The motion of a relativistic charged particle in a homogeneous electromagnetic field in De-Sitter space

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Received 8 December 2017; accepted 4 January 2018

We discuss the geometric characterization of the trajectory of a moving charged particle, for the case of a homogeneous electromagnetic field, in De-Sitter space when the motion is governed by the Lorentz equation. We employ totally relativistic approach during the discussion and it is based on a systematic use of the four-dimensional Frenet-Serret formulae, which is adapted to the De-Sitter space to determine the worldline geometry of the electromagnetic field acting on the particle in De-Sitter space, and of the Faraday antisymmetric tensor properties.

Keywords: De-Sitter space; homogeneous electromagnetic field; Faraday antisymmetric tensor; Frenet-Serret frame; trajectory of a charged particle.

PACS: 04.20.-q; 03.50.De; 02.40.-k.

1. Introduction

Principal least action governs the dynamics of a mechanical system between the times a and b. It is defined by the following integral and it takes the least possible value.

$$\mathcal{Q}=\int_a^b \mathcal{L}dt,$$

where $\mathcal{L}$ is used to describe positions and velocities of the system. The action is defined, for a free relativistic particle, by

$$\mathcal{Q}=-mc^2\int_a^b \sqrt{1-\frac{v^2}{c^2}}dt,$$

where m is a mass, v is a velocity, and c is the speed of light in a vacuum. The dynamics of the relativistic particle has been studied intensively in Minkowski spacetime for a long time. In Minkowski spacetime, an event is described by the point particle motion whose collection creates the worldline of the particle. The generalization of the action of the relativistic particle can be given by the curvatures $(e_1,e_2,...,e_n)$ of the worldline of the particle in $(n+1)$ dimensional Minkowski spacetime in the following manner.

$$\mathcal{Q}\left[\mathcal{X}\right]=\int \mathcal{L}\left(e_1,e_2,...,e_n\right)d\tau.$$

Here $\mathcal{X}^\gamma$ is embedding function of the particle given by $\mathcal{X}^\gamma=(ct,x,y,z),\ \gamma=0,1,2,3$ such that

$$d\tau=\sqrt{-\theta},$$

where

$$\theta=\sqrt{\eta_{\gamma\alpha}\frac{D\mathcal{X}^\gamma}{ds}\frac{D\mathcal{X}^\alpha}{ds}}$$

[1]. This approach has been effectively used to determine the dynamics of a system even though its internal form is not fully solved. For example, in the bosonic theory, the action and evolving of a supersymmetric particle can be understood via curvature dependent action of the relativistic particle [2]. It was proved by Polyakov in [3] that explicit solutions of the dynamics of a rigid body are divided into three types: tachyonic, massless, and massive depending on the value of the invariant of the particle. Kuznetsov and Plyushchay in [4,5,6,7] investigated the curvature and torsion dependent model of the action of the relativistic particle linearly. The relationship between the equation of the motion of the relativistic charged particle in the homogeneous electromagnetic field and the equation of the motion of the particle containing a linear term on the torsion of the trajectory are demonstrated by Plyushchay in [8] . It was proved by the authors in [9,10,11] that the correspondence between the dynamics of a relativistic charged particle and the geometry of a worldline described by the Frenet-Serret equations can be given by using the invariants of the electromagnetic field and the curvatures of the worldline.

Static solutions of the gravitational field equations were obtained with the help of the cosmological constant, which is introduced firstly by Einstein. The fundamental solution of the equations of Einstein is the Lorentz metric of Minkowski spacetime. In a vacuum, with a positive cosmological constant, the solutions of the modified equations of Einstein are the De-Sitter metrics. The introduction of the positive cosmological constant is responsible for replacing Minkowski spacetime by a De-Sitter spacetime for symbolizing absence of matter. There are many experimental and theoretical suggestions on the non-vanishing, positive or negative value of the cosmological constant. For instance, it was considered by Narlikar in [12] that a very high positive cosmological constant is required to the rapid growth at the early period of the evolution of the universe. Further, according to recent measurements and observations, it was presented by Ohanian and Ruffini in [13] that the positive cosmological constant is required to be at action to generate the observed proportion of expansion. Therefore, the interest in the formulations and characterizations of the kinematics and dynamics of a relativistic particle in De-Sitter space has grown [14,15,16]. Some formalisms of the electromagnetic field in non-static spherically-symmetric coordinates in extending De-Sitter universe was also investigated by Veko *et al*. in [17].

