Mathematical modeling of DNA vibrational dynamics and its solitary wave solutions

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In this work, the traveling wave solutions of a mathematical modeling of DNA vibration dynamics proposed by Peyrard-Bishop, that takes into consideration the inclusion of nonlinear interaction between adjacent displacements along the Hydrogen bonds, is investigated by both (G'/G)-expansion and F-expansion methods. Using these methods, some new explicit forms of traveling wave solutions of present nonlinear equation are given. The methods come in to be easier and faster by means of a symbolic computation and yield powerful mathematical tools for solving nonlinear evolution equations in many branches of sciences, especially Physics, Biology, etc.

Keywords: (G'/G)–expansion method; F–expansion method; generalized DNA vibration dynamics; hyperbolic function solutions; trigonometric function solutions; solitary wave solutions.

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1. Introduction

In recent decades, a new attitude with regard to the exploration of nonlinear evolution equations (NLEEs) has been actively progressing in various branches of Sciences. Nonlinear evolution equations have been the important subject of investigation in various branches of mathematical and physical sciences such as physics, fluid mechanics, chemistry, biology and etc. Obtaining the analytical solutions of NLEEs is an important topic in various areas of Science, since many of mathematical and physical models are explained by NLEEs. Among the possible solutions to NLEEs, specific form of solutions may contingent only on a single combination of variables such as solitons. In mathematics and physics, a soliton is a self-reinforcing solitary wave that sustains its form through it travels with constant speed. Solitons are the special solutions of certain nonlinear partial differential equations, with interesting properties. Because of a balance between nonlinear and linear effects, the shape of soliton wave pulses does not change during propagation in a medium. The soliton phenomenon firstly described by John Scott Russell (1808-1882). The main idea of this phenomenon occurred to him when he observed a solitary wave in the Union Canal in Scotland. Russell reproduced same phenomenon in a wave tank and named it the "Wave of Translation" (also known as solitary wave or soliton) [1].

The mathematical modeling of DNA vibration dynamics is proposed by Peyrard–Bishop, that takes into consideration the inclusion of nonlinear interaction between adjacent displacements along the Hydrogen bonds [8]. To study the main important properties of DNA structure, we have investigated the oscillator-chain of Peyrard–Bishop model (PB) [8] which has successfully predicated the appearance of solitonic structures. As it is well known, the balance between weak nonlinearity and dispersion in DNA dynamic model yields the typical derivation of model equations mathematically. This dispersion in many cases enters only in the linear level. Unfortunately, this approach eliminates other possibilities related to the presence of nonlinear dispersion [9,10]. Some mathematical structure of DNA has been described in various interesting lines of research, to see more structure and physical properties of DNA, the authors can Ref. to [2-6] and their references. The mathematical and physical modelling of DNA dynamics has reduced to an important nonlinear structures. The nonlinearity of the DNA dynamic model causes it to form localized waves. The localized waves have some interesting properties, such as the ability of transporting energy without dissipation [2-7].

Taking into consideration the inharmonic potential, Aguero *et al*. [6] and Najera *et al*. [11] have studied the following modified PB model:

$$u_{tt} - \left(\ell_1 + 3\ell_2 u_x^2\right) u_{xx} - 2\alpha\mathcal{D}e^{-\alpha u}(e^{-\alpha u} - 1) = 0, \quad (1)$$

where $\ell_1, \ell_2, \mathcal{D}$ and α are constants. Krumhansl *et al.*, [12], suggested a theory of soliton excitations as an explanation of the open states of DNA modeling system. In [12], they firstly developed the possibility that nonlinear effects might concentrate vibrational energy in DNA into localized soliton like objects. Some other similar worksare studied about solitons of DNA dynamics model, such as Yomosa in [13], proposed a soliton theory using a plane base-rotor model. This model was further refined by Takeno and Homma [14], who allowed discreteness effects to be taken into account, and by Zhang [15], who improved the model for base coupling. Zayed and Arnous also applied Homogeneous Balance Method in [35] and generalized Riccati equation mapping method in [36,37] to find traveling wave solutions of PB model (1.1).

