frac{\gamma_{\mu c}\gamma_{\mu c'}}{E_\mu - E} \tag{2.26a}$$

$c, c' = p, n$ and we have

$$\gamma_{\lambda p} \equiv \frac{1}{\sqrt{2T_0+1}} \sqrt{\frac{2\pi\hbar^2}{\mu}} C_{\lambda>} \ , \ \gamma_{\mu p} \equiv \sqrt{\frac{2T_0}{2T_0+1}} \sqrt{\frac{2\pi\hbar^2}{\mu}} C_{\mu<}$$

$$\gamma_{\lambda n} \equiv \sqrt{\frac{2T_0}{2T_0+1}} \sqrt{\frac{2\pi\hbar^2}{\mu}} C_{\lambda>} \ , \ \gamma_{\mu n} \equiv \frac{-1}{\sqrt{2T_0+1}} \sqrt{\frac{2\pi\hbar^2}{\mu}} C_{\mu<} \ . \tag{2.26b}$$

For the more general case when there are several compound nuclear states of $T_<$, we use the expression (2.24) of $\mathcal{R}(E)$ in the diagonal matrix (2.23b). In that case (2.26a) has a summation with respect to μ index and

$$\gamma_{\mu p} = \sqrt{\frac{2T_0}{2T_0+1}} \gamma_\mu \ , \quad \gamma_{\mu n} = -\frac{1}{\sqrt{2T_0+1}} \gamma_\mu \ . \tag{2.27}$$

To derive the $\boldsymbol{S}$-matrix from the $\boldsymbol{R}$-matrix, we follow the same procedure as in Ref. 3 and obtain

$$\boldsymbol{S} = \boldsymbol{K}^{\frac{1}{2}} (1 - i\boldsymbol{R}\boldsymbol{K})^{-1} (1 + i\boldsymbol{R}\boldsymbol{K}) \boldsymbol{K}^{-\frac{1}{2}} \tag{2.28a}$$

with $\boldsymbol{R}$ given by (2.26) and $\boldsymbol{K}$ by

$$\boldsymbol{K} = \begin{bmatrix} k_p & 0 \\ 0 & k_n \end{bmatrix} , \tag{2.28b}$$

where k_p , k_n were defined in (2.22).

From the previous expression for the $\boldsymbol{S}$-matrix, the analysis of the effects of external mixing can be carried out exactly as in Ref. 3; the poles of $\boldsymbol{S}$ retain all the properties discussed in Ref. 5 for the classical model.

The previous results have been obtained eliminating the Coulomb potential outside, so that all the sources of external mixing lie in the difference between the masses of the pC and the nA channels. The task of including the Coulomb potential between the proton and the target will be undertaken in the next section.

III. INCLUSION OF THE COULOMB POTENTIAL IN THE SCHEMATIC THEORY OF ISOBARIC ANALOGUE RESONANCES

We shall now indicate how the presence of the Coulomb potential in the proton channel can be included. Again we describe the system when it is in the form of two separated fragments by means of the vector Ψ in Fock's space, as in Eq. (2.1a), except that the Schrödinger equation which Ψ should satisfy, is no longer given by (2.2) but by the following differential equation

$$i\hbar \frac{\partial \Psi}{\partial t} = - \frac{\hbar^2}{2\mu} \nabla^2 \Psi + mc^2 \Psi + V_c(r)\left(\frac{1}{2} - t_z\right) \Psi , \qquad (3.1)$$

where

$$t_z = \begin{pmatrix} -1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

and

$$V_c(r) = \frac{Zze^2}{r}$$

is the Coulomb potential.

Again one describes the system when it is in the form of a compound state by the vector Φ of (2.1b) which, in the absence of a coupling between Φ and Ψ would satisfy (2.3). The analysis about the conservation in time of the scalar product of two vectors in Fock's space proceeds unaltered until we reach (2.13) where the Coulomb potential introduces a logarithmic divergence in the derivatives evaluated at $r = 0$, as we shall see below.