The main goal of the present study is to investigate the motion of a relativistic charged particle in a homogeneous (uniform and constant) electromagnetic field by emphasizing on the invariant geometric description of its trajectory in De-Sitter spacetime. We also aim to clarify the nature of the Frenet-Serret equations on the basis of a given physical system. This is achieved through the use of the Lorentz equation together with the Frenet-Serret formalism. We also establish a connection between the intrinsic scalars of the worldline of the curve and field invariants of the electromagnetic field in De-Sitter space.

2. Frenet-Serret equations for a timelike worldline in De-Sitter space

The intrinsic geometric features of a moving particle in space is determined mostly by using the Frenet-Serret formulae. These formulations are obtained by the Frenet-Serret (FS) tetrad, which is constructed by the tangent vector of the worldline, normal and binormal vectors together with a number of associated curvatures of the curve depending on a dimension of a space. In this work, the particle is assumed to follow a timelike worldline in De-Sitter spacetime. Thus, as a result of this motion, it is obtained a curve, which has a timelike tangent vector and spacelike normal and binormal vectors. Here, the arc-length parameter is also described to compute the distance traveled by the particle along its timelike worldline. When the main tetrad is stated initially and the associated curvatures are defined in terms of the functions of the arc-length parameter on the path, then the trajectory is found thanks to the FS relations. By the assumption, we restrict ourselves to a timelike curve $\beta^\gamma = \beta^\gamma(s)$, which corresponds to a moving timelike particle in special relativity. In this theory, the complete coordinate system for any event is defined by

$$(\beta^\gamma) = (ct, x, y, z), \quad \gamma = 0, 1, 2, 3. \tag{1}$$

The distance between two distinct events is computed by

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2, \tag{2}$$

where c is the velocity of light in the vacuum. Thus, for a timelike curve, we have

$$\eta_{\gamma\alpha}\left(\frac{D\beta^\gamma}{ds}\right)\left(\frac{D\beta^\alpha}{ds}\right) = -1, \tag{3}$$

where $\eta_{\gamma\alpha}$ is a metric tensor and s is the arc-length parameter. If we choose components of the unit speed timelike tangent vector as

$$\mathbf{T}^\gamma = \frac{d\beta^\gamma}{ds}, \quad \eta_{bc}\mathbf{T}^b\mathbf{T}^c = \mathbf{T}_\gamma\mathbf{T}^\gamma = -1, \tag{4}$$

then we have following normal and binormal vectors $\underset{\gamma=1,2}{\mathbf{E}}$ defined along the curve. So far it is described three orthogonal vectors along the worldline. However, De-Sitter spacetime is a four-dimensional space. Thus, there must be a fourth vector in addition to the tangent, normal, and binormal vectors for the complete framework construction. Therefore, it is considered that the curve β itself is a vector to establish FS equation system in De-Sitter spacetime. Finally, the main tetrad $(\mathbf{T}^\gamma, \underset{1}{\mathbf{E}}^\gamma, \underset{2}{\mathbf{E}}^\gamma, \underset{3}{\mathbf{E}}^\gamma)$ is defined by FS frame in the following manner.

$$\mathbf{T}_k\underset{\gamma}{\mathbf{E}}^k = 0, \ \underset{\gamma}{\mathbf{E}}_k\underset{\gamma}{\mathbf{E}}^k = 1, \quad \gamma = 1, 2, 3. \tag{5}$$

Here we exchange the timelike curve β with $\underset{3}{\mathbf{E}}^k$ for the simplicity purpose of the notation. This construction also obeys the following FS equation system [18,19].

$$\begin{aligned}
\frac{D\mathbf{T}^\gamma}{ds} &= e_1\underset{1}{\mathbf{E}}^\gamma + \underset{3}{\mathbf{E}}^\gamma, \\
\frac{D\underset{1}{\mathbf{E}}^\gamma}{ds} &= e_1\mathbf{T}^\gamma + e_2\underset{2}{\mathbf{E}}^\gamma, \\
\frac{D\underset{2}{\mathbf{E}}^\gamma}{ds} &= -e_2\underset{1}{\mathbf{E}}^\gamma, \\
\frac{D\underset{3}{\mathbf{E}}^\gamma}{ds} &= \mathbf{T}^\gamma.
\end{aligned} \tag{6}$$

where e_1 is the curvature and e_2 is the torsion of the timelike curve along the worldline.

