The Feynman-Dyson propagators for neutral particles (locality or non-locality?)*

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An analog of the $S = 1/2$ Feynman-Dyson propagator is presented in the framework of the $S = 1$ Weinberg's theory. The basis for this construction is the concept of the Weinberg field as a system of four field functions differing by parity and by dual transformations. Next, we analyze the recent controversy in the definitions of the Feynman-Dyson propagator for the field operator containing the $S = 1/2$ self/anti-self charge conjugate states in the papers by D. Ahluwalia *et al*. [11] and by W. Rodrigues Jr. *et al* [18, 19]. The solution to this mathematical controversy is obvious. It is related to the necessary doubling of the Fock Space (as in the Barut and Ziino works), thus extending the corresponding Clifford Algebra. However, the logical interrelations of different mathematical foundations with physical interpretations are not so obvious. We present some insights with respect to.

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1. The Weinberg propagators

We study the problem of construction of causal propagators in both higher-spin theories and the spin $S = 1/2$ Majorana-like theory. The hypothesis is: in order to construct the analogues of the Feynman-Dyson propagator we need actually four field operators connected by the dual and parity transformation. We use the standard methods of quantum field theory. So, the number of components in the causal propagators is enlarged accordingly. The conclusions are listed in the last Section: if we would not enlarge the number of components in the fields (in the propagator) we would not be able to obtain the causal propagator.

Accordingly to the Feynman-Dyson-Stueckelberg ideas, a causal propagator S_F has to be constructed by using the formula (*e.g.*, Ref. [1, p.91])

$$S_F(x_2, x_1) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{m}{E_p} \times \left[\theta(t_2 - t_1)\, a\; u^\sigma(p)\overline{u}^\sigma(p) e^{-ip\cdot x} + \theta(t_1 - t_2)\, b\; v^\sigma(p)\overline{v}^\sigma(p) e^{ip\cdot x}\right], \qquad (1)$$

where $x = x_2 - x_1$, m is the particle mass, $\hat{p} = p^\mu\gamma_\mu$, $p^\mu = (E_p, \vec{p})$, u^σ, v^σ are the 4-spinors, $\theta(t)$ is the Heaviside function. In the spin $S = 1/2$ Dirac theory, it results in

$$S_F(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \frac{\hat{p} + m}{p^2 - m^2 + i\epsilon}, \qquad (2)$$

provided that the constants a and b are determined by imposing

$$(i\hat{\partial}_2 - m) S_F(x_2, x_1) = \delta^{(4)}(x_2 - x_1), \qquad (3)$$

namely, $a = -b = 1/i$; $\partial_2 = \partial/\partial x_2$, ϵ defines the rules of work within poles.

However, attempts to construct the covariant propagator in this way have failed in the framework of the Weinberg theory, Ref. [2], which is a generalization of the Dirac ideas to higher spins. For instance, on the page B1324 of Ref. [2] Weinberg writes:

"*Unfortunately, the propagator arising from Wick's theorem is NOT equal to the covariant propagator except for $S = 0$ and $S = 1/2$. The trouble is that the derivatives act on the $\epsilon(x) = \theta(x) - \theta(-x)$ in $\Delta^C(x)$ as well as on the functions*[i] Δ *and* Δ_1. *This gives rise to extra terms proportional to equal-time δ functions and their derivatives. . . The cure is well known: . . . compute the vertex factors using only the original covariant part of the Hamiltonian $\mathcal{H}$; do not use the Wick propagator for internal lines; instead use the covariant propagator.*

The propagator proposed in Ref. [3] is the causal propagator. However, the old problem remains: the Feynman-Dyson propagator is not the Green function of the Weinberg equation. As mentioned, the covariant propagator proposed by Weinberg propagates kinematically spurious solutions [3].

The aim of my paper is to consider the problem of constructing the propagator in the framework of the model given in [4]. The concept of the Weinberg field 'doubles' has been proposed there. It is based on the equivalence between the Weinberg field and the antisymmetric tensor field, which can be described by both $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$. These field functions may be used to form a parity doublet. An essential ingredient of my consideration is the idea of combining the Lorentz and the dual transformation.

