SATURATION IN THE FERMI QUARK MODEL*

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ABSTRACT: The saturation problem in the one triplet Fermi quark model is treated, in the non-relativistic approximation, by a second quantization formalism with two-body and three-body forces, invariant under U(6). An approximate mass formula is derived based on which the saturation properties of the inter-quark forces are discussed, together with the gross features of the baryon spectrum.

I. INTRODUCTION

A most serious objection raised against the Fermi quark model[1], that is, a quark model built from one quark triplet satisfying Fermi statistics, is that the proton form factor, derived from an analytic antisymmetric wave-function with $L = 0$ has zeros, as shown by Mitra and Majundar[2]. Subsequently, Kreps and de Swart[3] have shown that the zeros might be shifted to

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a region where the form factor was not yet measured. But in a recent work, Meyer[4] presented a very simple example of a non-analytic wave function which does not give zeros at all, his form factor being in excellent agreement with experiment. It appears, then, that the Fermi quark model may still have its place as a meaningful model.

In the present work, we reexamine the saturation problem[5] of the inter-quark forces for a system of Fermionic quarks, in the non-relativistic approximation aiming to study in what conditions those forces saturate at some value h_0 of the quark number. This would, of course, mean that the forces are such that any configuration with a higher quark number would be energetically forbidden as a possible bound-state. The case $h_0 = 3$ corresponds to the case of strict saturation and as will be seen later on this may not necessarily exclude the existence of diquarks.

Thus, the saturation problem, in the Fermi quark model is intimately related to the problematical existence of fractionally charged particles[6], corresponding to quark configurations with triality different from zero.

In order to study the saturation properties, use is made in this paper of a convenient treatment of the second quantization formalism[7] to describe an assembly of quarks, interacting via U(6) invariant two-body and three-body forces, both of ordinary and exchange types. This is the analogue of the Wigner first approximation in the U(4) supermultiplet theory of nuclear structure[8], although in the nuclear case, three-body forces play only a minor role.

In quark models, three-body forces, together with two-body forces, have been introduced by Kuo and Radicati[9], some time ago, to bind three quarks into baryons. However, they have not discussed the saturation. Later, the same type of forces were discussed by Schiff[10] in a schematic treatment of the saturation.

In Section II, we develop the second quantization formalism and derive an approximate mass formula, in the "infinite range approximation", familiar in nuclear structure theory[11]. Based on this formula, we discuss, in Section III, the gross features of the baryon spectrum, together with the saturation properties. The main conclusions are discussed in Section IV.

II. THE SECOND-QUANTIZATION TREATMENT AND AN APPROXIMATE MASS FORMULA.

We consider a system of h Fermionic quarks, supposed to be very massive objects of mass M. In a non-relativistic treatment, we disregard their kinetic energy with respect to the total rest mass. The Hamiltonian of the quarks, interacting via two-body and three-body forces can then be written in this approximation, as

$$H = bM - \frac{1}{2!} \sum_{\substack{\rho_1 \rho_2 \\ \rho_1' \rho_2'}} < \rho_1 \rho_2 | V_{12} | \rho_1' \rho_2' > b^+_{\rho_2} b^+_{\rho_1} b^{\rho_1'} b^{\rho_2'}$$

$$- \frac{1}{3!} \sum_{\substack{\rho_1 \rho_2 \rho_3 \\ \rho_1' \rho_2' \rho_3'}} < \rho_1 \rho_2 \rho_3 | V_{123} | \rho_1' \rho_2' \rho_3' > b^+_{\rho_3} b^+_{\rho_2} b^+_{\rho_1} b^{\rho_1'} b^{\rho_2'} b^{\rho_3'} . \tag{1}$$

In Eq. (1), b^+_ρ and b^ρ are Fermi creation and annihilation operators satisfying

$$[b^+_g, b^{g'}]_+ = \delta^{\rho'}_\rho \quad ,$$

$$[b^+_\rho, b^+_{\rho'}]_+ = [b^\rho, b^{\rho'}]_+ = 0 . \tag{2}$$

The indices ρ are here composite indices, $\rho \equiv (\mu, r)$, where μ describes the spin-F spin of the quarks and r, their independent particle motion, $r \equiv (\nu \mathrm{lm})$. By an enumeration convention, the index μ runs from 1 to 6.