Eq. (3.1) for the proton wave function $(r\psi_p)$ has two linearly independent solutions, called F and G in the literature and $(r\psi_p)$ will be a linear combination of both; for $l = 0$, F and G have the following asymptotic behaviour[8]

$$\text{If } kr \to \infty , \quad \begin{cases} F \to \sin (kr - \eta \ln 2kr + \sigma_0) \\ G \to \cos (kr - \eta \ln 2kr + \sigma_0) \end{cases} \qquad (3.2)$$

$$\text{If } kr \to 0, \begin{cases} F \to Ckr \\ \\ G \to \frac{1}{C}\left\{1 + \frac{2r}{D}\left[\ln\frac{2r}{D} + \frac{1}{2}g(\eta) + \gamma - 1\right]\right\} \end{cases}, \tag{3.3}$$

where

$$D = \frac{\hbar^2}{\mu e^2 Z z}, \qquad \eta = \frac{1}{kD} \tag{3.4a, b}$$

$$\gamma = 0.5772\ldots \quad = \text{Euler's constant} \tag{3.4c}$$

$$C^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1}, \quad g(\eta) = -2\ln\eta + 2\eta^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+\eta^2)} \tag{3.4d, e}$$

$$e^{2i\sigma_0} = \frac{\Gamma(1+i\eta)}{\Gamma(1-i\eta)}. \tag{3.4f}$$

From (3.3) we observe that for $r \to 0$, $G \to C^{-1}$, while $\partial G/\partial r$ will diverge logarithmically.

In order to see how one should reformulate the bilinear expression (2.14), we shall consider, for the sake of simplicity, a one channel case first and then apply the results to our two channel problem. If in the one channel problem we call $\psi(r, t)$ the wave function for the two fragments (r being the relative vector) and $\varphi \equiv r\psi$, then for s-waves φ will satisfy the following equation in the stationary case

$$-\frac{\hbar^2}{2\mu}\frac{d^2\varphi}{dr^2} + \frac{Zze^2}{r}\varphi = E\varphi. \tag{3.5}$$

If instead of a potential of the type $1/r$ we had one of the type $1/(r+d)$ with $d > 0$, all the previous difficulties would not arise; Eq. (3.5) would read

$$-\frac{\hbar^2}{2\mu}\frac{d^2\varphi}{dr^2} + \frac{Zze^2}{r+d}\varphi = E\varphi, \tag{3.6}$$

with two linearly independent solutions $F(r+d)$ and $G(r+d)$. The more general solution can be written as a linear combination of F and G and we choose to write it as

$$\varphi = A(E)\, C\, [G(r+d) + F(r+d) \cot \delta] \,, \tag{3.7}$$

where $A(E)$ only depends on energy, C has been defined in Eq. (3.4) and δ is the phase shift. The R-function of the problem is given by

$$\left[\frac{1}{\varphi}\,\frac{d\varphi}{dr}\right]_{r=0} = \frac{1}{R(E)} \tag{3.8}$$

and it is important to realize that this $R(E)$, *being a property of the compound system only*[7], is exactly the same as the one we would have if in Eq. (3.6) the potential term were absent. For d small enough, φ and its derivative at $r = 0$ are given by

$$\varphi(r=0) \approx A\left\{1 + \frac{2d}{D}\left[\ln\frac{2d}{D} + \frac{1}{2}g(\eta) + \gamma - 1\right] + C^2 kd \cot\delta\right\}$$

$$\left[\frac{d\varphi}{dr}\right]_{r=0} \approx A\left\{\frac{2}{D}\left[\ln\frac{2d}{D} + \frac{1}{2}g(\eta) + \gamma\right] + C^2 k \cot\delta\right\}, \tag{3.9}$$

where the asymptotic relations (3.3) have been used. We can therefore write (3.8) for the R-function as

$$\frac{\frac{2}{D}\left[\ln\frac{2d}{D} + \frac{1}{2}g(\eta) + \gamma\right] + C^2 k \cot\delta}{1 + \frac{2d}{D}\left[\ln\frac{2d}{D} + \frac{1}{2}g(\eta) + \gamma - 1\right] + C^2 kd \cot\delta} = \frac{1}{R(E)} \,. \tag{3.9a}$$