The set of four equations has been proposed in Ref. [4]. For the functions $\psi_1^{(1)}$ and $\psi_2^{(1)}$, connected with the first one by the dual (chiral, $\gamma_5 = diag(1_{3\times3}), -1_{3\times3})$) transformation, the equations are

$$(\gamma_{\mu\nu} p_\mu p_\nu + m^2)\psi_1^{(1)} = 0\,, \qquad (4)$$

$$(\gamma_{\mu\nu} p_\mu p_\nu - m^2)\psi_2^{(1)} = 0\,, \qquad (5)$$

with $\mu,\nu = 1,2,3,4$. For the field functions connected with $\psi_1^{(1)}$ and $\psi_2^{(1)}$ by the $\gamma_5\gamma_{44}$ transformations the set of equations is written:

$$\left[\widetilde{\gamma}_{\mu\nu} p_\mu p_\nu - m^2\right]\psi_1^{(2)} = 0\,, \qquad (6)$$

$$\left[\widetilde{\gamma}_{\mu\nu} p_\mu p_\nu + m^2\right]\psi_2^{(2)} = 0\,, \qquad (7)$$

where $\widetilde{\gamma}_{\mu\nu} = \gamma_{44}\gamma_{\mu\nu}\gamma_{44}$ is connected with the $S=1$ Barut-Muzinich-Williams $\gamma_{\mu\nu}$ matrices [5,6].

In the cited paper I have used the plane-wave expansion:

$$\psi_1(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \times \left[u_1^\sigma(\vec{p}) a_\sigma(\vec{p}) e^{ip\cdot x} + v_1^\sigma(\vec{p}) b_\sigma^\dagger(\vec{p}) e^{-ip\cdot x}\right], \qquad (8)$$

$$\psi_2(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \times \left[u_2^\sigma(\vec{p}) c_\sigma(\vec{p}) e^{ip\cdot x} + v_2^\sigma(\vec{p}) d_\sigma^\dagger(\vec{p}) e^{-ip\cdot x}\right], \qquad (9)$$

where $E_p = \sqrt{\vec{p}^{\,2} + m^2}$; $a_\sigma(\vec{p}), c_\sigma(\vec{p}), b_\sigma^\dagger(\vec{p}), d_\sigma^\dagger(\vec{p})$ are annihilation/creation operators in the Fock space. This is in order to prove that one can describe an $S=1$ quantum particle with transversal components in the framework of the Weinberg and/or the antisymmetric tensor theory.

The corresponding 'bispinors' in the momentum space coincide with the Tucker-Hammer ones within a normalization^{ii}. Their explicit forms are

$$u_1^{\sigma\,(1)}(\vec{p}) = v_1^{\sigma\,(1)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[m + (\vec{S}\cdot\vec{p}) + \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \\ \left[m - (\vec{S}\cdot\vec{p}) + \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \end{pmatrix}, \qquad (10)$$

and

$$u_2^{\sigma\,(1)}(\vec{p}) = v_2^{\sigma\,(1)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[m + (\vec{S}\cdot\vec{p}) + \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \\ \left[-m + (\vec{S}\cdot\vec{p}) - \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \end{pmatrix}, \qquad (11)$$

where ξ_σ are the 3-component objects (the analogs of the Weyl spinors). Thus, $u_2^{(1)}(\vec{p}) = \gamma_5 u_1^{(1)}(\vec{p})$ and $\overline{u}_2^{(1)}(\vec{p}) = -\overline{u}_1^{(1)}(\vec{p})\gamma_5$.

