FIRST ORDER RELATIVISTIC KINETIC EQUATION FOR A TWO-COMPONENT INHOMOGENEOUS PLASMA

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ABSTRACT: The dynamical evolution of a two-component slightly relativistic plasma ($T \sim 10$ Kev) is discussed following Bogoliubov's method to solve Trubnikov's hierarchy of equations. The hierarchy is decoupled by means of the perturbation technique for a weakly interacting gas and solved up to second order. The final kinetic equation corresponds to the first order relativistic correction to the Fokker Planck equation for an inhomogeneous two-component plasma. The model is particularly adequate to discuss the transport properties of a fusion plasma.

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1. INTRODUCTION

The kinetic equation for a fully ionized, slightly relativistic plasma has been discussed by Trubnikov[1] using Bogoliubov's method. He assumes a Darwin lagrangian[2] for the interaction and obtains the collision operator for an homogeneous one component plasma, including dynamical shielding, using a perturbation expansion in the plasma parameter φ_0/kT. The resulting collision integral corresponds to the relativistic generalization of the Lenard Balescu equation[3] up to v^2/c^2 terms.

In classical plasma physics one usually deals with a simplified version of the Lenard Balescu equation: the Fokker Planck equation. The latter can be derived from the classical BBGKY hierarchy assuming that the plasma behaves as a moderately dense $(n_0 r_0^3 \sim 1)$, weakly interacting gas $(\varphi_0/kT << 1)$[4]. The perturbation expansion combined with Bogoliubov's method gives to first order the Vlasov equation and to second order the Fokker Planck collision operator[5].

In this paper we follow a similar procedure. We start with the generalized BBGKY hierarchy for a system of particles with velocity dependent interactions. We assume Darwin's lagrangian for the motion of the particles, and derive the first order relativistic correction to the Fokker Planck equation.

Two advantages of this equation over the one derived by Trubnikov should me mentioned. First it is much simpler; second, it is more general in that it is derived without restriction to a spatially homogeneous plasma.

In part II we solve the generalized hierarchy of equations for a two-component weakly interacting gas using Bogoliubov's method and arrive at the relativistic Fokker Planck equation correct up to v^2/c^2 terms.

Part III is devoted to the discussion of the equilibrium properties of the plasma and the derivation of the electron-ion radial distribution functions.

II. FOKKER-PLANCK EQUATION FOR SLIGHTLY RELATIVISTIC PLASMAS.

We consider a two component plasma with N particles interacting according to Darwin's lagrangian[2].

$$L_D = \sum_a \frac{m_a v_a^2}{2} + \sum_a \frac{m_a}{8} \frac{v_a^4}{c^2} - \sum_{a>b} \frac{e_a e_b}{r_{ab}} +$$

$$+ \sum_{a>b} \frac{e_a e_b}{2c^2} \frac{1}{r_{ab}} \left[\mathbf{v}_a \cdot \mathbf{v}_b + \left(\mathbf{v}_a \cdot \frac{r_{ab}}{|r_{ab}|}\right)\left(\mathbf{v}_b \cdot \frac{r_{ab}}{|r_{ab}|}\right) \right] \tag{1}$$

The generalized BBGKY hierarchy of equations can be written as[1]

$$\frac{\partial F_s}{\partial t} + \sum_{a=1}^{s} \left[\mathbf{v}_a \cdot \frac{\partial F_s}{\partial \mathbf{r}_a} + \frac{\partial}{\partial \mathbf{v}_a} \cdot I_s^{(a)} \right] \tag{2}$$

and

$$\left(I_s^{(a)}\right)_\alpha + \left(\frac{1}{2} \frac{v_a^2}{c^2} \delta_{\alpha\beta} + \frac{1}{c^2} v_{a\alpha} v_{a\beta}\right)\left(I_s^{(a)}\right)_\beta +$$

$$+ \frac{n_0 c^2}{m_a} \int (\tau_{as+1})_{\alpha\beta} \left(I_{s+1}^{(s+1)}\right)_\beta dx_{s+1} = \sum_{b=1}^{s} \left\{ - \frac{e_a e_b}{m_a} (\tau_{ab})_{\alpha\beta} \left(I_s^a\right)_\beta + \right.$$