We have, for instance, in an obvious notation:

$$1 \rightarrow p_0 \uparrow \;, \quad 2 \rightarrow n_0 \uparrow \;, \quad 3 \rightarrow \lambda_0 \uparrow \;,$$

$$4 \rightarrow p_0 \downarrow \;, \quad 5 \rightarrow n_0 \downarrow \;, \quad 6 \rightarrow \lambda_0 \uparrow \;. \tag{3}$$

Similar convention may be adopted for enumerating the available single-particle states, whose number is supposed to be finite.

The two-body and three-body potentials in Eq. (1) are symmetrical functions in their arguments and the corresponding matrix elements, with respect to the spin-F spin indices μ are taken between states of the fundamental representations of the U(6) group. Further, we assume that the forces have an ordinary part (W) and an exchange part (M), w.r.t. the U(6) group:

$$V_{12} = \sum_{t = W,\, M} V^t(12)\, P_{12}^t \,,$$

$$V_{123} = \sum_{t = W,\, M} V^t(123)\, P_{123}^t \,. \tag{4}$$

In the above equations P_{12}^W and P_{123}^W are unit operators in the spin-F spin space. It is convenient to express P_{12}^M in the form[8]

$$P_{12}^M = - E_\beta^\alpha(1)\, E_\alpha^\beta(2) \,, \tag{5}$$

in terms of the matrices $E_\alpha^\beta (\alpha, \beta = 1, 2, \ldots, 6)$ which are the well known, six by six matrices, forming the basis for the fundamental irreducible representation of the U(6) group. By definition, one has:

$$\left(E_\alpha^\beta\right)_\mu^\nu = \delta_{\alpha\mu}\delta^{\beta\nu} \,, \tag{6}$$

from which easily follows that

$$E_\alpha^\beta E_\gamma^\delta = \delta_\alpha^\delta\, E_\gamma^\beta \,. \tag{7}$$

Here, Greek indices run from 1 to 6.

The negative sign in Eq. (5) is a well known consequence of the Exclusion Principle. The physical content of that equation is more clearly seen by writing E_α^β as a direct product of σ_a^b $(a, b = 1, 2)$ and g_A^B $(A, B = 1,2,3)$ which, in turn, are bases for the fundamental representation of the U(2) and U(3) subgroups of U(6), respectively:

$$E_\alpha^\beta(i) = \sigma_a^b(i) \oplus g_A^B(i) \,, \tag{8}$$

the index i $(i = 1,2,3)$ being the particle index. Upon using the well known property

$$[a \oplus b] \cdot [c \oplus d] = [a \cdot c] \oplus [b \cdot d]$$

one immediately has

$$P^M_{12} = -\frac{1}{2}\,[1 + \vec{\sigma}(1)\cdot\vec{\sigma}(2)] \oplus \frac{1}{2}\,[\frac{2}{3} + \sum_{p=1}^{8} \vec{\lambda}_p(1)\cdot\vec{\lambda}_p(2)] \ , \tag{9}$$

where the first factor representsthe spin exchange force and the second, its F-spin analogue ($F_p = \lambda_p/2$, the λ_p being the Gell-Mann matrices).