In the recent decade, many powerful and direct methods have been proposed to find special solutions of nonlinear evolution equations (NLEEs), such as the Bácklund transformation [**?**] and Hirota bilinear method [**?**]. With the help of the computer implementations, some other algebraic method proposed, such as tanh/coth method [**?**], homogeneous balance method [**?**], the Miura transformation [**?**], sine/cosine method [**?**] and Exp–function method [**?**] and some other, see [**?**]. But most of the methods may sometimes fail or can only lead to a kind of special solution and the solution procedures become very complex as the degree of nonlinearity increases.

Since 2008, the (G'/G)–expansion method, firstly introduced by Wang *et al*. [**?**], has become widely used to search for various exact solutions of NLEEs [24-31]. The worth of the (G'/G)–expansion method is that it reduced the nonlinear PDEs to ODEs by some essentially linear methods, *i.e*. the method is based on the explicit linearization of NLEEs for traveling waves which leads to a second–order differential equation with constant coefficients. The F-expansion method is also an effective and direct algebraic method for constructing the exact solutions of nonlinear evolution equations [**?**]. To do this, it reduces the nonlinear evolution equations to a simple algebraic equation. Many of NLEEs have been solved by F-expansion method, (for a deeper discussion we refer the reader to [33,34] and the references given there).

In this paper, we first describe briefly the mathematical concept of both (G'/G)–expansion method and F-expansion method, then we use them to investigate the traveling wave solutions of a mathematical modeling of PB model (1.1).

2. Description of the (G'/G)–expansion method

In this section, we introduce briefly the (G'/G)–expansion method for solving certain nonlinear partial differential equations (PDEs). For a deeper discussion of (G'/G)–expansion method we refer the reader to [24-27]. Suppose we have a nonlinear PDE for $u(x,t)$, in the form

$$P(u, u_t, u_x, u_{xt}, u_{xx}, u_{tt}, \ldots) = 0, \tag{2}$$

where P is a polynomial in its arguments, which includes nonlinear terms and the highest order derivatives. Using the transformation $u(x,t) = U(\xi), \xi = x - \omega t$, Eq. (**??**) reduces to the ordinary differential equation (ODE)

$$P(U, -\omega U', U', -\omega U'', U'', \omega^2 U'', \ldots) = 0, \tag{3}$$

where $U = U(\xi)$, and prime denotes derivative with respect to ξ. We assume that the solution of Eq. (**??**) can be expressed by a polynomial in (G'/G) as follows:

$$U(\xi) = \sum_{n=1}^{m} \alpha_n \left(\frac{G'}{G}\right)^n + \alpha_0, \quad \alpha_m \neq 0. \tag{4}$$

where α_n, $n = 0, 1, 2, ..., m$, are constants to be determined later and $G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$\frac{d^2G(\xi)}{d\xi^2} + \lambda \frac{dG(\xi)}{d\xi} + \mu G(\xi) = 0. \tag{5}$$

where λ and μ are arbitrary constants. Using the general solutions of Eq. (**??**), we have

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\dfrac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right) - \frac{\lambda}{2}, & \lambda^2 - 4\mu > 0, \\ \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\dfrac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right) - \frac{\lambda}{2}, & \lambda^2 - 4\mu < 0, \end{cases} \tag{6}$$

and it follows, from (**??**) and (**??**), that

$$U' = -\sum_{n=1}^{m} n\,\alpha_n \left(\left(\frac{G'}{G}\right)^{n+1} + \lambda\left(\frac{G'}{G}\right)^n + \mu\left(\frac{G'}{G}\right)^{n-1}\right),$$

$$U'' = \sum_{n=1}^{m} n\,\alpha_n \left((n+1)\left(\frac{G'}{G}\right)^{n+2} + (2n+1)\lambda\left(\frac{G'}{G}\right)^{n+1} + n(\lambda^2+2\mu)\left(\frac{G'}{G}\right)^{n} + (2n-1)\lambda\mu\left(\frac{G'}{G}\right)^{n-1} + (n-1)\mu^2\left(\frac{G'}{G}\right)^{n-2} \right), \tag{7}$$

and so on, here the prime denotes the derivative with respective to ξ. To determine u explicitly, we take the following four steps:

Step 1. Determine the integer m by substituting Eq. (**??**) along with Eq. (**??**) into Eq. (**??**), and balancing the highest order nonlinear term(s) and the highest order partial derivative.