If at this point we take the limit $d \to 0$, we see that $[\partial\varphi/\partial r]_{r=0}$ diverges and therefore (3.8) is not correct in this limit. However the problem can be solved by taking the difference between two expressions like (3.8), for two

different energies, because the term $(2/D)\ln(2d/D)$ (which is the source of the divergence in $\partial\varphi/\partial r$), being energy independent, will cancel out. The following equation is therefore correct, even in the limit $d \to 0$

$$\left[\frac{1}{\varphi_E}\frac{d\varphi_E}{dr} - \frac{1}{\varphi_{E'}}\frac{d\varphi_{E'}}{dr}\right]_{r=0} = \frac{1}{R(E)} - \frac{1}{R(E')} \,. \tag{3.10}$$

Making use of (3.9)

$$\lim_{d\to 0}\left[\frac{1}{\varphi_E}\frac{d\varphi_E}{dr} - \left\{\frac{2}{D}\left[\ln\frac{2d}{D} + \frac{1}{2}g(\eta') + \gamma\right] + C'^2 k' \cot\delta'\right\}\right] = \frac{1}{R(E)} - \frac{1}{R(E')} \,,$$

where the prime on the various quantities means that they have to be evaluated at the energy E'. We can make, for example, the choice $E' = 0$. From the definition[9] (3.4e) for $g(\eta)$ one can see that

$$\lim_{E'\to 0} g(\eta') = \gamma \,. \tag{3.12}$$

We define

$$\lim_{d\to 0}\lim_{E'\to 0} C'^2 k' \cot\delta' \equiv 1/b \,.$$

We now recall that, in the absence of a Coulomb potential, the phase shift δ_0 and the R function are related by

$$k\cot\delta_0 = \frac{1}{R(E)} \tag{3.13}$$

In the vicinity of $E = 0$, one has the usual approximation involving the scattering length a and effective range r_0

$$k\cot\delta_0 \approx -\frac{1}{a} + \frac{1}{2}r_0 k^2 \tag{3.14}$$

so that, from (3.13) we see that

$$\frac{1}{R(0)} = -\frac{1}{a} \quad . \tag{3.15}$$

Introducing this result, together with (3.12) into (3.11) one gets

$$\lim_{d \to 0} \left[\frac{1}{\phi_E} \frac{d\varphi_E}{dr} - \left(\frac{2}{D} \ln \frac{2d}{D} + \frac{3\gamma}{D} + \frac{1}{b} + \frac{1}{a} \right) \right]_{r=0} = \frac{1}{R(E)} \tag{3.16}$$

or, if φ_E now denotes the wave function for the usual Coulomb potential

$$\lim_{r \to 0} \frac{\frac{d\varphi_E}{dr} - \frac{2}{D} \ln \frac{2r}{D} + A \quad \varphi_E}{\varphi_E} = \frac{1}{R(E)} \tag{3.18}$$

$$A \equiv \frac{3\gamma}{D} + \frac{1}{b} + \frac{1}{a} \quad .$$

This is therefore the appropriate generalization of Eq. (3.8), which gives the clue of how to reformulate the bilinear expression arising from the conservation of the scalar product.