Since the matrices $E^{\beta}_{\alpha}(i)$ commute for different values of the particle label (i), one may easily derive, from Eqs. (6) and (7), the well known properties

$$\left(P^M_{12}\right)^2 = 1\ , \qquad P^M_{12}\, E^{\beta}_{\alpha}(1)\, P^M_{12} = E^{\beta}_{\alpha}(2)\ . \tag{10}$$

By analogy we assume, without essential loss of generality, for the corresponding three-body exchange force the expression[9]

$$P^M_{123} = +\ E^{\alpha}_{\beta}(1)\, E^{\beta}_{\gamma}(2)\, E^{\gamma}_{\alpha}(3)\ . \tag{11}$$

The positive sign in the above equation, as printed out by Kuo and Radicati[9], is a consequence of the property

$$P^M_{123} = P^M_{12} \cdot P^M_{13}\ , \tag{12}$$

as can be seen from Eqs. (5),(6) and (7).

The solution of Eq. (1) presents, of course, considerable difficulties. Besides, the spatial dependence of $V^t(1,2)$ and $V^t(1,2,3)$ is actually not known. However, it is possible to extract some physics from Eq. (1) taking as a very first approximation, the potentials $V^t(1,2)$ and $V^t(1,2,3)$ at the limit of a square well of infinite range, the so called infinite range square well limit[11], well known in nuclear structure theory. Although the inter-quark forces are, of course, short-range forces, the approximation appears to be physically relevant, providing some semi-quantitative information on the dynamics of the problem.

In this approximation, the Hamiltonian in Eq. 1 is exactly diagonalizable, for each irreducible representation(IR) $[h_1, h_2, \ldots, h_6]$ of U(6).

In fact, in the limit considered, one has

$$< \rho_1 \; \rho_2 \mid V^t(12) \; P^t_{12} \mid \rho_1' \, \rho_2' > \approx$$

$$V^{\mathrm{II}}_t < \mu_1 \, \mu_2 \, \mu_3 \mid P^t_{123} \mid \mu_1' \, \mu_2' \, \mu_3' > \delta_{r_1 r_1'} \, \delta_{r_2 r_2'} , \tag{13}$$

and similarly,

$$< \rho_1 \, \rho_2 \, \rho_3 \mid V^t(123) \; P^t_{123} \mid \rho_1' \, \rho_2' \, \rho_3' > \approx$$

$$V^{\mathrm{III}}_t < \mu_1 \, \mu_2 \, \mu_3 \mid P^t_{123} \mid \mu_1' \, \mu_2' \, \mu_3' > \delta_{r_1 r_1'} \, \delta_{r_2 r_2'} \, \delta_{r_3 r_3'} \quad , \tag{14}$$

where V^{II}_t and V^{III}_t are constant. The deltas in the orbital labels r, in the above equations, allow Eq. (1) to be written in the form

$$M_{[h]} \approx hM - \frac{1}{2!} \sum_{t = M, M} V^{\mathrm{II}}_t < \mu_1 \; \mu_2 \mid P^t_{12} \mid \mu_1' \; \mu_2' > \left[C^{\mu_1'}_{\mu_1} C^{\mu_2'}_{\mu_2} - \delta^{\mu_1'}_{\mu_2} C^{\mu_2'}_{\mu_1} \right]$$

$$- \frac{1}{3!} \sum_{t = W, M} V^{\mathrm{III}}_t < \mu_1 \mu_2 \mu_3 \mid P^t_{123} \mid \mu_1' \, \mu_2' \, \mu_3' > \Big[C^{\mu_1'}_{\mu_1} C^{\mu_2'}_{\mu_2} C^{\mu_3'}_{\mu_3}$$

$$- (\delta^{\mu_1'}_{\mu_2} C^{\mu_2'}_{\mu_1} C^{\mu_3'}_{\mu_3} + \delta^{\mu_1'}_{\mu_3} C^{\mu_3'}_{\mu_1} C^{\mu_2'}_{\mu_2}) + (\delta^{\mu_1'}_{\mu_2} \delta^{\mu_2'}_{\mu_3} C^{\mu_3'}_{\mu_1} + \delta^{\mu_1'}_{\mu_3} \delta^{\mu_3'}_{\mu_2} C^{\mu_2'}_{\mu_1}) \Big]$$

(15)