3. Faraday Antisymmetric Tensor in De-Sitter space

In the previous section, we introduce prerequisite information to determine kinematical properties of the moving particle along the timelike worldline by using FS frame construction in De-Sitter spacetime. If we also assume that the particle is charged by q and it has a mass m, then it produces an electromagnetic field. This field can be seen as the combination of a magnetic field and an electric field. Faraday antisymmetric tensor F_{uv} represents the electromagnetic field of a physical mechanism in the given spacetime. This tensor is written in the complete coordinate system $\beta^\gamma = \beta^\gamma(s)$ and following form of representation is obtained.

$$[F_{uv}] = \begin{bmatrix} 0 & S_x & S_y & S_z \\ -S_x & 0 & -R_z & R_y \\ -S_y & R_z & 0 & -R_x \\ -S_z & -R_y & R_x & 0 \end{bmatrix}, \tag{7}$$

$$[F^{uv}] = \begin{bmatrix} 0 & -S_x & -S_y & -S_z \\ S_x & 0 & -R_z & R_y \\ S_y & R_z & 0 & -R_x \\ S_z & -R_y & R_x & 0 \end{bmatrix}, \tag{8}$$

where $\mathbf{S} = (S_x, S_y, S_z)$ is an electric field 3-vectors and $\mathbf{R} = (R_x, R_y, R_z)$ is a magnetic field 3-vectors.

Dual of the antisymmetric Faraday tensor F_{uv} is computed by considering totally antisymmetric tensor σ of Levi-Civita. Its value depends on the number of permutation $uvab$ of 0123.

i. If it has an even number of permutation, then $\sigma = 1$,

ii. If it has an odd number of permutation, then $\sigma = -1$.

Thus, simple dual of the antisymmetric Faraday tensor F_{uv} is computed by

$$[{}^*F_{uv}] = \frac{1}{2}\sigma_{uvab}F^{ab}, \tag{9}$$

which has the following matrix representation

$$[{}^*F_{uv}] = \begin{bmatrix} 0 & -R_x & -R_y & -R_z \\ R_x & 0 & -S_z & S_y \\ R_y & S_z & 0 & -S_x \\ R_z & -S_y & S_x & 0 \end{bmatrix}. \tag{10}$$

Lorentz invariant of the electromagnetic field is expressed by using the identities given in Eqs. $(7, 8, 10)$ as the following.

$$\begin{aligned} F_{uv}F^{uv} &= 2\left(R^2 - S^2\right), \\ {}^*F_{uv}F^{uv} &= 4\left(\mathbf{S}\cdot\mathbf{R}\right), \end{aligned} \tag{11}$$

where S is the magnitude of the electric field and R is the magnitude of the magnetic field.

Now, we adapt given FS frame construction of the De-Sitter spacetime to investigate the motion of the accelerated charged particle within the context of general and special relativity in that space. This leads us to determine intrinsic geometric features of the trajectory of the moving charged particle in the electromagnetic field, ultimately. The first step in that process is to observe the behavior of the F_{uv} in the main tetrad.

We firstly write F^{uv} in terms of its bases, which are the class of antisymmetric tensors at each point of the timelike curve.

$$\begin{aligned} \underset{\gamma}{C}^{uv} &= \mathbf{T}^u \underset{\gamma}{\mathbf{E}}^v - \mathbf{T}^v \underset{\gamma}{\mathbf{E}}^u, \quad \gamma = 1, 2, 3, \\ \underset{4}{C}^{uv} &= \underset{1}{\mathbf{E}}^u \underset{2}{\mathbf{E}}^v - \underset{1}{\mathbf{E}}^v \underset{2}{\mathbf{E}}^u, \\ \underset{5}{C}^{uv} &= \underset{1}{\mathbf{E}}^u \underset{3}{\mathbf{E}}^v - \underset{1}{\mathbf{E}}^v \underset{3}{\mathbf{E}}^u, \\ \underset{6}{C}^{uv} &= \underset{2}{\mathbf{E}}^u \underset{3}{\mathbf{E}}^v - \underset{2}{\mathbf{E}}^v \underset{3}{\mathbf{E}}^u. \end{aligned} \tag{12}$$