Step 2. Substitute Eq. (**??**) give the value of m determined in *Step 1*, along with Eq. (**??**) into Eq. (**??**) and collect all terms with the same order of (G'/G) together, the left-hand side of Eq. (**??**) is converted into a polynomial in (G'/G). Then set each coefficient of this polynomial to zero to derive a set of algebraic equations for $\omega, \lambda, \mu, \alpha_n$ for $n = 0, 1, 2, ..., m$.

Step 3. Solve the system of algebraic equations obtained in *Step 2*, for $\omega, \lambda, \mu, \alpha_0, ..., \alpha_m$ by use of computer programs such Matlab, Maple and Mathematica.

Step 4. Use the results obtained in above steps to derive a series of fundamental solutions $u(\xi)$ of Eq. (**??**) depending on (G'/G), since the solutions of Eq. (**??**) have been well known for us, then we can obtain exact solutions of Eq. (**??**).

2.1. Application

Let us consider the Peyrard–Bishop DNA dynamic model equation

$$u_{tt} - (\ell_1 + 3\ell_2 u_x^2)u_{xx} - 2\alpha\mathcal{D}e^{-\alpha u}(e^{-\alpha u} - 1) = 0, \tag{8}$$

where ℓ_1, ℓ_2, α and $\mathcal{D}$ are constants.

We would like to use our method to obtain new and more general, travelling wave solutions of Eq. (**??**) by assuming the solution in the following frame:

$$u = U(\xi), \quad \xi = x - \omega t, \tag{9}$$

where ω is arbitrary constants generally termed the *wave velocity*, and prime denotes derivative with respect to ξ. Using the wave variable ξ in (**??**) we find

$$\omega^2 U'' - \left(\ell_1 + 3\ell_2 (U')^2\right) U'' - 2\alpha D e^{-\alpha U}(e^{-\alpha U} - 1) = 0, \tag{10}$$

Multiplying Eq. (**??**) by U' and integrating once with respect to ξ, we obtain

$$\frac{1}{2}(\omega^2 - \ell_1)(U')^2 - \frac{3}{4}\ell_2 (U')^4 + \mathcal{D}e^{-\alpha U}(e^{-\alpha U} - 2) + \mathcal{R} = 0, \tag{11}$$

where $\mathcal{R}$ is an integration constant. Introducing the new variable

$$\phi(\xi) = e^{-\alpha U(\xi)}, \tag{12}$$

reduces Eq. (**??**) into the following form:

$$\frac{1}{2\alpha^2}(\omega^2-\ell_1)\phi^2(\phi')^2-\frac{3}{4\alpha^4}\ell_2(\phi')^4 + \mathcal{D}\phi^5(\phi-2)+\mathcal{R}\phi^4=0. \tag{13}$$

According to *Step 1*, considering the homogeneous balance between $(\phi')^4$ and ϕ^6 we get $4m+4=6m$, hence $m=2$. We then suppose that Eq. (**??**) has the following formal solutions:

$$\phi=\alpha_2(G'/G)^2+\alpha_1(G'/G)+\alpha_0, \tag{14}$$

where $\alpha_2\neq 0, \alpha_1$, and α_0, are constants which are unknown, to be determined later.