In Eq. (15), the operators $C_{\mu}^{\mu'}$ are defined as

$$C_{\mu}^{\mu'} = \sum_{r} b_{\mu r}^{+} b^{\mu' r} \quad , \tag{16}$$

and they obey the following commutation relations, as one can easily derive from Eq. (2):

$$[C_{\mu}^{\mu'}, C_{\mu''}^{\mu'''}] = C_{\mu}^{\mu'''} \delta_{\mu''}^{\mu'} - C_{\mu''}^{\mu'} \delta_{\mu}^{\mu'''} \quad . \tag{17}$$

Thus, one sees that the $C_{\mu}^{\mu'}$ are U(6) (or SU(6)) generators. The use of U(6), instead of the usual SU(6), implies that the invariant operator

$$\sum_{\mu=1}^{6} C_{\mu}^{\mu}$$

gives the quark number operator, whose eigenvalues, for an IR characterized by $[h_1, h_2, \ldots h_6]$ are given by

$$\sum_{\mu=1}^{6} h\mu \quad .$$

Further, we note that, by using U(6), as done here, the F-spin algebra may be taken as $U_F(3)$ or $SU_F(3)$, both of which are subalgebras of U(6). Thus, either quarks with fractional charges or quarks with integral charges[12] are both possible cases.

It remains to evaluate the matrix elements of P_{12}^{t} and P_{123}^{t} in Eq. (15). Having in mind the enumeration convention, Eq. (3), and Eq. (6) one has

$$\langle \mu_1 | E_{\alpha}^{\beta} | \mu_1' \rangle \equiv (E_{\alpha}^{\beta})_{\mu_1}^{\mu_1'} = \delta_{\alpha\mu_1} \delta^{\beta\mu_1'} \quad .$$

Therefore

$$< \mu_1 \mu_2 | P_{12}^{W} | \mu_1' \mu_2' > = \delta_{\mu_1}^{\mu_1'} \delta_{\mu_2}^{\mu_2'} ,$$

$$< \mu_1 \mu_2 \mu_3 | P_{123}^{W} | \mu_1' \mu_2' \mu_3' > = \delta_{\mu_1}^{\mu_1'} \delta_{\mu_2}^{\mu_2'} \delta_{\mu_3}^{\mu_3'} ,$$

$$< \mu_1 \mu_2 | P_{12}^{M} | \mu_1' \mu_2' > = - \delta_{\mu_1}^{\mu_2'} \delta_{\mu_2}^{\mu_1'} ,$$

$$< \mu_1 \mu_2 \mu_3 | P_{123}^{M} | \mu_1' \mu_2' \mu_3' > = \delta_{\mu_1}^{\mu_3'} \delta_{\mu_2}^{\mu_1'} \delta_{\mu_3}^{\mu_2'} . \tag{18}$$

Upon substitution in Eq. (15) one finally gets

$$M_{[h]} \approx hM - V_0 \left[\binom{h}{2} A_W^{II} + \binom{h}{3} A_W^{III} - \right.$$

$$\left. - \frac{1}{2} (C_2 - 6h) A_M^{II} + \frac{1}{6} \{ C_3 - 6C_2 + h(37 - 2h) \} A_M^{III} \right] , \tag{19}$$

where we put $V_t^{II} = V_0 A_t^{II}$ and $V_t^{III} = V_0 A_t^{III}$, with $V_0 > 0$. In Eq. (19), C_3 and C_2 are the third and second order Casimir invariants of U(6)

$$C_2 = \sum_{\alpha , \beta} C_{\alpha}^{\beta} C_{\beta}^{\alpha} ,$$

$$C_3 = \sum_{\alpha , \beta , \gamma} C_{\beta}^{\alpha} C_{\gamma}^{\beta} C_{\delta}^{\gamma} , \tag{20}$$

whose eigenvalues, for a given IR of U(6) characterized by the maximum weights vector $[h_1 , h_2 , \ldots , h_6]$, are respectively given by