Thus, we have

$$F^{uv} = \sum_{\gamma=1}^{6} b_\gamma \underset{\gamma}{C}^{uv}, \tag{13}$$

where b_γ are some sufficiently smooth functions along the worldline of the timelike curve. Using simple algebraic properties and FS frame construction given in Eqs. (4-6) we reach following equalities.

$$\begin{aligned} \underset{\gamma}{C}_{uv}\underset{\gamma}{C}^{uv} &= -2, \quad \text{when} \quad \gamma = 1, 2, 3, \\ \underset{\gamma}{C}_{uv}\underset{\gamma}{C}^{uv} &= 2, \quad \text{when} \quad \gamma = 4, 5, 6, \\ \underset{\gamma}{C}_{uv}\underset{\alpha}{C}^{uv} &= 0, \quad \text{when} \quad \gamma \neq \alpha. \end{aligned} \tag{14}$$

Further, we are allowed to write

$$F_{uv}F^{uv} = \sum_{\gamma=1}^{6} b_\gamma \underset{\gamma}{C}_{uv} \sum_{\gamma=1}^{6} b_\gamma \underset{\gamma}{C}^{uv}.$$

If we also use Eqs. $(12, 14)$, the first part of the Eq. (11) is induced to

$$\left(R^2 - S^2\right) = \left(-b_1^2 - b_2^2 - b_3^2 + b_4^2 + b_5^2 + b_6^2\right). \tag{15}$$

With the aid of the Eq. (9) we can also define the dual of each $\underset{\gamma=1,\ldots,6}{C_{uv}}$ in the following manner.

$$\underset{\gamma=1,\ldots,6}{{}^*C_{uv}} = \frac{1}{2}\sigma_{uvab}\underset{\gamma=1,\ldots,6}{C^{ab}}. \tag{16}$$

Here we assume that the FS tetrad $(\mathbf{T}^\gamma, \underset{1}{\mathbf{E}}^\gamma, \underset{2}{\mathbf{E}}^\gamma, \underset{3}{\mathbf{E}}^\gamma)$ is positively oriented when

$$\sigma_{uvab}\mathbf{T}^u \underset{1}{\mathbf{E}}^v \underset{2}{\mathbf{E}}^a \underset{3}{\mathbf{E}}^b = 1. \tag{17}$$

Hence, we obtain that

$${}^*F_{uv}F^{uv} = \sum_{\gamma=1}^{6} b_\gamma {}^*\underset{\gamma}{C}_{uv} \sum_{\gamma=1}^{6} b_\gamma \underset{\gamma}{C}^{uv}. \tag{18}$$

To expand the Eq. (18) we firsly need to give the following equalities, which are obtained by Eqs. $(16, 17)$.

$$\begin{aligned} {}^*\underset{\gamma}{C}_{uv}\underset{\gamma}{C}^{uv} &= 0, \quad \text{when} \quad \gamma = 1, \ldots 6, \\ {}^*\underset{1}{C}_{uv}\underset{6}{C}^{uv} &= -{}^*\underset{2}{C}_{uv}\underset{5}{C}^{uv} = {}^*\underset{3}{C}_{uv}\underset{4}{C}^{uv} = 2, \\ {}^*\underset{\gamma}{C}_{uv}\underset{\alpha}{C}^{uv} &= 0 \quad \text{for other cases.} \end{aligned} \tag{19}$$

Now, if we follow Eqs. $(18, 19)$, the second part of the Eq. (11) is induced to

$$(\mathbf{S}\cdot\mathbf{R}) = b_1 b_6 - b_2 b_5 + b_3 b_4. \tag{20}$$

As a consequence, Eqs. (15,20) give the invariant of the electromagnetic field in terms of arbitrary functions defined along the worldline of the curve. To observe the behavior of the F^{uv} along the timelike worldline we also need to compute

$$\frac{DF^{uv}}{ds} = \sum_{\gamma=1}^{6} \frac{D}{ds}(b_\gamma \underset{\gamma}{C}^{uv}). \tag{21}$$