$$\left. + \frac{e_a e_b}{m_a} F_s \left[\frac{(r_{ab})_\alpha}{r_{ab}^3} + \left(\sigma_a^{(b)}\right)_\alpha \right] \right\} + \frac{n_0 e^2}{m_a} \int dr_{s+1} \left\{ \frac{(r_{as+1})_\alpha}{|r_{as+1}|^3} \left[\int F_{s+1} dv_{s+1} - F_s \right] + \right.$$

$$\left. + \int \left(\sigma_a^{s+1}\right)_\alpha F'_{s,+1} dv_{s+1} \right\} \tag{3}$$

where

$$(\tau_{ab})_{\alpha\beta} = \frac{1}{2c^2} \quad \frac{\delta_{\alpha\beta}}{|r_{ab}|} + \frac{(r_{ab})_\alpha (r_{ab})_\beta}{|r_{ab}|^3}$$

and

$$\left(\sigma_a^b\right)_\alpha = \frac{(r_{ab})_\alpha}{2c^2\left|r_{ab}\right|^3}\left[v_b^2 - 3\,\frac{(\mathbf{r}_{ab}\cdot\mathbf{v}_b)^2}{\left|r_{ab}\right|^2}\right] + \frac{1}{c^2 r_{ab}^3}\left[\mathbf{v}_a \times(\mathbf{v}_b \times \mathbf{r}_{ab})\right]_\alpha \tag{4}$$

$I_s^{(a)}$ is defined in terms of the N particle distribution function f_N as

$$I_s^{(a)} = V^s \int \dot{\mathbf{v}}_a f_N\, dx_{s+1} \ldots dx_N \tag{5}$$

Equation (3) differs from the one given by Trubnikov in the form of the left-hand side. The difference is due to the v^4/c^2 term taken into account in the expression for the kinetic energy in equation (1). As will be shown in Appendix I these terms are necessary in order to obtain the correct Vlasov equation to first order in v^2/c^2.

We look for a perturbation solution of equations (2) and (3) in the small parameter e^2/kT.

To zeroth order we have from (3)

$$\left(I_s^{(0)(a)}\right)_\alpha = 0$$

and from (2) we get $F_s^0 = f_1^0(x_1)\ldots f_1^0(x_s)$, assuming the initial condition

$$F_S^0\,(t=0) = f_1^0(t=0\,,x_1)\ldots f_1^0(t=0\,,x_s)$$

To first order we get

$$\left(I_s^{(1)(a)}\right)_\alpha + \left(\frac{1}{2}\,\frac{v_a^2}{c^2}\,\delta_{\alpha\beta} + \frac{1}{c^2}\,v_{a\alpha}\,v_{a\beta}\right)\left(I_s^{(1)(a)}\right)_\beta = \left(J_s^{(a)}\right)_\alpha \,.$$

$$= \frac{n_0 e^2}{m_a}\int dr_{s+1}\left\{\frac{(r_{as+1})_\alpha}{\left|ra_{s+1}\right|^3}\left[\int F_{s+1}^0\, dv_{s+1}^3 - F_s^0\right] + \int\left(\sigma_a^{s+1}\right)_\alpha F_{s+1}^0\, dv_{s+1}\right\}$$

$$+ \sum_{\substack{b=1\\ b\neq a}} \frac{e_a e_b}{m_a}\, F_s^0\left[\frac{(r_{ab})_\alpha}{\left|r_{ab}\right|^3} + \left(\sigma_a^{(b)}\right)_\alpha\right] \tag{6}$$

From (6) we invert and obtain

$$\left(I_s^{(1)(a)}\right)_\alpha = \left(J_s^{(a)}\right)_\alpha - \frac{1}{2}\frac{v_a^2}{c^2}\left(J_s^{(a)}\right)_\alpha - \frac{1}{c^2} v_{a\alpha} v_{a\beta}\left(J_s^{(a)}\right)_\beta \tag{7}$$