Let us now go back to our two channel problem. We note that if in the proton channel we have to deal with the derivative

$$\left[\frac{\partial r\psi_p}{\partial r} - \left(\frac{2}{D} \ln \frac{2r}{D} + A \right) (r\psi_p) \right]_{r=0} , \tag{3.19a}$$

while in the neutron channel we have only

$$\left[\frac{\partial (r\psi_n)}{\partial r} \right]_{r=0} , \tag{3.19b}$$

then when carrying out the transformation (2.10) for the derivatives to put them in terms of $(r\psi_>)$ and $(r\psi_<)$ we obtain

$$\begin{bmatrix} \frac{1}{\sqrt{2T_0+1}} & \sqrt{\frac{2T_0}{2T_0+1}} \\ \sqrt{\frac{2T_0}{2T_0+1}} & -\frac{1}{\sqrt{2T_0+1}} \end{bmatrix} \begin{bmatrix} \left[\frac{\partial r\psi_p}{\partial r} - \left(\frac{2}{D}\ln\frac{2r}{D} + A\right)(r\psi_p)\right]_{r=0} \\ \left[\frac{\partial r\psi_n}{\partial r}\right]_{r=0} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\partial r\psi_>}{\partial r} - \frac{1}{\sqrt{2T_0+1}}\left(\frac{2}{D}\ln\frac{2r}{D} + A\right)(r\psi_p) \\ \frac{\partial r\psi_<}{\partial r} - \sqrt{\frac{2T_0}{2T_0+1}}\left(\frac{2}{D}\ln\frac{2r}{D} + A\right)(r\psi_p) \end{bmatrix}_{r=0}, \qquad (3.20)$$

where we made use of the fact that from (2.10)

$$\frac{1}{\sqrt{2T_0+1}}\frac{\partial r\psi_p}{\partial r} + \sqrt{\frac{2T_0}{2T_0+1}}\frac{\partial r\psi_n}{\partial r} = \frac{\partial r\psi_>}{\partial r} \qquad (3.21a)$$

and similarly for $r\psi_<$.

The wave equations for the problem where we have a Coulomb potential could then be obtained by the same conservation of probability arguments of the previous Section. The only change would be that (3.20) replaces the derivatives

$$\left[\begin{matrix} \frac{\partial r\psi_>}{\partial r} \\ \frac{\partial r\psi_<}{\partial r} \end{matrix}\right]_{r=0} \qquad (3.21b)$$

appearing in (2.19c, d), and a term $-Zze^2/r$ must be added to (2.19a).

The diagonal form (2.23) of the $\boldsymbol{R}$-matrix will now connect the wave functions and derivatives

$$\begin{bmatrix} r\psi_> \\ r\psi_< \end{bmatrix}_{r=0} = \boldsymbol{D} \begin{bmatrix} \dfrac{\partial r\psi}{\partial r}_> - \dfrac{1}{\sqrt{2T_0+1}} \left(\dfrac{2}{D} \ln \dfrac{2r}{D} + A \right) (r\psi_p) \\ \dfrac{\partial r\psi}{\partial r}_< - \sqrt{\dfrac{2T_0}{2T_0+1}} \left(\dfrac{2}{D} \ln \dfrac{2r}{D} + A \right) (r\psi_p) \end{bmatrix}_{r=0} \tag{3.22}$$

and so, applying the transformation O of (2.10), we see that the $\boldsymbol{R} = \tilde{\boldsymbol{O}}\boldsymbol{D}\boldsymbol{O}$ matrix now relates

$$\begin{bmatrix} r\psi_p \\ r\psi_n \end{bmatrix}_{r=0} = \boldsymbol{R} \begin{bmatrix} \dfrac{\partial r\psi_p}{\partial r} - \left(\dfrac{2}{D} \ln \dfrac{2r}{D} + A \right) r\psi_p \\ \dfrac{\partial r\psi_n}{\partial r} \end{bmatrix}_{r=0} . \tag{3.23}$$

Equation (3.23) contains therefore the generalized definition (3.18) of the $\boldsymbol{R}$-matrix for the proton channel and the usual one for the neutron channel.