Substituting Eq. (**??**) along with Eq. (**??**) into Eq. (**??**) and collecting all terms with the same order of (G'/G), together, the left–hand sides of Eq. (**??**) are converted into a polynomial in (G'/G). Equating each coefficient of this polynomial to zero yields a set of simultaneous algebraic equations for $\lambda, \mu, \omega, \alpha_0, \alpha_1$ and α_2. Solving the system of algebraic equations with the aid of Maple 12, we obtain the following general results

$$\begin{aligned}
&\mu=\pm\frac{\sqrt{3}\alpha^2}{6\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\pm\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}\right),\\
&\lambda=0,\\
&\omega=\pm\sqrt{\mp 2\sqrt{3}\sqrt{\frac{\ell_2}{\mathcal{D}}}\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}+\ell_1},\\
&\alpha_0=\pm\frac{\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}}{\mathcal{D}},\\
&\alpha_1=0,\\
&\alpha_2=\pm\frac{2\sqrt{3}}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}.
\end{aligned} \tag{15}$$

Therefore, substitute the obtained results (**??**) in (**??**), we get

$$\phi(\xi)=\pm\frac{2\sqrt{3}}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{G'}{G}\right)^2_{\pm}\pm\frac{\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}}{\mathcal{D}}. \tag{16}$$

Substituting the general solutions (**??**) into Eq. (**??**), we obtain following two types of traveling wave solutions of Eq. (**??**):

2.1.1. *Hyperbolic type function solutions*

When $\mu<0$, using the relationship $u=(-1/\alpha)\ln\phi$, we obtain hyperbolic type function solution $u_{\mathcal{H}}$, of Peyrard-Bishop DNA dynamic model (**??**) as follows:

$$u_{\pm\mathcal{H}}(x,t)=\frac{-1}{\alpha}\ln\left(\pm\frac{2\sqrt{3}}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{G'}{G}\right)^2_{\pm}\pm\frac{\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}}{\mathcal{D}}\right), \tag{17}$$

where

$$\left(\frac{G'}{G}\right)_{\pm}=\frac{\sqrt{-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{-4\mu}}{2}\,\xi\right)+C_2\cosh\left(\frac{\sqrt{-4\mu}}{2}\,\xi\right)}{C_1\cosh\left(\frac{\sqrt{-4\mu}}{2}\,\xi\right)+C_2\sinh\left(\frac{\sqrt{-4\mu}}{2}\,\xi\right)}\right),$$

and

$$\xi=x\mp\left(\sqrt{\mp 2\sqrt{3}\sqrt{\frac{\ell_2}{\mathcal{D}}}\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}+\ell_1}\right)t,\quad \mu=\pm\frac{\sqrt{3}\alpha^2}{6\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\pm\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}\right),\quad C_1,\quad C_2$$

are arbitrary constants and $\mathcal{R}$ is integration constant. It is easy to see that the hyperbolic type solution (**??**) can be rewritten at $C_1^2>C_2^2$, as follows

$$u_{\pm\mathcal{H}1}(x,t)=\frac{-1}{\alpha}\ln\left(\frac{\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}}{\mathcal{D}\cosh^2\left(-\frac{\sqrt{6}}{6}\sqrt{-\frac{\sqrt{3}\alpha^2}{\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}(\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}})}\xi+\eta_{\mathcal{H}}\right)}\right), \tag{18a}$$

while at $C_1^2<C_2^2$, one can obtain

$$u_{\pm\mathcal{H}2}(x,t)=\frac{-1}{\alpha}\ln\left(\frac{-\mathcal{D}-\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}}}{\mathcal{D}\cosh^2\left(-\frac{\sqrt{6}}{6}\sqrt{-\frac{\sqrt{3}\alpha^2}{\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}(\mathcal{D}+\sqrt{\mathcal{D}^2-\mathcal{D}\mathcal{R}})}\xi+\eta_{\mathcal{H}}\right)-1}\right). \tag{18b}$$

where

$$\xi = x \mp \sqrt{\mp 2\sqrt{3}\sqrt{\frac{\ell_2}{\mathcal{D}}}\sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}} + \ell_1)}\, t, \quad \mu = \pm\frac{\sqrt{3}\alpha^2}{6\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\pm\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}}\right), \quad \eta_{\mathcal{H}} = \tanh^{-1}\left(\frac{C_1}{C_2}\right),$$

are arbitrary constants and $\mathcal{R}$ is integration constant. By setting appropriate parameters, the solution (**??**) can be reduce to solution (31) in [**?**].

