THE CHARGE FORM FACTOR OF THE MASS 3 NUCLEI†

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ABSTRACT: In this note we derive the theoretical expression for the charge form factor of the mass 3 nuclei, using an expansion of the ground state wave function in terms of translationally invariant harmonic oscillator states.

1. INTRODUCTION

The problems involved in the use of basis of translationally invariant harmonic oscillator states for the calculation of the properties of the three and four nucleon systems have been studied extensively, taking ad-

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vantage as much as possible of the group theoretical classification of the states[1].

Recently Jackson[2] has been able to carry out a calculation of various properties of the tritium nucleus with the Reid potential[3] up to a very large number of oscillator quanta. The agreement he gets for the binding energy is good in comparison with other calculations. It seems therefore interesting to find other tests of the goodness of his ground state wave function. One of these is the charge form factor, whose theoretical expression we derive below for the general case of the mass 3 nuclei, using results of ref. 1.

2. THE GROUND STATE OF THE MASS 3 NUCLEI

The ground state of the mass 3 nuclei is characterized by a definite parity π, an angular momentum J of projection M, an isospin $T = 1/2$ (neglecting the Coulomb force in the case of ^{3}He), and an isospin component $M_T = 1/2$ for ^{3}H and $- 1/2$ for ^{3}He. As was shown in ref. 1 it can be expanded in terms of products of orbital and spin-isospin states of definite permutational symmetry combined so as to satisfy the Pauli principle:

$$|\pi JMT = \tfrac{1}{2}M_T >$$

$$= \sum_{\alpha L f S} a(\alpha L f S) \sum_r \frac{1}{\sqrt{d_f}} (-1)^r \left[|\alpha \pi L f r > | ST = \tfrac{1}{2} M_T \tilde{f} \tilde{r} > \right]_{JM} \qquad (2.1)$$

Here L is the total orbital angular momentum, S the total spin, and the square bracket stands for vector coupling of these two angular momenta to the total value J and projection M. The symmetry of the orbital states under $S(3)$ is specified by the partition f and the Yamanouchi symbol r, and that of the spin isospin states by $\tilde{f}$ and $\tilde{r}$, associated with f and r. The symbol d_f refers to the dimension of the representation f of $S(3)$ ($d_f = 1$ for $f = \{3\}$ or $\{111\}$, $d_f = 2$ for $f = \{21\}$), and the phase $(-1)^r$ is defined in such a way that $(-1)^r = +1, +1, +1, -1$ for $r = (111), (321), (211), (121)$, respectively. The quantum number α serves to complete the classification of the orbital states. The coefficients of the expansion $a(\alpha L f S)$ are determined by the diagonalization of the hamiltonian matrix.

The three particle spin-isospin states of definite permutational symmetry are easily constructed from the one and two particle states using the well-known one particle spin-isospin fractional parentage coefficients.[4] The easiest way of constructing the three-particle orbital states of definite permutational symmetry[1] is by writing them in terms of the states

$$|n_1 l_1 n_2 l_2 LM_L > = [P_{n_1 l_1}(\eta^1) P_{n_2 l_2}(\eta^2)]_{LM_L} |0> , \qquad (2.2)$$

where $P_{nl}(\eta)$ is the polynomial in the creation operator η which creates a one particle harmonic oscillator state characterized by n, l, $|0>$ is the vacuum state, and the creation operators η^1, η^2 are defined by[1]

$$\begin{bmatrix} \eta^1 \\ \\ \eta^2 \end{bmatrix} = \begin{bmatrix} -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \dot{\eta}^1 \\ \\ \dot{\eta}^2 \end{bmatrix} \qquad (2.3)$$

in terms of the creation operators $\dot{\eta}^1$, $\dot{\eta}^2$, associated with the Jacobi relative coordinates

$$\dot{\mathbf{x}}^1 = \frac{1}{\sqrt{2}}(\mathbf{x}^1 - \mathbf{x}^2) ,$$

$$\dot{\mathbf{x}}^2 = \frac{1}{\sqrt{6}}(\mathbf{x}^1 + \mathbf{x}^2 - 2\mathbf{x}^3) . \qquad (2.4)$$