In particular for $s = 1$ we have

$$I_1^{(1)} = \frac{n_0 e^2}{m_1}\int dr_2\, dv_2\, \frac{r_{12}}{|r_{12}|^3} f_1(x_1)\, f_1(x_2) + \frac{n_0 e^2}{m_1}\int dx_2\, \sigma_1^{(2)} f_1(x_1)\, f_1(x_2) -$$

$$- \frac{1}{2}\frac{v_1^2}{c^2}\frac{n_0 e^2}{m_1}\int dr_2\, dv_2\, \frac{r_{12}}{|r_{12}|^3} f_1(x_1)\, f_1(x_2) - \frac{1}{c^2}\frac{n_0 e^2}{m_1} \mathbf{v}_1 \int dr_2\, dv_2\, \frac{\mathbf{v}_1 \cdot \mathbf{r}_{12}}{|r_{12}|^3} f_1(x_1)\, f_1(x_2)$$

(8)

In order to compute $F_s^{(1)}(x_1, \ldots, x_s, t)$ we make Bogoliubov's assumption[6] that the F_s are synchronized with F_1. Following Bogoliubov's notation we have from (2)

$$D_1 f_s^0 + D_0 F_s^{(1)} + \sum_{a=1}^{s} \mathbf{v}_a \cdot \frac{\partial}{\partial \mathbf{r}_a} F_s^{(1)} = - \sum_a \frac{\partial}{\partial \mathbf{v}_a} \cdot I_s^{(1)(a)} \tag{9}$$

where as usual $D_1 F_s^0$ is short for $\frac{\delta F_s^0}{\delta f_1} \cdot \frac{\partial f_1}{\partial t}$ with successive replacement of $\frac{\partial f_1}{\partial t}$ by the first order collision operator of the kinetic equation satisfied by f_1. f_1.

From equation (9) we deduce

$$D_0 F_s^{(1)} + \sum_{a=1}^{s} \mathbf{v}_a \cdot \frac{\partial}{\partial \mathbf{r}_a} F_s^{(1)} = - \sum_a \frac{\partial}{\partial \mathbf{v}_a} \cdot I_s^{*(a)} \tag{10}$$

where

$$\sum_a \frac{\partial}{\partial v_a} I_s^{*(a)} = D_1 F_s^0 + \sum_a \frac{\partial}{\partial \mathbf{v}_a} \cdot I_s^{(1)(a)} \tag{11}$$

Using equations (6) and (8) we get

$$\left(I_s^{*(a)}\right)_\alpha = \sum_{b=1\, b\neq a} \frac{e_a e_b F_s^0}{m_a} \left[\frac{(r_{ab})_\alpha}{|r_{ab}|^3} + (\sigma_a^b)_\alpha - \frac{1}{2}\frac{v_a^2}{c^2}\frac{(r_{ab})_\alpha}{|r_{ab}|^3} - \frac{v_{a\alpha} v_{a\beta}}{c^2}\frac{(r_{ab})_\alpha}{|r_{ab}|^3} \right] \tag{12}$$

The solution of equation (10) can be obtained in the usual way. Using the evolution operator

$$S_\tau^{(s)} = \exp\left[\tau \sum_{a=1}^{s} \mathbf{v}_a \cdot \nabla_a\right] \tag{13}$$

we can write

$$F_s^{(1)}(x_1 \ldots x_s, f_1) = \int_0^\infty d\tau\, S_{-\tau}^{(s)} \left\{ -\sum_a \frac{\partial}{\partial \mathbf{v}_a} \cdot I_s^{*a}(x_1, \ldots, x_s, S_\tau^{(1)} f_1) \right\} \tag{14}$$

Making use of the relation

$$\exp[-\tau \mathbf{v}_a \cdot \nabla_a] \frac{\partial}{\partial \mathbf{v}_a} = \frac{\partial}{\partial \mathbf{v}_b} \exp[-\tau \mathbf{v}_a \cdot \nabla_a] + \tau \nabla_a \exp[-\tau \mathbf{v}_a \cdot \nabla_a]\, \delta_{ab} \tag{15}$$

one gets for $s = 2$

$$F_2^{(1)} = \int_0^\infty d\tau \left(-\frac{\partial}{\partial \mathbf{v}_1} - \tau \frac{\partial}{\partial \mathbf{r}_1}\right) \cdot \left\{ \frac{e_1 e_2}{m_1} f_1(x_1) f_1(x_2) \left[\frac{\tilde{r}_{12}}{|\tilde{r}_{12}|^3} + \tilde{\sigma}_1^{(2)} \right] - \right.$$