2.1.2. *Trigonometric type function solutions*

Now when $\mu > 0$, using the relationship $(-1/\alpha)\ln\phi$, the trigonometric type function solution $u_{\mathcal{T}}$, of Peyrard–Bishop DNA dynamic model (**??**) can be obtain as follows:

$$u_{\pm\mathcal{T}}(x,t) = \frac{-1}{\alpha}\ln\left(\pm\frac{2\sqrt{3}}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{G'}{G}\right)_{\pm}^2 \pm \frac{\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}}}{\mathcal{D}}\right), \tag{19}$$

where

$$\left(\frac{G'}{G}\right)_{\pm} = \frac{\sqrt{-4\mu}}{2}\left(\frac{-C_1\sin(\frac{\sqrt{-4\mu}}{2}\,\xi) + C_2\cos(\frac{\sqrt{-4\mu}}{2}\,\xi)}{C_1\cos(\frac{\sqrt{-4\mu}}{2}\,\xi) + C_2\sin(\frac{\sqrt{-4\mu}}{2}\,\xi)}\right),$$

and

$$\xi = x \mp (\sqrt{\mp 2\sqrt{3}\sqrt{\frac{\ell_2}{\mathcal{D}}}\sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}} + \ell_1)}\, t, \quad \mu = \pm\frac{\sqrt{3}\alpha^2}{6\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\pm\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}}\right), \quad C_1, \quad C_2$$

are arbitrary constants and $\mathcal{R}$ is integration constant. It is easy to see that the trigonometric type solution (**??**) can be rewritten at $C_1^2 > C_2^2$, as follows

$$u_{\pm\mathcal{T}1}(x,t) = \frac{-1}{\alpha}\ln\left(\frac{\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}}}{\mathcal{D}}\left(\tan^2\left(\frac{\sqrt{6}}{6}\sqrt{\frac{\sqrt{3}\alpha^2}{\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}(\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}})}\xi + \eta_{\mathcal{T}}\right) + 1\right)\right), \tag{20a}$$

and for $C_1^2 < C_2^2$, one can obtain

$$u_{\pm\mathcal{T}2}(x,t) = \frac{-1}{\alpha}\ln\left(\frac{\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}}}{\mathcal{D}}\left(\cot^2\left(\frac{\sqrt{6}}{6}\sqrt{\frac{\sqrt{3}\alpha^2}{\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}(\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}})}\xi + \eta_{\mathcal{T}}\right) + 1\right)\right). \tag{20b}$$

where

$$\xi = x \mp (\sqrt{\mp 2\sqrt{3}\sqrt{\frac{\ell_2}{\mathcal{D}}}\sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}} + \ell_1)}\, t, \quad \mu = \pm\frac{\sqrt{3}\alpha^2}{6\ell_2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\pm\mathcal{D} + \sqrt{\mathcal{D}^2 - \mathcal{D}\mathcal{R}}\right), \quad \eta_{\mathcal{T}} = \tan^{-1}\left(\frac{C_1}{C_2}\right),$$

are arbitrary constants and $\mathcal{R}$ is integration constant.