The evaluation of the $\boldsymbol{S}$-matrix proceeds now exactly as before. Following Lane and Thomas (Ref. 7, page 269) we define for channel c the incoming and outgoing waves $I_c(r)$ and $O_c(r)$ for s-waves

$$I_c = G_c - iF_c \ , \quad O_c = G_c + iF_c \tag{3.24}$$

with asymptotic behaviour, at large distances

$$I_c \sim e^{-i(k_c r - \eta_c \ln 2k_c r + \sigma_{0c})}$$

$$O_c \sim e^{i(k_c r - \eta_c \ln 2k_c r + \sigma_{0c})} , \tag{3.25}$$

where k_c is the wave number for channel c and σ_{0c} has been defined in Eq. (3.4f); note that if c denotes the neutron channel, $\sigma_{0c} = 0$ and $\eta_c = 0$.

The wave function in channel c', when one has incident waves in channel c only, can be written as

$$\psi^{(c)}_{c'}(r) = \frac{I_{c'}(r)}{\sqrt{k_{c'} r}} \delta_{c'c} - \frac{O_{c'}(r)}{\sqrt{k_{c'} r}} \delta_{c'c} \tag{3.26}$$

or, in matrix notation, at $r = 0$

$$\begin{bmatrix} r\psi_p^{(p)} & r\psi_p^{(n)} \\ r\psi_n^{(p)} & r\psi_n^{(n)} \end{bmatrix} = \begin{bmatrix} I_p/\sqrt{k_p} & 0 \\ 0 & I_n/\sqrt{k_n} \end{bmatrix} - \begin{bmatrix} O_p/\sqrt{k_p} & 0 \\ 0 & O_n/\sqrt{k_p} \end{bmatrix} \begin{bmatrix} S_{pp} & S_{pn} \\ S_{np} & S_{nn} \end{bmatrix} \equiv K^{-\frac{1}{2}}(I - OS)$$

where K was defined in Eq. (2.28b) and the matrices I and O are defined as

$$I \equiv \begin{bmatrix} I_p & 0 \\ 0 & I_n \end{bmatrix}_{r=0} , \quad O \equiv \begin{bmatrix} O_p & 0 \\ 0 & O_n \end{bmatrix}_{r=0} \tag{3.28}$$

One can also write the appropriate expression for the derivatives at $r = 0$ in matrix form

$$\begin{bmatrix} \dfrac{\partial r\psi_p^{(p)}}{\partial r} - \left(\dfrac{2}{D}\ln\dfrac{2r}{D} + A\right) r\psi_p^{(p)} & \dfrac{\partial r\psi_p^{(n)}}{\partial r} - \left(\dfrac{2}{D}\ln\dfrac{2r}{D} + A\right) r\psi_p^{(n)} \\ \dfrac{\partial r\psi_n^{(p)}}{\partial r} & \dfrac{\partial r\psi_n^{(n)}}{\partial r} \end{bmatrix}_{r=0} = K^{-\frac{1}{2}}(I' - O'S) , \tag{3.29}$$

$$I' \equiv \begin{bmatrix} \dfrac{dI_p}{dr} - \left(\dfrac{2}{D}\ln\dfrac{2r}{D} + A\right) I_p & 0 \\ 0 & \dfrac{dI_n}{dr} \end{bmatrix}_{r=0} , \quad O' \equiv \begin{bmatrix} \dfrac{dO_p}{dr} - \left(\dfrac{2}{D}\ln\dfrac{2r}{D} + A\right) O_p & 0 \\ 0 & \dfrac{dO_n}{dr} \end{bmatrix}_{r=0} \tag{3.30}$$

The R-matrix relation (3.23) can now be written as

$$K^{-\frac{1}{2}}(I - OS) = RK^{-\frac{1}{2}}(I' - O'S) , \tag{3.31}$$

which can be solved for the S-matrix to give

$$S = O^{-1}K^{\frac{1}{2}}(1 - RL)^{-1}(1 - RL^*)K^{-\frac{1}{2}}I , \tag{3.32}$$

which is clearly the generalization of Eq. (2.28a) when there is a Coulomb potential in the proton channel. The matrix L is a modification of the logarithmic derivative of the outgoing wave function of Ref. 9), i.e.