$$\left. - \frac{1}{2}\frac{v_1^2}{c^2}\frac{e_1 e_2}{m_1} f_1(x_1) f_1(x_2) \frac{\tilde{r}_{12}}{|\tilde{r}_{12}|^3} - \frac{1}{c^2} \mathbf{v}_1 \frac{e_1 e_2}{m_1} f_1(x_1) f_1(x_2) \frac{\mathbf{v}_1 \cdot \tilde{r}_{12}}{|\tilde{r}_{12}|^3} \right\} -$$

$$-\left[\frac{\partial}{\partial \mathbf{v}_2} + \tau \frac{\partial}{\partial \mathbf{r}_2}\right] \cdot \left\{ \frac{e_1 e_2}{m_2} f_1(x_1) f_1(x_2) \left[\frac{\tilde{r}_{21}}{|\tilde{r}_{21}|^3} + \tilde{\sigma}_2^{(1)} \right] - \right.$$

$$\left. - \frac{1}{2}\frac{v_2^2}{c^2}\frac{e_1 e_2}{m_2} f_1(x_1) f_1(x_2) \frac{\tilde{r}_{21}}{|\tilde{r}_{21}|^3} - \frac{e_1 e_2}{m_2} \frac{1}{c^2} \mathbf{v}_2 f_1(x_1) f_1(x_2)\, \mathbf{v}_2 \cdot \frac{\tilde{r}_{21}}{|\tilde{r}_{21}|^3} \right\} \tag{16}$$

where

$$\tilde{\boldsymbol{r}}_{12} = S^{(2)}_{-\tau}\, \boldsymbol{r}_{12} = \boldsymbol{r}_{12} - \boldsymbol{v}_{12}\, \tau$$

and

$$\tilde{\sigma}^{(b)}_{a} = S^{(2)}_{-\tau}\, \sigma^{(b)}_{a} = \sigma^{(b)}_{a}(\boldsymbol{r}_{12} - \boldsymbol{v}_{12}\, \tau)$$

We next go to second order and search for the expression for $I^{(2)(1)}_{1}$. We have

$$\left(I^{(2)}_{1}\right)_{\alpha} + \left(\frac{1}{2}\frac{v_1^2}{c^2}\,\delta_{\alpha\beta} + \frac{1}{c^2}\, v_{1\alpha} v_{1\beta}\right)\left(I^{(2)}_{1}\right)_{\beta} + \frac{n_0 e_1}{m_1}\int e_2\, (\tau_{12})_{\alpha\beta}\left(I^{(1)(2)}\right)_{\beta} d\boldsymbol{x}_2 =$$

$$= \frac{n_0 e_1}{m_1}\int e_2\, d\boldsymbol{x}_2\, \frac{(r_{12})_{\alpha}}{|r_{12}|^3}\, F^{(1)}_{2} + \frac{n_0 e_1}{m_1}\int e_2\, d\boldsymbol{x}_2\, (\sigma^{(2)}_{1})_{\alpha}\, F^{(1)}_{2} \tag{18}$$

In equation (18) we have to use the non relativistic expression for $I^{(2)}_{2}$, namely

$$\left(I^{(1)(2)}_{2}\right)_{\alpha} = \frac{e_1 e_2}{m_2}\, f_1(\boldsymbol{x}_1)\, f_1(\boldsymbol{x}_2)\, \frac{(r_{21})_{\alpha}}{|r_{21}|^3} + \frac{n_0 e_2}{m_2}\int e_3\, d\boldsymbol{x}_3\, \frac{(r_{23})_{\alpha}}{|r_{23}|^3}\, f_1(\boldsymbol{x}_1)\, f_1(\boldsymbol{x}_2)\, f_1(\boldsymbol{x}_3) \tag{19}$$

so that

$$\left(I^{(2)}_{i}\right)_{\alpha} = \frac{n_0 e_1}{m_1}\int e_2\, d\boldsymbol{x}_2\, \frac{(r_{12})_{\alpha}}{|r_{12}|^3}\, F^{(1)}_{2} + \frac{n_0 e_1}{m_1}\int e_2\, d\boldsymbol{x}_2\, (\sigma^{(2)}_{1})_{\alpha}\, F^{(1)}_{2} -$$

$$- \frac{n_0 e^4}{m_1 m_2}\int (\tau_{12})_{\alpha\beta}\, f_1(\boldsymbol{x}_1)\, f_1(\boldsymbol{x}_2)\, \frac{(r_{21})_{\beta}}{|r_{21}|^3}\, d\boldsymbol{x}_2 -$$