The transformation brackets from the states (2.2) to the states $|\alpha\pi LM_L fr>$ are well known[1] and show that we can replace α by a single set of quantum numbers $n_1 l_1 n_2 l_2$, the summation in (2.1) being restricted to those values of the latter such that $(-1)^{2n_1 + l_1 + 2n_2 + l_2}$ is equal to the parity π . The possibility of forming states corresponding to a definite representation f of $S(3)$ is is connected with the value taken by the number μ defined by the congruence relation

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The symbols η and ξ are vectors as no bold face type for them was available.

$$2n_1 + l_1 - 2n_2 - l_2 \equiv \mu \,(\text{mod } 3)\,, \tag{2.5}$$

as is shown in Table 1 .

TABLE 1

The coefficient $A(\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2\,,\, n_1 l_1 n_2 l_2 f r)$ of formula (2.8)

	μ	f r	L	$A(\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2\,,\, n_1 l_1 n_2 l_2 f r)$
$(n_1 l_1) \neq (n_2 l_2)$	0	$\{3\}\,(111)$		$\frac{1}{\sqrt{2}}(-i)^{2\dot{n}_1+\dot{l}_1}[1+(-1)^{\dot{l}_1}]$
	0	$\{111\}\,(321)$		$\frac{1}{\sqrt{2}}(-i)^{2\dot{n}_1+\dot{l}_1}[1-(-1)^{\dot{l}_1}]$
	1, 2	$\{21\}\,(211)$		$\frac{1}{\sqrt{2}}(-i)^{2\dot{n}_1+\dot{l}_1}[1+(-1)^{\dot{l}_1}]$
	1, 2	$\{21\}\,(121)$		$\frac{1}{\sqrt{2}}(-i)^{2\dot{n}_1+\dot{l}_1}[1-(-1)^{\dot{l}_1}]$
$(n_1 l_1) = (n_2 l_2)$	0	$\{3\}\,(111)$	even	$(-i)^{2\dot{n}_1+\dot{l}_1}$
	0	$\{111\}\,(321)$	odd	$(-i)^{2\dot{n}_1+\dot{l}_1}$

In the computation of the form factor we shall have to deal with the expectation value of an operator with respect to the state (2.1). The calculus is greatly simplified if the orbital states $|n_1 l_1 n_2 l_2 L M_L fr>$ are expressed in terms of the Jacobi coordinates (2.4). To accomplish this it is necessary to pass from the states (2.2) to the states

$$|\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 L M_L > = [P_{\dot{n}_1 \dot{l}_1}(\dot{\eta}_1)\, P_{\dot{n}_2 \dot{l}_2}(\dot{\eta}_2)]_{LM_L} |0> \;, \qquad (2.6)$$

so that we finally get

$$|n_1 l_1 n_2 l_2 L M_L fr> =$$

$$= \sum_{\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2} |\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 L M_L >< \dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 L | n_1 l_1 n_2 l_2 L fr> \;. \qquad (2.7)$$

It has been shown in refs. 1, 5 that the transformation brackets in (2.7) are given by

$$< \dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 L | n_1 l_1 n_2 l_2 L fr>$$

$$= A(\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 \,,\, n_1 l_1 n_2 l_2 fr) < \dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 L | n_1 l_1 n_2 l_2 L > \;, \qquad (2.8)$$

where $< \dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 L | n_1 l_1 n_2 l_2 L >$ is a standard two particle transformation bracket[6], and $A(\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 \,,\, n_1 l_1 n_2 l_2 fr)$ is a coefficient whose values are given in Table 1 for all possible values of the quantum numbers.

In the next section, we derive the form factor of the mass 3 nuclei using the wave function (2.1), together with (2.7) and (2.8).