$$\boldsymbol{L} \equiv \begin{bmatrix} \frac{1}{O_p}\left[\frac{dO_p}{dr} - \left(\frac{2}{D}\ln\frac{2r}{D} + A\right)O_p\right] & 0 \\ 0 & \frac{1}{O_n}\frac{dO_n}{dr} \end{bmatrix}_{r=0} \tag{3.33}$$

which, from the behaviour (3.3) of F and G in the vicinity of $r = 0$, can be written explicitly as

$$\boldsymbol{L} = \begin{bmatrix} \left[\frac{g(\eta) - \gamma}{D} - \frac{1}{b} - \frac{1}{a}\right] + ik_p C^2 & 0 \\ 0 & ik_n \end{bmatrix}. \tag{3.34}$$

In the absence of Coulomb forces, $C \to 1$, $D \to \infty$, $1/b \to -1/a$, so that $\boldsymbol{L} \to i\boldsymbol{K}$, $\boldsymbol{O} \to \boldsymbol{1}$, $\boldsymbol{I} \to \boldsymbol{1}$ and the $\boldsymbol{S}$-matrix of Eq. (3.32) reduces to that of Eq. (2.28a).

CONCLUSIONS

We have shown how the schematic theory of nuclear reactions can be extended to include the isotopic spin formalism, in the specific example of a two-channel problem involving isobaric analogue resonances. The quantum mechanical problem is mathematically identical and it has physically many points in common with a classical problem discussed in an earlier paper[3]. The analysis was first carried out in the absence of a Coulomb potential between the incoming proton and the incident target, the only source of "external mixing" being provided in this case by the barrier, equal to the Q of the (p, n) reaction, felt by the neutron in the field of the analog of the target. The Coulomb potential felt by the proton was then included and it was shown what modifications one has to introduce in order to avoid the problem of the singularity introduced at the origin by the Coulomb field.

The complete time-dependent behaviour of the classical model was studied in an earlier publication[5] and the time dependence of the nuclear problem could be studied using the ideas developed in Ref. 2).

REFERENCES

1. M. Moshinsky, Phys. Rev. 81 (1951) 347.
2. M. Moshinsky, Phys. Rev. 84 (1951) 525.
3. P. A. Mello and M. Moshinsky, (Am. J. Phys., in press).
4. D. Robson, Phys. Rev. 137 (1965) 535.
5. M. Moshinsky and P. A. Mello, Rev. Mex. Fís. 19 (1970) 157.
6. J.D. Anderson and C. Wong, Phys. Rev. Lett. 7 (1961) 250.
7. A.M. Lane and R.G. Thomas, Rev. Mod. Phys. 30 (1958) 257.
8. M. Moshinsky, Rev. Mex. Fís. 2 (1953) 245.
9. Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematical Series 55, 1964, Ed. by M. Abramowitz and I. A. Stegun, p. 259.

RESUMEN

En un trabajo previo, uno de los autores discutió una teoría esquemática de las reacciones nucleares, en la cual se tratan las partículas incidentes y el núcleo compuesto como partículas puntuales. La principal ventaja del modelo consiste en que el análisis de la dependencia temporal del problema se simplifica considerablemente. En el presente trabajo se extiende esta teoría esquemática, para poder incluir el formalismo de isospín, en el ejemplo específico de un problema de dos canales con resonancias análogas isobáricas.

TABLE 1

Relations between the variables in the mechanical and the nuclear reaction problems.