$$- \frac{n_0 e^4}{m_1 m_2}\int d\boldsymbol{x}_2\, d\boldsymbol{x}_3\, (\tau_{12})_{\alpha\beta}\, \frac{(r_{23})_{\beta}}{|r_{23}|^3}\, f_1(\boldsymbol{x}_1)\, f_1(\boldsymbol{x}_2)\, f_1(\boldsymbol{x}_3)$$

$$- \frac{1}{2}\frac{v_1^2}{c^2}\frac{n_0 e_1}{m_1}\int e_2\, d\boldsymbol{x}_2\, \frac{(r_{12})_{\alpha}}{|r_{12}|^3}\, F^{(1)}_{2} - \frac{1}{c^2}\frac{n_0 e_1}{m_1}\, v_{1\alpha}\int e_2\, d\boldsymbol{x}_2\, v_{1\beta}\, \frac{(r_{12})_{\beta}}{|r_{12}|^3}\, F^{(1)}_{2} \tag{20}$$

Equations (20) and (16) give the expression for a weakly relativistic inhomogeneous plasma collision operator between particles of masses m_1 and m_2. This kinetic equation must be used to compute the transport properties of a fusion plasma.

For an homogeneous system, equation (16) for $F_2^{(1)}$ becomes

$$F_2^{(1)} = \int_0^\infty d\tau \Big\{ - \frac{e_1 e_2}{m_1} \frac{\partial f_1(v_1)}{\partial v_{1\beta}} f_1(v_2) \frac{(\tilde{r}_{12})_\beta}{|r_{12}|^3} + \frac{e_1 e_2}{m_1} \frac{\partial f_1(v_1)}{\partial v_{1\beta}} f_1(v_2)(\tilde{\sigma}_1^{(2)})_\beta +$$

$$+ 5 \frac{e_1 e_2}{m_1} \frac{\mathbf{v}_1}{c^2} \cdot \frac{\tilde{r}_{12}}{|\tilde{r}_{12}|^3} f_1(v_1) f_1(v_2) + \frac{1}{2} \frac{e_1 e_2}{m_1} \frac{v_1^2}{c^2} \frac{\partial}{\partial v_{1\beta}} f_1(v_1) f_1(v_2) \frac{(\tilde{r}_{12})_\beta}{|\tilde{r}_{12}|^3} +$$

$$+ \frac{e_1 e_2}{m_1} \frac{v_{1\beta}}{c^2} \mathbf{v}_1 \cdot \frac{\tilde{r}_{12}}{|\tilde{r}_{12}|^3} \frac{\partial f_1(v_1)}{\partial v_{1\beta}} f_1(v_2) \Big\} + [1 \rightarrow 2]$$

where $[1 \rightarrow 2]$ indicates the previous expression with indices 1 and 2 interchanged.

To get the form of the collision operator in the homogeneous case, we make use of the relations

$$\int_0^\infty d\tau \, dr_2 \, \frac{(r_{12})_\alpha}{|r_{12}|^3} \cdot \frac{(\tilde{r}_{12})_\beta}{|\tilde{r}_{12}|^3} = 2\pi \int \frac{dk}{k} \frac{P_{\alpha\beta}}{|v|}$$

and

$$\int_0^\infty d\tau \, dr_2 \, \frac{(r_{12})_\alpha}{|r_{12}|^3} (\tilde{\sigma}_1^{(2)})_\beta = \int_0^\infty d\tau \, dr_2 \, \frac{(\tilde{r}_{12})_\alpha}{|\tilde{r}_{12}|^3} (\sigma_1^{(2)})_\beta =$$

$$= 2\pi \int \frac{dk}{k} \Big(\frac{v_2^2}{2c^2} - \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{c^2} \Big) \frac{P_{\alpha\beta}}{|v|} + 2\pi \int \frac{dk}{k} \frac{v_{2\beta} v_{1\gamma}}{c^2} \frac{P_{\alpha\gamma}}{|v|} -$$