The bispinors

$$u_1^{\sigma\,(2)}(\vec{p}) = v_1^{\sigma\,(2)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[m - (\vec{S}\cdot\vec{p}) + \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \\ \left[-m - (\vec{S}\cdot\vec{p}) - \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \end{pmatrix}, \qquad (12)$$

$$u_2^{\sigma\,(2)}(\vec{p}) = v_2^{\sigma\,(2)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[-m + (\vec{S}\cdot\vec{p}) - \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \\ \left[-m - (\vec{S}\cdot\vec{p}) - \frac{(\vec{S}\cdot\vec{p})^2}{(E+m)}\right]\xi_\sigma \end{pmatrix} \qquad (13)$$

satisfy Eqs. (6) and (7) written in the momentum space. Thus, $u_1^{(2)}(\vec{p}) = \gamma_5\gamma_{44} u_1^{(1)}(\vec{p})$, $\overline{u}_1^{(2)} = \overline{u}_1^{(1)}\gamma_5\gamma_{44}$, $u_2^{(2)}(\vec{p}) = \gamma_5\gamma_{44}\gamma_5 u_1^{(1)}(\vec{p})$ and $\overline{u}_2^{(2)}(\vec{p}) = -\overline{u}_1^{(1)}\gamma_{44}$.

Let me check, if the sum of four equations

$$\begin{aligned}
&\left[\gamma_{\mu\nu}\partial_\mu\partial_\nu - m^2\right] \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1)\, a\;\; u_1^{\sigma\,(1)}(p)\overline{u}_1^{\sigma\,(1)}(p) e^{ip\cdot x} + \theta(t_1 - t_2)\, b\;\; v_1^{\sigma\,(1)}(p)\overline{v}_1^{\sigma\,(1)}(p) e^{-ip\cdot x}\right] \\
&+ \left[\gamma_{\mu\nu}\partial_\mu\partial_\nu + m^2\right] \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1)\, a\;\; u_2^{\sigma\,(1)}(p)\overline{u}_2^{\sigma\,(1)}(p) e^{ip\cdot x} + \theta(t_1 - t_2)\, b\;\; v_2^{\sigma\,(1)}(p)\overline{v}_2^{\sigma\,(1)}(p) e^{-ip\cdot x}\right] \\
&+ \left[\widetilde{\gamma}_{\mu\nu}\partial_\mu\partial_\nu + m^2\right] \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1)\, a\;\; u_1^{\sigma\,(2)}(p)\overline{u}_1^{\sigma\,(2)}(p) e^{ip\cdot x} + \theta(t_1 - t_2)\, b\;\; v_1^{\sigma\,(2)}(p)\overline{v}_1^{\sigma\,(2)}(p) e^{-ip\cdot x}\right] \\
&+ \left[\widetilde{\gamma}_{\mu\nu}\partial_\mu\partial_\nu - m^2\right] \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1)\, a\;\; u_2^{\sigma\,(2)}(p)\overline{u}_2^{\sigma\,(2)}(p) e^{ip\cdot x} + \theta(t_1 - t_2)\, b v_2^{\sigma\,(2)}(p)\overline{v}_2^{\sigma\,(2)}(p) e^{-i\cdot px}\right] \\
&= \delta^{(4)}(x_2 - x_1)
\end{aligned} \qquad (14)$$

can be satisfied by the definite choice of a and b. The relation $u_i(p) = v_i(p)$ for bispinors in the momentum space had been used in Ref. [4]. In the process of calculations I assume that the 3-'spinors' are normalized to $\delta_{\sigma\sigma'}$.

The simple calculations give

$$\partial_\mu\partial_\nu \left[a\,\theta(t_2-t_1)\,e^{ip(x_2-x_1)} + b\,\theta(t_1-t_2)\,e^{-ip(x_2-x_1)}\right] = -\big[a\,p_\mu p_\nu\theta(t_2-t_1)\exp\left[ip(x_2-x_1)\right] + b\,p_\mu p_\nu\theta(t_1-t_2)\exp\left[-ip(x_2-x_1)\right]\big] + a\big[-\delta_{\mu4}\delta_{\nu4}\delta\,'(t_2-t_1) + i(p_\mu\delta_{\nu4}+p_\nu\delta_{\mu4})\delta(t_2-t_1)\big] \times \exp\left[i\vec{p}(\vec{x}_2-\vec{x}_1)\right] + b\,\left[\delta_{\mu4}\delta_{\nu4}\delta\,'(t_2-t_1) + i(p_\mu\delta_{\nu4}+p_\nu\delta_{\mu4})\delta(t_2-t_1)\right]\exp\left[-i\vec{p}(\vec{x}_2-\vec{x}_1)\right]\;; \tag{15}$$