Mechanical Problem	Nuclear Reaction Problem
x	r
$\partial^2/\partial t^2$	$-i\hbar\partial/\partial t$
$u_1(x,t)$	$r\psi_p(r,t)$
$u_2(x,t)$	$r\psi_n(r,t)$
$v_1(x,t)$	$r\psi_>(r,t)$
$v_2(x,t)$	$r\psi_<(r,t)$
$w_1(t)$	$\bar{\phi}_\lambda(t)$
$w_2(t)$	$\bar{\phi}_\mu(t)$
τ/ρ	$\hbar^2/(2\mu)$
λ/ρ	$\lvert Q\rvert$
λ_1/M	$[M_\lambda-(m_p+m_C)]\,c^2\equiv E_\lambda$
λ_2/M	$[M_\mu-(m_p+m_C)]\,c^2\equiv E_\mu$
τ/M	$\frac{2\pi\hbar^2}{\mu}C^2_{\lambda>}$ and $\frac{2\pi\hbar^2}{\mu}C^2_{\mu<}$
ω^2	E
k_1	k_p
k_2	k_n

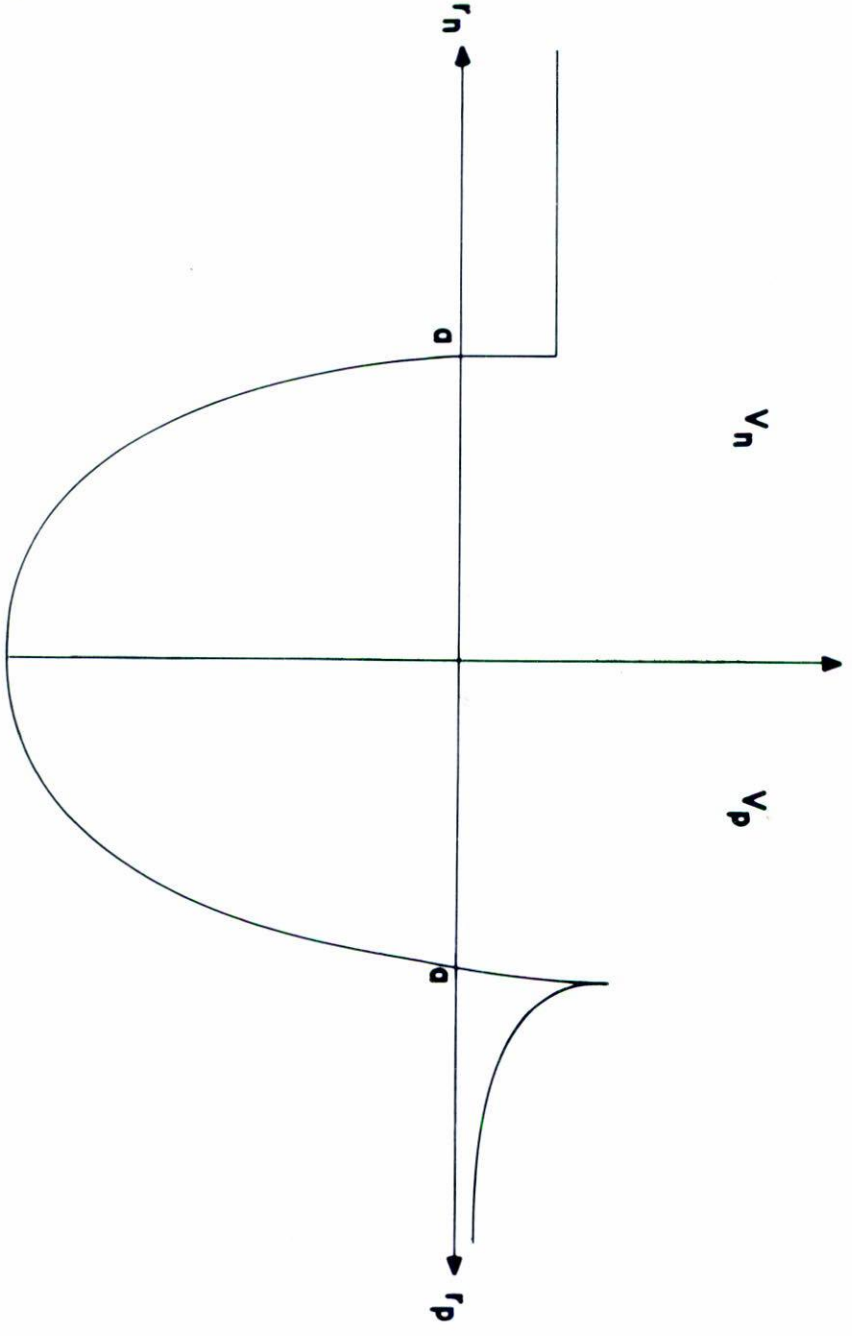

Fig. 1 The potentials of the proton in the (pC) and the neutron in the (nA) channels.