$$- \pi \int \frac{dk}{k} \frac{1}{|v|} \frac{v_{2\gamma} v_{2\nu}}{c^2} \Big[P_{\alpha\beta} \delta_{\gamma\nu} + P_{\alpha\gamma} \delta_{\beta\nu} + P_{\alpha\nu} \delta_{\beta\gamma} - \frac{1}{2} (P_{\alpha\beta} P_{\gamma\nu} + P_{\alpha\gamma} P_{\beta\nu} + P_{\alpha\nu} P_{\beta\gamma}) \Big] \tag{21}$$

where

$$P_{\alpha\beta} = \delta_{\alpha\beta} - \frac{(v_1 - v_2)_\alpha (v_1 - v_2)_\beta}{v^2} \text{ and } \mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$$

and the divergent integral $\int dk/k$ is made finite following the usual prescription of introducing arbitrary cut-offs at large and small k numbers (closest approach distance and Debye length respectively).

The expression for the collision operator between particles of masses m_1 and m_2 becomes

$$\begin{aligned}(I_1^{(2)})_\alpha = \frac{2\pi n_0 e^4}{m_1}\int\frac{dk}{k}\int dv_2 \Bigg\{ & \frac{1}{m_1}\frac{\partial f_1(v_1)}{\partial v_{1\beta}} f_1(v_2)\Bigg[-\frac{P_{\alpha\beta}}{|v|} + \frac{v_1^2}{c^2}\frac{P_{\alpha\beta}}{|v|} - \\ & - \frac{1}{2c^2|v|}(P_{\alpha\beta}P_{\gamma\nu} + 2P_{\alpha\gamma}P_{\beta\nu})v_{2\gamma}v_{2\nu} + \frac{2\mathbf{v}_1\cdot\mathbf{v}_2}{c^2}\frac{P_{\alpha\beta}}{|v|} + \frac{v_{1\beta}v_{1\gamma}}{c^2}\frac{P_{\alpha\gamma}}{|v|} + \\ & + \frac{v_{1\alpha}v_{1\gamma}}{c^2}\frac{P_{\beta\gamma}}{|v|}\Bigg] + \frac{1}{m_2}f_1(v_1)\frac{\partial f_1(v_2)}{\partial v_{2\beta}}\Bigg[\frac{P_{\alpha\beta}}{|v|} - \frac{v_2^2}{2c^2}\frac{P_{\alpha\beta}}{|v|} - \frac{v_1^2}{2c^2}\frac{P_{\alpha\beta}}{|v|} - \\ & - \frac{v_{2\beta}v_{2\gamma}}{c^2}\frac{P_{\alpha\gamma}}{|v|} - \frac{v_{1\alpha}v_{1\gamma}}{c^2}\frac{P_{\beta\gamma}}{|v|} - \frac{2\mathbf{v}_1\cdot\mathbf{v}_2}{c^2}\frac{P_{\alpha\beta}}{|v|} + \frac{1}{2c^2|v|}(P_{\alpha\beta}P_{\gamma\nu} + 2P_{\alpha\gamma}P_{\beta\nu})v_{2\gamma}v_{2\nu}\Bigg] + \\ & + \left(\frac{1}{m_1} - \frac{1}{m_2}\right) 5 f_1(v_1) f_1(v_2)\frac{v_{1\beta}}{c^2}\frac{P_{\alpha\beta}}{|v|}\Bigg\}\end{aligned} \tag{23}$$

and the kinetic equation for a two component interacting charged gas is

$$\frac{\partial f_1(v_1)}{\partial t} = -\frac{\partial}{\partial \mathbf{v}_1}\cdot \mathbf{I}_1^{(2)} \tag{24}$$

where $\mathbf{I}_1^{(2)}$ is given by the sum of two operators similar to (23) corresponding to equal particle interaction and different particle interaction. Equation (24) gives the Fokker Planck term to zeroth order plus the first order relativistic corrections to the Fokker-Planck equation.

III. EQUILIBRIUM SOLUTIONS

The equilibrium solution for an electron-ion plasma can be obtained by requiring that the collision operator vanishes.

We proceed to annul the electron-electron and ion-ion collision operators and afterwards check whether the solution annuls the electron-ion operator.