3. THE FORM FACTOR OF THE MASS 3 NUCLEI

The charge form factor of a nucleus is defined by

$$F_{\mathrm{ch}}(\mathbf{q}) \equiv \frac{1}{Z} \int e^{i\mathbf{q} \cdot \mathbf{x}} \rho(\mathbf{x})\, d\mathbf{x} \;, \qquad (3.1)$$

where Z is the number of protons of the nucleus, $\rho(\mathbf{x})$ the charge density referred to the centre of mass of the nucleus, and $\hbar \mathbf{q}$ the momentum transfer (in units of $[\hbar m\omega]^{1/2}$). When the ground state wave function of the nucleus is given by (2.1), the form factor can be written as

$$F_{ch}(\mathbf{q}) = f_1(\mathbf{q})\, F_1(\mathbf{q}) + f_0(\mathbf{q})\, F_0(\mathbf{q}) \ , \tag{3.2}$$

where $f_\nu(\mathbf{q}) = f_\nu(q^2)$ is the form factor of the proton ($\nu = 1$) or the neutron ($\nu = 0$), which is known experimentally, and $F_\nu(\mathbf{q})$ is given by

$$F_\nu(\mathbf{q}) = F_\nu(q^2) = \frac{1}{Z} \sum_{\substack{ff' \\ rr'}} \ \sum_{\substack{n_1 l_1 n_2 l_2 \\ n'_1 l'_1 n'_2 l'_2}} \ \sum_{LS} \left\{ \frac{1}{4}\ [1 + (-1)^{\pi + 2n_1 + l_1 + 2n_2 + l_2}] \right.$$

$$\times\ [1 + (-1)^{\pi + 2n'_1 + l'_1 + 2n'_2 + l'_2}]\ \frac{(-1)^r (-1)^{r'}}{\sqrt{d_f d_{f'}}}$$

$$\times\ \langle n'_1 l'_1 n'_2 l'_2 L f' r' |\ \frac{\sin \kappa |\dot{\mathbf{x}}^2|}{\kappa |\dot{\mathbf{x}}^2|}\ | n_1 l_1 n_2 l_2 L f r \rangle$$

$$\times\ \langle ST = \tfrac{1}{2} M_T \tilde{f}' \tilde{r}' \,|\, \tfrac{1}{2} + (-1)^\nu t_0^3 \,|\, ST = \tfrac{1}{2} M_T \tilde{f} \tilde{r} \rangle\ a^*(n'_1 l'_1 n'_2 l'_2 L f' S)$$

$$\left. \times\ a(n_1 l_1 n_2 l_2 L f S) \right\} . \tag{3.3}$$

Here π takes the value 0 for positive parity and 1 for negative parity, κ is given by

$$\kappa = \sqrt{\frac{2}{3}}\, q \ , \tag{3.4}$$

and t_0^3 is the third component of the isospin of particle 3 .

The matrix element of

$$\frac{\sin \kappa |\dot{x}^2|}{\kappa |\dot{x}^2|}$$

can be reduced to a one particle matrix element using expansion (2.7) in bra and ket, and is found to be equal to

$$< n_1' l_1' n_2' l_2' L f' r' | \frac{\sin \kappa |\dot{x}^2|}{\kappa |\dot{x}^2|} | n_1 l_1 n_2 l_2 L f r >$$

$$= \sum_{\dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 \dot{n}_2'} \{ < \dot{n}_1 \dot{l}_1 \dot{n}_2 \dot{l}_2 L | n_1 l_1 n_2 l_2 L f r > < \dot{n}_1 \dot{l}_1 \dot{n}_2' \dot{l}_2 L | n_1' l_1' n_2' l_2' L f' r' >^*$$

$$\times \sum_{p = \dot{l}_2}^{\dot{l}_2 + \dot{n}_2 + \dot{n}_2'} [B(\dot{n}_2' \dot{l}_2, \dot{n}_2 \dot{l}_2, p) \, I_p(\kappa^2)] \} , \qquad (3.5)$$

where the coefficient $B(\dot{n}_2' \dot{l}_2, \dot{n}_2 \dot{l}_2, p)$ has been tabulated by Brody and Moshinsky[6], and $I_p(\kappa^2)$ is given by[1]

$$I_p(\kappa^2) = \frac{1}{2} e^{-\frac{\kappa^2}{4}} \sum_{s=0}^{p} \binom{2p+1}{2s} \frac{\Gamma(s + \frac{1}{2})}{\Gamma(p + \frac{1}{2})} \left(- \frac{\kappa^2}{4} \right)^{p-s} \qquad (3.6)$$