$$g_2 = 7h + \sum_{\alpha=1}^{6} h_\alpha (h_\alpha - 2\alpha) ,$$

$$g_3 = \sum_{\alpha=1}^{6} h_\alpha^3 + \sum_{\alpha=1}^{6} (8-3\alpha) h_\alpha^2 - \sum_{\alpha=1}^{6} h_\alpha \sum_{\beta=\alpha}^{6} h_\beta$$

$$+ \sum_{\alpha=1}^{6} h_\alpha (3\alpha^2 - 15\alpha) + h(7+h) , \qquad (21)$$

where $h = \sum_{\alpha=1}^{6} h_\alpha$.

III . THE GROSS FEATURES OF THE BARYON SPECTRUM AND THE SATURATION PROPERTIES.

We begin this section by specializing Eq. (19), for the baryon case ($h = 3$). Upon using Eq. (21), one easily gets for the masses corresponding to the representations [300000] , [210000] and [111000] , of dimensions 56, 70 and 20 ,

$$M_{56} = M_0 - V_0 \left[-3A_M^{II} + A_M^{III} \right] ,$$

$$M_{70} = M_0 + V_0 \frac{1}{2} A_M^{III} ,$$

$$M_{20} = M_0 - V_0 \left[3A_M^{II} + A_M^{III} \right] , \qquad (22)$$

where we separated the common contribution, due to the ordinary forces, in the term

$$M_0 = 3M - V_0 \left[3A_W^{II} + A_W^{III} \right] . \qquad (23)$$

On empirical grounds, one knows that[13]

$$M_{56} < M_{70} \quad .$$

with no evidence for the 20 up to about 2.5 GeV. Therefore, either (I) the 20 appears at a higher energy or (II) its absence is due to some peculiar dynamical feature of the forces.

In case I, by imposing the condition

$$M_{56} < M_{70} < M_{20} \quad , \tag{24}$$

in Eq. (20), one obtains

$$- A_M^{II} \pm \frac{1}{2} A_M^{III} > 0 \quad , \tag{25}$$

which implies $A_M^{II} < 0$. Thus the two-body exchange forces are attractive in the 56 and repulsive in the 20, as can be seen from Eq. (22). If one further imposes the positiveness of the masses in Eq. (22), one sees that the possible solutions for the parameters $A_M^{II} = x$ and $A_M^{III} = y$, in a (x, y) plane, correspond to the points in the interior of a triangle, denoted by I, in Fig. 1, the vertices of which are $(0, 0)$, $(-\xi/5, 2\xi/5)$ and $(-\xi, -2\xi)$, where $\xi = M_0/V_0$. Note that in this case there exist solutions with A_M^{III} either positive or negative. Taking $(x, y) \equiv (-\xi/5, 2\xi/5)$, for instance, one gets

$$\frac{M_{20}}{M_{56}} = 3 \quad \text{and} \quad \frac{M_{70}}{M_{56}} = 1.5 \quad . \tag{26}$$

If $A_M^{III} \leqslant 0$, one has

$$M_{20} - M_{70} \geqslant M_{70} - M_{56} \quad , \tag{27}$$

that is, the mean masses will be equally spaced only if $A_M^{III} = 0$.

In case II, we have $M_{70} - M_{56} > 0$, $M_{70} > 0$ but $M_{20} < 0$. The possible solutions correspond to the internal points of the region II, Fig. 1. One sees that $A_M^{II} > 0$ and $A_M^{III} > 0$ in this case.