By using Eqs. (6,12) differential properties of the tensor $\underset{\gamma=1,\ldots,6}{C^{uv}}$ is calculated as

$$\begin{aligned}
&\frac{D}{ds}\underset{1}{C}^{uv} = e_2 \underset{2}{C}^{uv} - \underset{5}{C}^{uv},\\
&\frac{D}{ds}\underset{2}{C}^{uv} = e_1 \underset{4}{C}^{uv} - \underset{6}{C}^{uv} - e_2 \underset{1}{C}^{uv},\\
&\frac{D}{ds}\underset{3}{C}^{uv} = e_1 \underset{5}{C}^{uv}, \frac{D}{ds}\underset{4}{C}^{uv} = e_1 \underset{2}{C}^{uv},\\
&\frac{D}{ds}\underset{5}{C}^{uv} = e_1 \underset{3}{C}^{uv} + e_2 \underset{6}{C}^{uv} - \underset{1}{C}^{uv},\\
&\frac{D}{ds}\underset{6}{C}^{uv} = -e_2 \underset{5}{C}^{uv} - \underset{2}{C}^{uv}.
\end{aligned} \tag{22}$$

Thus, the evolution of F^{uv} on the timelike worldline is stated by using Eqs. $(21, 22)$ as

$$\begin{aligned}
\frac{DF^{uv}}{ds} =& \left(\frac{Db_1}{ds} - b_2 e_2 - b_5\right) \underset{1}{C}^{uv}\\
&+ \left(\frac{Db_2}{ds} + b_1 e_2 + b_4 e_1 - b_6\right) \underset{2}{C}^{uv}\\
&+ \left(\frac{Db_3}{ds} + b_5 e_1\right) \underset{3}{C}^{uv} + \left(\frac{Db_4}{ds} + b_2 e_1\right) \underset{4}{C}^{uv}\\
&+ \left(\frac{Db_5}{ds} - b_1 + b_3 e_1 - b_6 e_2\right) \underset{5}{C}^{uv}\\
&+ \left(\frac{Db_6}{ds} - b_2 + b_5 e_2\right) \underset{6}{C}^{uv}.
\end{aligned} \tag{23}$$

4. Lorentz equation of homogeneous electromagnetic field in De-Sitter space

Lorentz equation is effectively used in an electromagnetic field to govern the motion of a charged particle having a positive mass m and charge q. It is described by

$$\frac{D\mathbf{T}^\gamma}{ds} = tF^{\gamma\alpha}\mathbf{T}_\alpha, \quad \text{where} \quad t = \frac{q}{mc^2}. \tag{24}$$

If Faraday electromagnetic tensor is constant and uniform, we can investigate the relation between FS scalars and the invariants of the electromagnetic field. If we consider the FS formalism given by the Eq. (6), then we can write the above statement as

$$e_1 \underset{1}{\mathbf{E}}^\gamma + \underset{3}{\mathbf{E}}^\gamma = tF^{\gamma\alpha}\mathbf{T}_\alpha. \tag{25}$$

If we also use the Eq. (13), we obtain that

$$e_1 \underset{1}{\mathbf{E}}^\gamma + \underset{3}{\mathbf{E}}^\gamma = t\sum_{u=1}^{6} b_u \underset{u}{C}^{\gamma\alpha}\mathbf{T}_\alpha. \tag{26}$$

By using the above equality and the Eq. (13), we have

$$b_1 = \frac{e_1}{t}, \quad b_2 = 0, \quad b_3 = \frac{1}{t}. \tag{27}$$

By the assumption of constancy and uniformity of the Faraday tensor, we know that each statement in the parenthesis of the Eq. (23), must be equal to zero. Using this fact together with the equalities given by the Eq. (27), we find that

$$\frac{Db_4}{ds} = b_5 = b_6 = 0. \tag{28}$$

We improve another formula to demonstrate the invariants given from Eqs. (11,15,20), by using the FS scalars. If we plug each component found in Eqs. (27,28), into Eqs. (15,20), then we get that

$$\left(R^2 - S^2\right) = b_4^2 - \frac{1}{t^2}(1 + e_1^2), \tag{29}$$

$$(\mathbf{S}\cdot\mathbf{R}) = \frac{b_4}{t}, \tag{30}$$

where b_4 is constant and e_1 is the curvature of the worldline.

Corollary 1. The worldline of the moving charged particle in the homogeneous electromagnetic field is a circular helix in De-Sitter spacetime.

Proof. From Eqs. (23,27,28) we have

$$\frac{De_1}{ds} = \frac{De_2}{ds} = 0.$$

The rest is evident from the Lancret theorem [20].