Here $\binom{2p+1}{2s}$ is a binomial coefficient, and Γ is the gamma function. The matrix element

$$< ST = \frac{1}{2} M_T \tilde{f}' \tilde{r}' | \frac{1}{2} + (-1)^{\nu} t_0^3 | ST = \frac{1}{2} M_T \tilde{f} \tilde{r} > \qquad (3.7)$$

can be easily evaluated using one particle spin-isospin fractional parentage coefficients[4], and is tabulated in Table 2 for $S = 1/2$ and in Table 3 for $S = 3/2$

TABLE 2

The matrix element (3.7) for $S = 1/2$

$\bar{f}'$ $\bar{r}'$ \ $\bar{f}$ $\bar{r}$	{111} (321)	{21} (211)	{21} (121)	{3} (111)
{111} (321)	$\frac{1}{2} + (-1)^\nu \frac{1}{3} M_T$		$-(-1)^\nu \frac{2}{3} M_T$	
{21} (211)		$\frac{1}{2} + (-1)^\nu \frac{1}{3} M_T$		$(-1)^\nu \frac{2}{3} M_T$
{21} (121)	$-(-1)^\nu \frac{2}{3} M_T$		$\frac{1}{2} + (-1)^\nu \frac{1}{3} M_T$	
{3} (111)		$(-1)^\nu \frac{2}{3} M_T$		$\frac{1}{2} + (-1)^\nu \frac{1}{3} M_T$

TABLE 3

The matrix element for $S = 3/2$

$\bar{f}'$ $\bar{r}'$ \ $\bar{f}$ $\bar{r}$	{21} (211)	{21} (121)
{21} (211)	$\frac{1}{2} - (-1)^\nu \frac{1}{3} M_T$	
{21} (121)		$\frac{1}{2} + (-1)^\nu M_T$

Now the knowledge of the mixing coefficients $a(n_1 l_1 n_2 l_2 L f S)$ of the ground state wave function allows us to calculate $F_\nu(q^2)$ and finally the form factor (3.2). We expect that in a near future this will be applied to the form factor of the tritium nucleus.

Before concluding we wish also to give a procedure for determining the matrix element (3.5) without the use of transformation brackets.

4. DETERMINATION OF THE FORM FACTOR WITHOUT THE USE OF TRANSFORMATION BRACKETS.

In the determination of the matrix element (3.5) we let the operator $(\kappa|\dot{\mathbf{x}}^2|)^{-1} \sin \kappa|\dot{\mathbf{x}}^2|$ remain unmodified, but changed the bra and ket to Jacobi coordinates with the help of transformation brackets. In this way the evaluation of the matrix element became trivial. Another procedure to carry out this evaluation, which may be more practical for numerical calculations, is to express $\dot{\mathbf{x}}^2$ in terms of the coordinates and momenta associated with η^2, η^2 defined by (2.3), and leave the bra and ket as they stand.

From the relation (2.3), and the corresponding one between the an ni nihilation operators ξ^1, ξ^2 and $\dot{\xi}'$, $\dot{\xi}^2$ which is obtained from (2.3) by hermitian conjugation, we see, from the definitions

$$\eta^s = \frac{1}{\sqrt{2}}(\mathbf{x}^s - i\,\mathbf{p}^s), \quad \xi^s = \frac{1}{\sqrt{2}}(\mathbf{x}^s + i\,\mathbf{p}^s), \quad s = 1, 2, \tag{4.1}$$

and similar ones for $\dot{\eta}^s$, $\dot{\xi}^s$, that we have the linear canonical transformation

$$\dot{\mathbf{x}}^1 = \frac{1}{\sqrt{2}}(\mathbf{p}^1 - \mathbf{p}^2), \quad \dot{\mathbf{p}}^1 = -\frac{1}{\sqrt{2}}(\mathbf{x}^1 - \mathbf{x}^2), \tag{4.2a, b}$$

$$\dot{\mathbf{x}}^2 = \frac{1}{\sqrt{2}}(\mathbf{x}^1 + \mathbf{x}^2), \quad \dot{\mathbf{p}}^2 = \frac{1}{\sqrt{2}}(\mathbf{p}^1 + \mathbf{p}^2). \tag{4.3a, b}$$