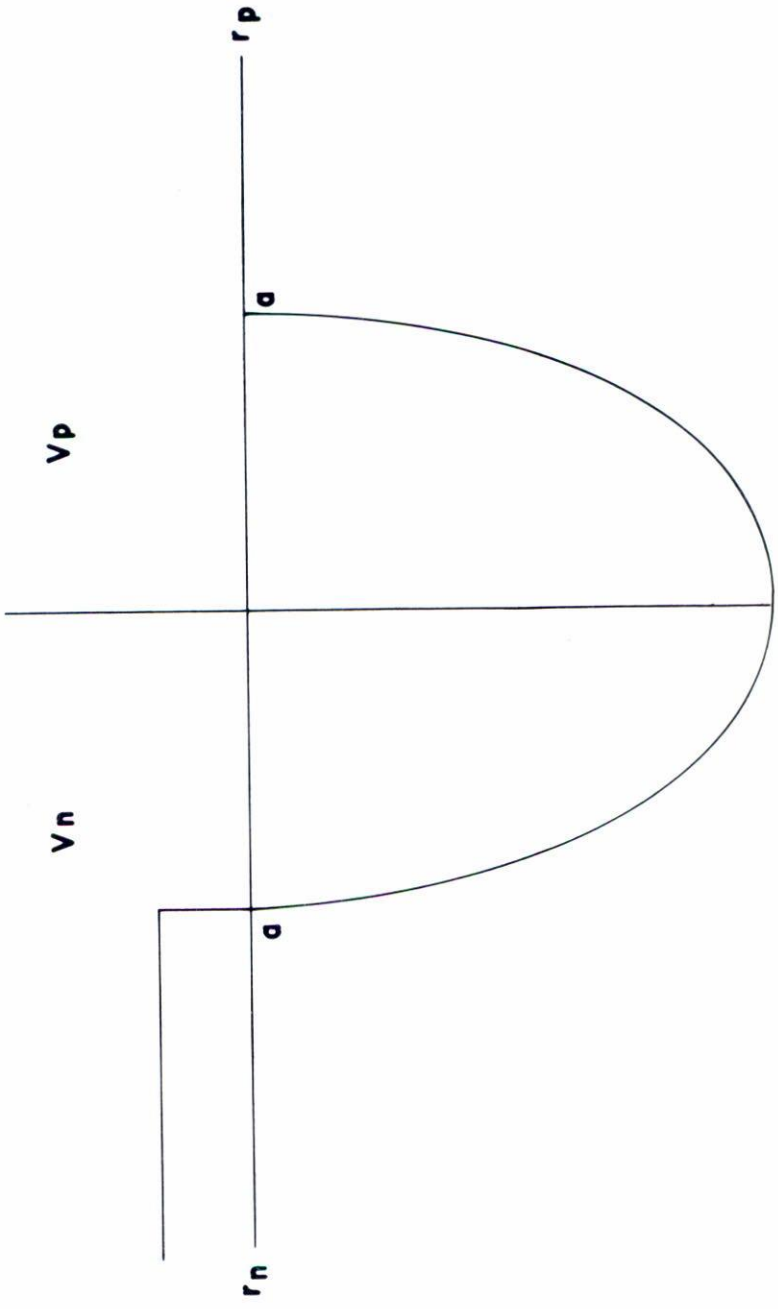

Fig. 2 A simplified version of the potentials of neutron and proton discussed in Fig. 1.