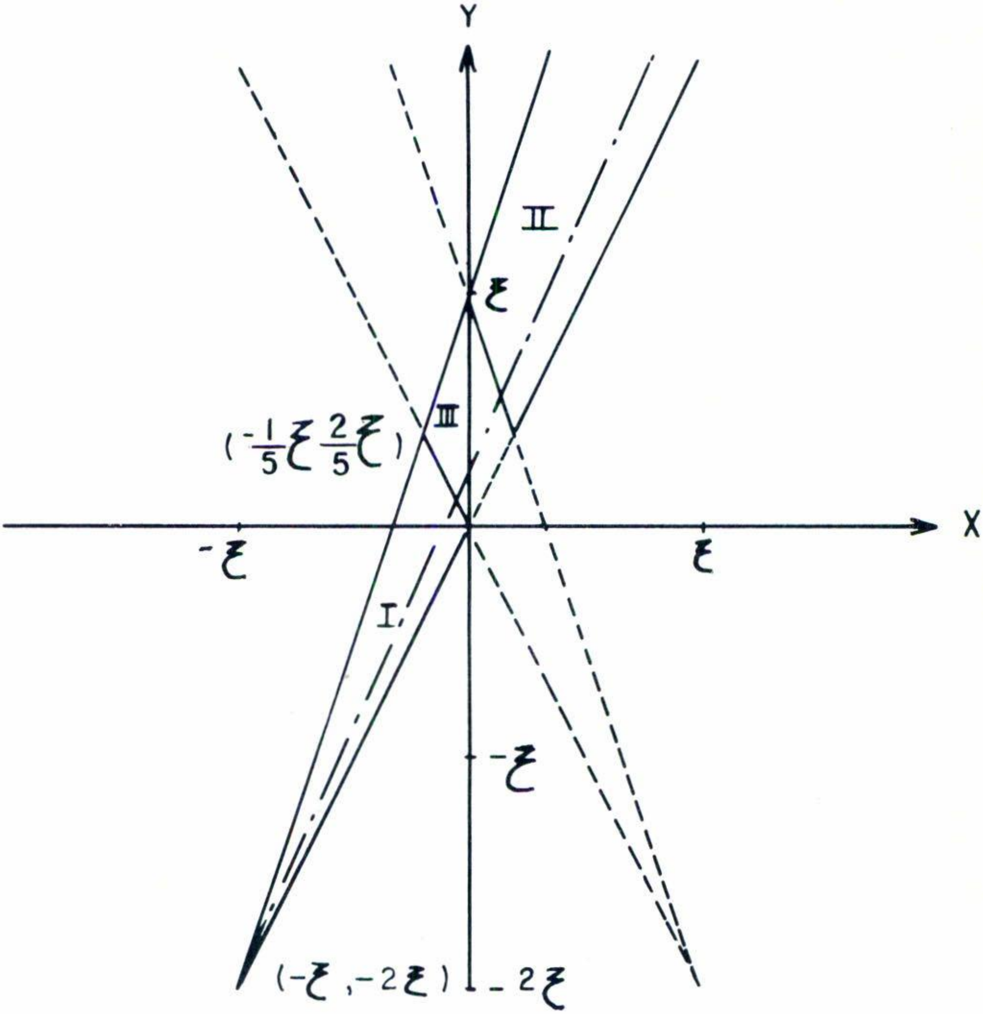

Fig. 1. In solid lines are indicated the regions in the plane $(x, y) \equiv (A_M^{II}, A_M^{III})$, corresponding to cases I,II, and IIIas discussed in the text. The straight line crossing those regions correspondsto the ratio $\frac{M_{70}}{M_{56}} = 1.5$

Finally, in Fig. 1 is represented, in region III, the solution considered by Kuo and Radicati[9]. In this region, one has $M_{56} < M_{20} < M_{70}$, which, however, is not empirically favored. And is also shown, in Fig. 1, the straight line corresponding to the ratio $M_{70}/M_{56} = 1.5$.

Some consequence of the above discussion for the cases $h = 2$ (diquarks) and $h = 4$ are now briefly discussed. Here, diquarks appears in U(6) supermiltiplets of dimensions **21** and **15** and one easily gets the result that $M_{15} > M_{21}$ for cases I and III and $M_{21} > M_{15}$ for case II.

For $b = 4$, the U(6) supermultiplets have dimensions **126**, **210**, **105**, **35** and **15**. From a more detailed analysis of this case, one arrives at the result that there is no region in the (x, y) plane for which all those masses are positive. Thus, at least one of the supermultiplets is to be excluded as non-physical.