Thus $(\kappa|\dot{\mathbf{x}}^2|)^{-1} \sin(\kappa|\dot{\mathbf{x}}^2|)$ can be expanded[8] in terms of Bessel functions and spherical harmonics associated with $\mathbf{x}^1$, $\mathbf{x}^2$ in the form

$$(\kappa\,|\,\dot{x}^2\,|)^{-1}\sin(\kappa\,|\,\dot{x}^2\,|)$$

$$= \sum_{\lambda=0}^{\infty} 4\pi(-1)^{\lambda}\, j_\lambda(\beta r_1)\, j_\lambda(\beta r_2) \sum_\mu Y^*_{\lambda\mu}(\theta_1\varphi_1)\, Y_{\lambda\mu}(\theta_2\varphi_2)\,, \tag{4.4}$$

where $\beta = (\kappa/\sqrt{2})$ and r_s, θ_s, φ_s are the spherical coordinates related with x^s, $s = 1, 2$.

It seems then that the matrix element (3.5) can be calculated straightforwardly. A word of caution is required though. We recall that in the ket (2.2) the ground state $|0>$ is defined[1] by

$$|0> = \pi \exp\{-\tfrac{1}{2}[(\dot{x}^1)^2 + (\dot{x}^2)]\} \tag{4.5}$$

The normal evaluation procedure[6] of the matrix elements of the terms in the expansion (4.4) can proceed only if in (2.2) the ground state $|0>$ is replaced by

$$|\overline{0}> = \pi^{-\frac{1}{2}} \exp\{-\frac{1}{2}[(x^1)^2 + (x^2)^2]\}\ . \tag{4.6}$$

We have shown elsewhere[7] that under the linear canonical transformation (4.2), (4.3) the state $|0>$ transforms into $|\overline{0}>$, and thus the replacement is justified. A more elementary way of seeing that we can replace $|0>$ by $|\overline{0}>$ in the matrix elements of $(\kappa\,|\,\dot{x}^2|)^{-1}\sin(\kappa\,|\,\dot{x}^2\,|)$, is to expand this operator in series and then use (2.3) and the adjoint equation, to express the terms of the series in terms of η^1, η^2, ξ^1, ξ^2. From this expansion and (2.2), the matrix element (3.5) becomes the vacuum expectation value of a function of η^1, η^2, ξ^1, ξ^2 which we can put in time ordered form with all annihilation operators to the right of the creation ones. As the ξ^s are linear functions of $\dot{\xi}$ $s = 1, 2$, we have

$$\xi^s\,|0> = 0\ , \tag{4.7}$$

and thus the effect of this operator on $|0>$ is the same as on $|\overline{0}>$. Therefore the replacement of $|0>$ by $|\overline{0}>$ in (2.2) does not alter the value of the matrix element (3.5) and we are justified in following the normal evaluation procedure.

The matrix element (3.5) reduces then essentially[1] to

$$I \equiv \langle n_1' l_1' n_2' l_2' L \mid (\kappa \mid \dot{x}^2 \mid)^{-1} \sin (\kappa \mid \dot{x}^2 \mid) \mid n_1 l_1 n_2 l_2 L \rangle$$

$$= \sum_{\lambda=0}^{\infty} 4\pi(-1)^{\lambda} \langle n_1' l_1' \mid j_\lambda(\beta r_1) \mid n_1 l_1 \rangle \langle n_2' l_2' \mid j_\lambda(\beta r_2) \mid n_2 l_2 \rangle$$

$$\langle l_1' l_2' L \mid \sum_{\mu} Y^*_{\lambda\mu}(\theta_1 \varphi_1) \, Y_{\lambda\mu}(\theta_2 \varphi_2) \mid l_1 l_2 L \rangle \tag{4.8}$$

where the last matrix element in (4.8) has the well known form[9]

$$\langle l_1' l_2' L \mid \sum_{\mu} Y^*_{\lambda\mu}(\theta_1 \varphi_1) \, Y_{\lambda\mu}(\theta_2 \varphi_2) \mid l_1 l_2 L \rangle$$