Corollary 2. If each FS vector satisfies the Lorentz equation in the homogeneous electromagnetic field then the FS scalars are written in terms of $\mathbf{F}^{\gamma\alpha}$ as follows.

$$e_1 = \pm\, i(1 + t^2(F^2)^{\gamma\alpha}\mathbf{T}_\gamma\mathbf{T}_\alpha)^{\frac{1}{2}}, \tag{31}$$

$$e_2 = \pm\, it((F^2)^{\gamma\alpha}\underset{2}{\mathbf{E}}_\gamma\underset{2}{\mathbf{E}}_\alpha)^{\frac{1}{2}}, \tag{32}$$

where $i^2 = -1$.

Proof. We should first remind that for a given arbitrary vector $\mathcal{A}^\gamma$, if we define $\mathcal{B}_\gamma = (F^n)_{\gamma\alpha}\mathcal{A}^\alpha$, then we have

$$\mathcal{B}_\gamma\mathcal{B}^\gamma = (-1)^n(F^{2n})_{\gamma\alpha}\mathcal{A}^\gamma\mathcal{A}^\alpha, \tag{33}$$

where $(F^n)^\gamma_{\ \alpha} = F^\gamma_{\ u}F^u_{\ v}...F^v_{\ \alpha}$. By the assumption

$$e_1\underset{1}{\mathbf{E}}^\gamma + \underset{3}{\mathbf{E}}^\gamma = tF^{\gamma\alpha}\mathbf{T}_\alpha, \quad t = \frac{q}{mc^2}.$$

If we square both sides of the above statement by using the Eq. (33) ; then it leads to

$$e_1^2 + 1 = -t^2(F^2)^{\gamma\alpha}\mathbf{T}_\gamma\mathbf{T}_\alpha.$$

This implies that

$$e_1^2 = -1 - t^2(F^2)^{\gamma\alpha}\mathbf{T}_\gamma\mathbf{T}_\alpha.$$

Since Lorentz equation is assumed to satisfy each FS vector, we are allowed to write

$$\frac{D\underset{2}{\mathbf{E}}^{\gamma}}{ds} = tF^{\gamma\alpha}\underset{2}{\mathbf{E}}_{\alpha}.$$

Again using the Eq. (6) we observe that

$$-e_2\underset{1}{\mathbf{E}}^{\gamma} = tF^{\gamma\alpha}\underset{2}{\mathbf{E}}_{\alpha}.$$

Squaring both sides of the statement we find that

$$e_2^2 = -t^2(F^2)^{\gamma\alpha}\underset{2}{\mathbf{E}}_{\gamma}\underset{2}{\mathbf{E}}_{\alpha}.$$

Considering the similar argument it can also be obtained that

$$1 = t^2(F^2)^{\gamma\alpha}\underset{3}{\mathbf{E}}_{\gamma}\underset{3}{\mathbf{E}}_{\alpha}. \tag{34}$$

Corollary 3. The invariant of the homogeneous electromagnetic field is defined by the antisymmetric Faraday tensor and FS vectors as the following.

$$\left(R^2 - S^2\right) = b_4^2 - (F^2)^{\gamma\alpha}\underset{3}{\mathbf{E}}_{\gamma}\underset{3}{\mathbf{E}}_{\alpha}(-t^2(F^2)^{\gamma\alpha}\mathbf{T}_{\gamma}\mathbf{T}_{\alpha}),$$

$$(\mathbf{S}\cdot\mathbf{R}) = \pm b_4((F^2)^{\gamma\alpha}\underset{3}{\mathbf{E}}_{\gamma}\underset{3}{\mathbf{E}}_{\alpha})^{\frac{1}{2}}.$$

Proof. It is evident if we follow the Eqs. (29,30) first and then the Eqs. (31,32,34) latter.

5. Conclusion

In the present paper, we introduce a geometric approach to investigate the motion of a timelike relativistic charged particle subjected to a homogeneous electromagnetic field in De-Sitter space. Aside from the geometric characterization of the timelike wordline of the charged particle in De-Sitter space we also correlate the intrinsic scalars of the worldline of the charged particle and field invariants of the electromagnetic field in De-Sitter space.

This study will also lead up to further research on the investigation of the dynamics of the moving charged particles when they are experienced some well-known external forces beside the electromagnetic field *i.e.* the frictional force, the gravitational force, the normal force, and the resultant force in De-Sitter space. Consequently, we aim to obtain more applicable and widely acceptable results to comprehend the exact movement of the charged particle in a given homogeneous electromagnetic field in De-Sitter space when the motion is governed by the Lorentz equation.

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