If the pattern of the baryon spectrum repeats itself in this case, the **15** supermultiplet is the probable candidate and besides, the **126** would appear in the lowest mass, this last condition being found possible for points in the (x, y) plane, with $x < 0$, limited by the lines $x = 0$ and $x - y = 0$.

At this point, it appears clear the distinct role played by the two types of force, ordinary and exchange.

For an estimate of the relative magnitude of those forces consider, for instance, the case

$$\frac{A_M^{III}}{A_M^{II}} = \frac{y}{x} = -1 ,$$

with $A_M^{II} < 0$. Then

$$M_{70} - M_{56} = \frac{9}{2} V_0 \mid x \mid .$$

On the other hand, for $A_W^{III}/A_W^{II} = -1$ with $A_W^{II} > 0$, and a quark mass $M \gtrsim 10$ GeV, one gets

$$\left| \frac{A_W^{II}}{A_M^{II}} \right| \approx 10^2 . \qquad (28)$$

One sees that the ordinary forces are about two order of magnitude stronger than the exchange forces. The role of the ordinary forces, in this treatment, is to provide the large reduction of mass from $3M$ to M_0. Instead, the exchange forces play their main role in the form of the low-lying baryon spectrum and in the mass difference of the spin-F spin supermultiplets.

Therefore, one is justified in neglecting altogether the exchange forces, in confront with the ordinary ones, in a first discussion of the saturation[10]. In this case, Eq. (19) reads

$$M_{[b]} \simeq bM - V_0 \left[\frac{1}{2} b(b-1) A_W^{II} + \frac{1}{6} b(b-1)(b-2) A_W^{III} \right] . \quad (29)$$

The first and second terms in this equation* are proportional to the number of two-body and three-body "bonds", respectively. Since the number of three-body bonds increases more rapidly than the number of two-body "bonds" as b increases, it is clear that, in order to achieve saturation at some value $b = b_0$, one has to assume three-body repulsive forces and attractive two-body forces, as proposed at first time by Schiff[10]. As a consequence, the saturative solution necessarily allows the existence of diquarks. Following de Swart[14], Eq. (29) for a possible state may be written as

$$\frac{M_{[b]}}{bM} \cong (1-\epsilon) b^2 + \left(2\epsilon - \frac{5}{2}\right) b + 3\left(1 - \frac{1}{2}\right) < 1 \quad (30)$$

where we wrote $V_0 A_W^{II} = 3M\left(1 - \frac{1}{2}\epsilon\right)$ and $V_0 A_W^{III} = 3M(1-\epsilon)$, in terms of the parameter $\epsilon < 1$.

As pointed out by de Swart, there are several possibilities for configurations to exist with triality different from zero and masses substantially lower than the quark mass M.

In Fig. 2, it is shown $M_{[b]}/(bM)$ against b and one has

$$M_{2q} < M_{4q} < M_q \ ,$$

the diquark having the lowest mass and then it would be energetically possible a super-strong decay of the M_q and M_{4q} into diquarks by processes such as

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* We note that, for the IR of the type $[b, 0, \ldots, 0]$, the contribution of the exchange and ordinary forces in Eq. (19) is of the same form, so that Eq. (29), in this case, is valid with coefficients $A_W^{II} - A_M^{II}$, $A_W^{III} + A_M^{III}$.

$$q \rightarrow (\overline{q}\,\overline{q}) + (qqq) \;,$$

$$qqqq \rightarrow (\overline{q}\,\overline{q}) + 2\,(qqq) \;,$$

which preserve charge and baryonic number, as proposed in the original paper by Gell-Mann[1, 14].