3. F-expansion method

In order to get more solutions of (**??**), we use the F-expansion method [**?**] to deal with (**??**). Supposing that Eq. (**??**) has the following formal solutions:

$$\phi = \alpha_2(\varphi)^2 + \alpha_1(\varphi) + \alpha_0, \tag{21}$$

where $\alpha_2 \neq 0, \alpha_1$, and α_0, are constants to be determined further. And φ satisfy

$$\varphi' = C(\varphi)^2 + B(\varphi) + A. \tag{22}$$

Substituting (**??**) and (**??**) into (**??**) and collecting all terms with the same order of φ together equating each coefficient of this polynomial to zero, one can get

Case 1:

$$R=-4\,AC\left(-\alpha^2\,(D)\sqrt{\frac{l_2}{D}}\sqrt{3}+3\,ACl_2\right)\alpha^{-4},$$

$$\alpha=\alpha,\quad \omega=\omega,\quad \alpha_0=2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2},$$

$$l_1=\left(12\,ACl_2-2\,\alpha^2\,(D)\sqrt{\frac{l_2}{D}}\sqrt{3}+\alpha^2\omega\right)\alpha^{-2},$$

$$l_2=l_2,\quad A=A,\quad C=C,\quad D=D,\quad B=0,$$

$$\alpha_1=0,\quad \alpha_2=\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}. \tag{23}$$

Case 2:

$$R=-4\,AC\left(\alpha^2\,(D)\sqrt{\frac{l_2}{D}}\sqrt{3}+3\,ACl_2\right)\alpha^{-4},$$

$$\alpha=\alpha,\quad \omega=\omega,\quad \alpha_0=-2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2},$$

$$l_1=\left(12\,ACl_2+2\,\alpha^2\,(D)\sqrt{\frac{l_2}{D}}\sqrt{3}+\alpha^2\omega\right)\alpha^{-2},$$

$$l_2=l_2,\quad A=A,\quad C=C,\quad D=D,\quad B=0,$$

$$\alpha_1=0,\quad \alpha_2=-\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}. \tag{24}$$

Thus, substitute the obtained results (**??**) and (**??**) in (**??**), we get

$$\phi(\xi)=\pm\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\,(\varphi)^2\pm 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}. \tag{25}$$

Substituting the general solutions (**??**) into Eq. (**??**), we obtain the following two types of traveling wave solutions of Eq. (**??**):

In view of (**??**) has a lot of fundamental solutions (twenty seven solutions) [**?**], one can find a number of exact travelling wave solutions for (**??**), which are listed some special solutions as follows.

Family 1: When $-4AC>0$ and $AC\neq 0$,

$$u=-\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left(-\frac{1}{2C}\left[\sqrt{-4AC}\tanh\left(\frac{\sqrt{-4AC}}{2}\xi\right)\right]\right)\right)^2+2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{26}$$

$$u=-\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left(-\frac{1}{2C}\left[\sqrt{-4AC}\coth\left(\frac{\sqrt{-4AC}}{2}\xi\right)\right]\right)\right)^2+2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{27}$$

$$u=-\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(-\frac{1}{2C}\left[\sqrt{-4AC}\left(\tanh\left(\sqrt{-4AC}\xi\right)\pm i\mathrm{sech}\left(\sqrt{-4AC}\xi\right)\right)\right]\right)^2+2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{28}$$

$$u=-\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(-\frac{1}{2C}\sqrt{-4AC}\left(\coth\left(\sqrt{-4AC}\xi\right)\pm i\mathrm{csch}\left(\sqrt{-4AC}\xi\right)\right)\right)^2+2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{29}$$

$$u=-\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(-\frac{1}{4C}\sqrt{-4AC}\left(\tanh\left(\frac{\sqrt{-4AC}}{4}\xi\right)+\coth\left(\frac{\sqrt{-4AC}}{4}\xi\right)\right)\right)^2+2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{30}$$

$$u=-\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{2C}\left[\frac{\sqrt{(E^2+F^2)(-4AC)}-E\sqrt{-4AC}\cosh(\sqrt{-4AC}\xi)}{E\sinh(\sqrt{-4AC}\xi)+F}\right]\right)^2+2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{31}$$

$$u=-\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{2C}\left[-\frac{\sqrt{(F^2-E^2)(-4AC)}+E\sqrt{-4AC}\sinh(\sqrt{-4AC}\xi)}{E\cosh(\sqrt{-4AC}\xi)+F}\right]\right)^2+2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{32}$$

where E and F are two non-zero real constants and satisfies $F^2 - E^2 > 0$.