The equilibrium solution is of the form

$$f = f_0(1 + h) \tag{25}$$

where f_0 is Maxwellian and h satisfies the equation

$$P_{\alpha\beta}\left\{ \frac{\partial h_2}{\partial v_{2\beta}} + \frac{3}{2}\,\frac{m}{kTc^2}\,v_2^2\,v_{2\beta} - \frac{\partial h_1}{\partial v_{1\beta}} - \frac{3}{2}\,\frac{m}{kTc^2}\,v_1^2\,v_{1\beta} \right\} = 0 \tag{26}$$

with solution

$$h = -\frac{3}{8}\,\frac{m}{kTc^2}\,v^4 + \frac{\alpha}{c^2}\,v^2 + \beta\ , \tag{27}$$

where α and β are arbitrary constants. The condition that the electron-ion operator vanishes fixes the value of $\alpha = 5/2$.

Finally the normalization condition

$$\int d^3v\ h = 0 \tag{28}$$

gives

$$\beta = -\frac{15}{8}\,\frac{kT}{mc^2}\ . \tag{29}$$

Thus the equilibrium solutions are

$$f_e = f_{0e}\left\{ 1 - \frac{3m_e}{8kTc^2}\,v^4 + \frac{5}{2}\,\frac{v^2}{c^2} - \frac{15}{8}\,\frac{kT}{m_e c^2} \right\}$$

$$f_i = f_{0i} \left\{ 1 - \frac{3m_i}{8kTc^2} v^4 + \frac{5}{2} \frac{v^2}{c^2} - \frac{15}{8} \frac{kT}{m_i c^2} \right\} \tag{30}$$

The two particle equilibrium solution can be obtained from equation 21) using the solutions given by equation (30). Writing

$$f_2(x_1, x_2) = f_1(x_1) f_1(x_2) g_{12} \tag{31}$$

we obtain

$$g_{ab} = g^0_{ab} \left\{ 1 + \frac{1}{2} \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{c^2} + \frac{1}{2c^2} \frac{\mathbf{v}_1 \cdot \mathbf{r}_{12}}{|r_{12}|} \cdot \frac{\mathbf{v}_2 \cdot \mathbf{r}_{12}}{|r_{12}|} \right\} \tag{32}$$

where g^0_{ab} is the non-relativistic radial distribution function, and a and b stand for electron or ion.

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RESUMEN

Se discute en este trabajo la evolución dinámica de un plasma de dos componentes levemente relativista ($T \sim 10$ Kev), siguiendo el método de Bogoliubov para resolver la jerarquía de ecuaciones de Trubnikov. Se desacopla la cadena usando un método perturbativo para un gas en interacción débil, y se resuelve hasta segundo orden. La ecuación cinética final corresponde a la corrección relativista a primer orden de la ecuación de Fokker Planck para un plasma inhomogéneo de dos componentes. El modelo es particularmente adecuado para discutir las propiedades de transporte de un plasma de fusión.

APPENDIX I

We will show here that the collision operator given by equation (8) is in agreement with the relativistic Vlasov's equation to first order in v^2/c^2.

The relativistic Vlasov's equation can be written in the form[7]

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{v}} (\mathbf{g} f) = 0 \tag{A.1}$$

where

$$g_\alpha = \frac{e}{m\gamma} \left(\delta_{\alpha\beta} - \frac{v_\alpha v_\beta}{c^2} \right) \left(E_\beta + \frac{1}{c} (\mathbf{v} \times \mathbf{B})_\beta \right) \tag{A.2}$$

and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the self consistent fields.

Up to first order in v^2/c^2 (and first order in the ratio of electrostatic to rest energy), we can express the self consistent fields in the form[1]

$$E_\alpha = n_0 \int dx_2 \, f_1(x_2) \, \frac{e(\mathbf{r}_{12})_\alpha}{|r_{12}|^3} \left\{ 1 + \frac{v_2^2 - 3(\mathbf{r}_{12} \cdot \mathbf{v}_2)^2}{2c^2 |r_{12}|^2} \right\} \tag{A.3}$$

and

$$B_\alpha = \frac{n_0 e}{c} \int \frac{(\mathbf{v} \times \mathbf{r}_{12})_\alpha}{|r_{12}|^3} f_1(x_2) \, dx_2 \tag{A.4}$$