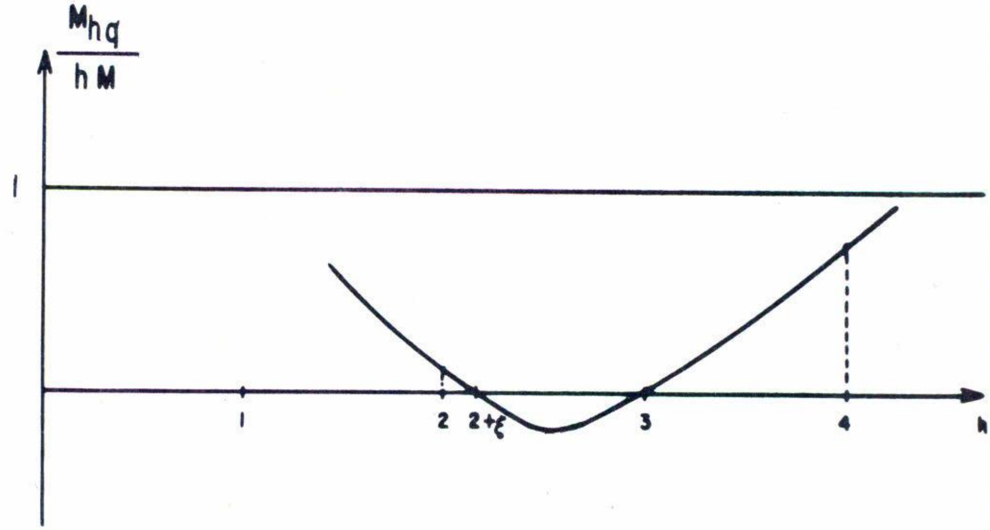

Fig. 2. $\frac{M_{hq}}{hM}$ as a function of the quark number h.
The parameter[14] ϵ is such that $M_{2q} = \epsilon M$ and $M_{3q} \simeq$

CONCLUSIONS

The theoretical importance of the quark models, in view of their remarkable success in coordinating a large amount of data on particles and resonances, dwells on the indication of dynamical ideas which may be relevant for a future theory. Among the dynamical problems suggested by the quark models one finds that of the saturation. Since the quark models present a still unresolved problem of statistics, the solution of the saturation problem depends on the specific model considered[5, 15]. In this paper, we discussed saturation in the one triplet model of Fermionic quarks, together with the gross features of the baryonic spectrum. Although the diagonalization of Eq. (1) would be desirable for a definite class of potentials, the complexity of the analysis indicates the necessity of a preliminary treatment allowing a semiquantitative discussion of the problem, specially the role of the different forces involved and their relative importance. In such an approximate treatment, we arrived at mass formula Eq. (19), based on which the main conclusions of this work are derived. The mass formula contains the contributions of ordinary and Majorana exchange two-body and three-body forces. The ordinary forces are almost two orders of magnitude stronger than the exchange forces. They are dominant in an approximate discussion of the saturation, which is achieved by a mixture of 3-body repulsive and 2-body attractive forces, as first discussed by Schiff[10]. On the other hand, the Majorana exchange forces are mainly responsible for the mass differences of the U(6) supermultiplets. As far as the baryon spectrum is concerned the mixture of two and three body Majorana exchange forces allows solutions corresponding to the ordering $M_{56} < M_{70} < M_{20}$. However, it is equally possible to have solutions which excludes the existence of the 20 supermultiplet, with $M_{56} < M_{70}$, as seems to be indicated by the empirical evidence.

The diagonalization of the Hamiltonian Eq. (1), beyond the present approximation, aiming to a proper treatment of the orbital excitations would be the natural step towards a closer solution of the problem. We hope to return to this interesting topic at a later date.

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RESUMEN

Se trata el problema de la saturación en el modelo de un triplete de quarks fermiónicos, dentro de una aproximación no relativista, mediante un formalismo de segunda cuantización con fuerzas de dos y tres cuerpos, invariantes frente a U(6). Se obtiene una fórmula de masas aproximada, en términos de la cual se discuten las propiedades de saturación de las fuerzas entre los quarks, así como las características gruesas del espectro bariónico.