$$= (-)^{l_1' + l_2 - L} \, W(l_1 l_2 l_1' l_2' ; L\lambda) \, [(2l_1' + 1)(2l_2' + 1)]^{\frac{1}{2}}$$

$$\left[\frac{(2l_1 + 1)(2\lambda + 1)}{4\pi(2l' + 1)}\right]^{\frac{1}{2}} \langle l_1 \lambda \, 00 \mid l_2' 0 \rangle \left[\frac{(2l_2 + 1)(2\lambda + 1)}{4\pi(2l' + 1)}\right]^{\frac{1}{2}} \langle l_2 \lambda \, 00 \mid l_2' 0 \rangle \tag{4.9}$$

while the first is given by[6]

$$\langle n' l' \mid j_\lambda(\beta r) \mid nl \rangle =$$

$$\sum_{p = \frac{1}{2}(l + l')}^{n + n' + \frac{1}{2}(l + l')} B(nl, n'l', p) \, I_p(\beta, \lambda) \quad , \tag{4.10}$$

with

$$I_p(\beta, \lambda) = \frac{2}{\Gamma(p + 3/2)} \int_0^\infty r^{2p+2} j_\lambda(\beta r) \, e^{-r^2} dr \ . \tag{4.11}$$

We note that the sum in (4.8) is actually finite as the Clebsch-Gordan coefficients in (4.9) restrict λ by the inequalities

$$\max \{ |l_1 - l_1'|, |l_2 - l_2'| \} \leqslant \lambda \leqslant \min \{ l_1 + l_1', l_2 + l_2' \} \quad . \tag{4.12}$$

The radial integral (4.11) can be evaluated with the help of the well known relation[10]

$$\int_0^\infty x^\mu e^{-\alpha x^2} J_\nu(\beta x)\, dx = \frac{\beta^\nu \Gamma(\frac{\nu+\mu+1}{2})}{2^{\nu+1} \alpha^{\frac{1}{2}(\mu+\nu+1)} \Gamma(\nu+1)} {}_1F_1\left(\frac{\nu+\mu+1}{2}, \nu+1, -\frac{\beta^2}{4\alpha}\right) \tag{4.13}$$

where ${}_1F_1(a, b, z)$ is the confluent hypergeometric function

$${}_1F_1(a, b, z) = \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b)}{\Gamma(a)\Gamma(b+n)} \frac{z^n}{n!} \tag{4.14}$$

Writing

$$j_\lambda(\beta r) = (\pi/2\beta r)^{\frac{1}{2}} J_{\lambda+\frac{1}{2}}(\beta r) \ , \tag{4.15}$$

we see that the radial integral becomes

$$I_p(\beta, \lambda) = \frac{\sqrt{\pi}\beta^\lambda \Gamma(\frac{\lambda+2p+3}{2})}{\Gamma(p+3/2)\, 2^{\lambda+1} \Gamma(\lambda+3/2)} {}_1F_1\left(\frac{\lambda+2p+3}{2}), \lambda+\frac{3}{2}, -\frac{\beta^2}{4}\right) \tag{4.16}$$

The confluent hypergeometric function is given by an infinite sum, but if we make the transformation[10]

$${}_1F_1(a, b, z) = e^z {}_1F_1(b-a, b, -z) \ , \tag{4.17}$$

our integral can also be written as

$$I_p(\beta,\lambda) = \frac{\sqrt{\pi}\,\beta^\lambda\,\Gamma\left(\frac{\lambda+2p+3}{2}\right)}{\Gamma(p+3/2)\,2^{\lambda+1}\,\Gamma(\lambda+3/2)}\,e^{-\beta^2/4}\;{}_1F_1\left(\frac{\lambda}{2}-p,\,\lambda+\frac{3}{2},\,\frac{\beta^2}{4}\right). \tag{4.18}$$

As $1/2\lambda - p$ is always a negative integer the function ${}_1F_1$ is in this case a finite polynomial.

We have thus determined the form factor of a three particle system in a way that does not involve transformation brackets.

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RESUMEN

En esta nota se obtiene la expresión teórica del factor de forma de la distribución de carga de los núcleos de masa 3, utilizando un desarrollo de la función de onda del estado base en términos de estados de oscilador armónico invariantes bajo traslaciones.