Thus

$$g_\alpha = \frac{e^2}{m} \int \frac{(r_{12})_\alpha}{|r_{12}|^3} f(x_2) \, dx_2 + \frac{e^2}{m} \int f(x_2) \frac{(r_{12})_\alpha}{|r_{12}|^3} \left\{ \frac{v_2^2}{2c^2} - \frac{3(\mathbf{r}_{12} \cdot \mathbf{v}_2)^2}{2|r_{12}|^2 c^2} \right\} +$$

$$+ \frac{e}{mc^2} \frac{(\mathbf{v}_1 \times (\mathbf{v}_2 \times \mathbf{r}_{12}))_\alpha}{|r_{12}|^3} f(x_2) \, dx_2 - \frac{e}{mc^2} v_{1\alpha} \int f(x_2) \frac{\mathbf{v}_2 \cdot \mathbf{r}_{12}}{|r_{12}|^3} -$$

$$- \frac{e^2}{2mc^2} \int f(x_2) \frac{(r_{12})_\alpha}{|r_{12}|^3} \, dx_2 \tag{A.5}$$

Replacing (A.5) into (A.1) we get a kinetic equation with a collision operator identical to equation (8).

APPENDIX II

It is possible to obtain the Fokker Planck equation from the Lenard Balescu equation in the low density limit ($n_0 r_0^3 \varphi_0 \ll kT$). Accordingly it should be possible to derive Eq. (23) from Trubnikov's equation in the same limit. Actually this is not so due to two reasons:

1) In Trubnikov's lagrangian, (Eq. (1.1) pag. 51, ref. (1)) is missing the term $\Sigma(mv^4/8c^2)$.

2) In his derivation of the kinetic equation, (following his Eq. (3.3), pag. 62), he makes the choice $D_0 = -\nu$, which is precisely the opposite to the one made by Bogoliubov, and in the non-relativistic limit describes a system evolving to equilibrium when $t \to -\infty$ instead of $t \to +\infty$ as it should be[8].

On the other hand it is possible to compare the equilibrium solutions, Eq. (30) and (32) with those derived in reference (9) Eq. (21) and (28) respectively.

We consider the result derived in reference (9) in the limit of infinite relativistic Debye radius ($d_c \to \infty$), which corresponds to the Fokker Planck situation of no shielding. From Eq. (21) of ref. (9) we get

$$dW_1 \to f_0^{(R)} d^3p \tag{B.1}$$

where $f_0^{(R)}$ is the relativistic Maxwell distribution function

$$f_0^{(R)'} = \frac{1}{4\pi m^3 c^3} \frac{mc^2}{kT} \left[K_2\left(\frac{mc^2}{kT}\right)\right]^{-1} \exp\left[-\frac{c}{kT}\sqrt{p^2 + m^2c^2}\right]. \tag{B.2}$$

The ordinary Maxwell distribution is related to $f_0^{(R)}$ by[10]

$$f_0 = \lim_{c\to\infty} \left[m^3\left(1 - \frac{v^2}{c^2}\right)^{-5/2} f_0^{(R)}\right] \tag{B.3}$$

It is simple to check that, up to v^2/c^2 terms, the expression

$$f_0^{(R)} m^3 \left[1 - \frac{v^2}{c^2}\right]^{-5/2}$$

reduces exactly to Eq. (30).

To compare the two-particle equilibrium distribution function we take the limit $d_c \to \infty$ in Eq. (28) of ref. (9). Since

$$\lim_{d_c \to \infty} \frac{1}{d_c} T_{ij} \left(\frac{r_{12}}{d_c}\right) = \frac{(r_{12})_i (r_{12})_j}{|r_{12}|^2} \frac{1}{2|r_{12}|} + \delta_{ij} \frac{1}{2|r_{12}|}, \qquad \text{(B.4)}$$

we get

$$g^{\text{Trub}} \xrightarrow[d_c \to \infty]{} -\frac{e_1 e_2}{kT|r_{12}|} \left\{ 1 + \beta_{1i} \beta_{2j} |r_{12}| \left[\frac{(r_{12})_i (r_{12})_j}{2|r_{12}|^3} + \frac{\delta_{ij}}{2|r_{12}|} \right] \right\}$$

$$= g_{12}^0 \left[1 + \frac{v_1 \cdot v_2}{2c^2} + \frac{(v_1 \cdot r_{12})(v_2 \cdot r_{12})}{2c^2 |r_{12}|^2} \right], \qquad \text{(B.5)}$$

which is the same as Eq. (32).