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left[\frac{2A\cosh(\sqrt{-4AC}\xi)}{\sqrt{-4AC}\sinh(\sqrt{-4AC}\xi) \pm i\sqrt{-4AC}\xi}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{33}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left[\frac{2A\sinh(\sqrt{-4AC}\xi)}{\sqrt{-4AC}\cosh(\sqrt{-4AC}\xi) \pm \sqrt{-4AC}\xi}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{34}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left[\frac{4A\sinh(\frac{\sqrt{-4AC}}{4}\xi)\cosh(\frac{\sqrt{-4AC}}{4}\xi)}{2\sqrt{-4AC}\cosh^2(\frac{\sqrt{-4AC}}{4}\xi) - \sqrt{-4AC}}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{35}$$

Family 2 : When $-4AC < 0$ and $AC \neq 0$,

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{2C}\left[\sqrt{4AC}\tan\left(\frac{\sqrt{4AC}}{2}\xi\right)\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{36}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{2C}\left[\sqrt{4AC}\cot\left(\frac{\sqrt{4AC}}{2}\xi\right)\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{37}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{2C}\left[\sqrt{4AC}\left(\tan\left(\sqrt{4AC}\xi\right) \pm \sec\left(\sqrt{4AC}\xi\right)\right)\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{38}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(-\frac{1}{2C}\left[\sqrt{4AC}\left(\cot\left(\sqrt{4AC}\xi\right) \pm \csc\left(\sqrt{4AC}\xi\right)\right)\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{39}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{4C}\left[\sqrt{4AC}\left(\tan\left(\frac{\sqrt{4AC}}{4}\xi\right) - \cot\left(\frac{\sqrt{4AC}}{4}\xi\right)\right)\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{40}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{2C}\left[\frac{\pm\sqrt{(F^2-E^2)(4AC)} - E\sqrt{4AC}\cos(\sqrt{4AC}\xi)}{E\sin(\sqrt{4AC}\xi) + F}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{41}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\frac{1}{2C}\left[\frac{\pm\sqrt{(F^2-E^2)(4AC)} + E\sqrt{4AC}\sin(\sqrt{4AC}\xi)}{E\cos(\sqrt{4AC}\xi) + F}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{42}$$

where E and F are two non-zero real constants and satisfies $F^2 - E^2 > 0$.

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left[\frac{-2A\cos(\sqrt{4AC}\xi)}{\sqrt{4AC}\sin(\sqrt{4AC}\xi) \pm i\sqrt{4AC}\xi}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{43}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left[\frac{2A\sin(\sqrt{4AC}\xi)}{\sqrt{4AC}\cos(\sqrt{4AC}\xi) \pm \sqrt{4AC}\xi}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{44}$$

$$u = -\frac{1}{\alpha}\ln\left(\frac{2\sqrt{3}C^2}{\alpha^2}\sqrt{\frac{\ell_2}{\mathcal{D}}}\left(\left[\frac{4A\sin(\frac{\sqrt{4AC}}{4}\xi)\cos(\frac{\sqrt{4AC}}{4}\xi)}{2\sqrt{4AC}\cos^2(\frac{\sqrt{4AC}}{4}\xi) - \sqrt{4AC}}\right]\right)^2 + 2\sqrt{\frac{l_2}{D}}AC\sqrt{3}\alpha^{-2}\right). \tag{45}$$

Remark 1. Using Case. 2, one can get other exact solutions of Eq. (**??**). The details are omitted here.

4. Conclusions

This study shows that both (G'/G)–expansion method and F–expansion method are quite efficient and practically well suited for use in finding exact solutions for a mathematical modeling of DNA vibration dynamics. The reliability of the methods and the reduction in the size of computational domain give these methods a wider applicability. With the aid of Maple 12, we have assured the correctness of the obtained solutions by putting them back into the original equation. We hope that they will be useful for further studies in applied sciences and engineering.

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