and

$$u_1^{(1)}\overline{u}_1^{(1)} = \frac{1}{2}\begin{pmatrix} m^2 & S_p\otimes S_p \\ \overline{S}_p\otimes\overline{S}_p & m^2\end{pmatrix},$$

$$u_2^{(1)}\overline{u}_2^{(1)} = \frac{1}{2}\begin{pmatrix} -m^2 & S_p\otimes S_p \\ \overline{S}_p\otimes\overline{S}_p & -m^2\end{pmatrix}, \tag{16}$$

$$u_1^{(2)}\overline{u}_1^{(2)} = \frac{1}{2}\begin{pmatrix} -m^2 & \overline{S}_p\otimes\overline{S}_p \\ S_p\otimes S_p & -m^2\end{pmatrix},$$

$$u_2^{(2)}\overline{u}_2^{(2)} = \frac{1}{2}\begin{pmatrix} m^2 & \overline{S}_p\otimes\overline{S}_p \\ S_p\otimes S_p & m^2\end{pmatrix}, \tag{17}$$

where

$$S_p = m + (\vec{S}\cdot\vec{p}) + \frac{(\vec{S}\cdot\vec{p})^2}{E+m}\,, \tag{18}$$

$$\overline{S}_p = m - (\vec{S}\cdot\vec{p}) + \frac{(\vec{S}\cdot\vec{p})^2}{E+m} \tag{19}$$

are the Lorentz boost matrices. Due to

$$\left[E_p - (\vec{S}\cdot\vec{p})\right] S_p\otimes S_p = m^2\left[E_p + (\vec{S}\cdot\vec{p})\right]\,, \tag{20}$$

$$\left[E_p + (\vec{S}\cdot\vec{p})\right] \overline{S}_p\otimes\overline{S}_p = m^2\left[E_p - (\vec{S}\cdot\vec{p})\right]\,, \tag{21}$$

one can conclude: the generalization of the notion of causal propagators is admitted by using the 'Wick's formula' for the time-ordered particle operators provided that $a = b = 1/4im^2$. It is necessary to consider all four equations, Eqs. (4)-(7). Obviously, this is related to the 12-component formalism, which I presented in [4].

The $S = 1$ analogues of the formula (2) for the Weinberg propagators follow immediately. In the Euclidean metrics they are:[iii]

$$S_F^{(1)}(p) \sim -\frac{1}{i(2\pi)^4(p^2+m^2-i\epsilon)}\left[\gamma_{\mu\nu}p_\mu p_\nu - m^2\right], \tag{22}$$

$$S_F^{(2)}(p) \sim -\frac{1}{i(2\pi)^4(p^2+m^2-i\epsilon)}\left[\gamma_{\mu\nu}p_\mu p_\nu + m^2\right], \tag{23}$$

$$S_F^{(3)}(p) \sim -\frac{1}{i(2\pi)^4(p^2+m^2-i\epsilon)}\left[\widetilde{\gamma}_{\mu\nu}p_\mu p_\nu + m^2\right], \tag{24}$$

$$S_F^{(4)}(p) \sim -\frac{1}{i(2\pi)^4(p^2+m^2-i\epsilon)}\left[\widetilde{\gamma}_{\mu\nu}p_\mu p_\nu - m^2\right]. \tag{25}$$

We should use the obtained set of Weinberg propagators (22,23,24,25) in the perturbation calculus of scattering amplitudes. In Ref. [7] the amplitude for the interaction of two $2(2S+1)$ bosons has been obtained on the basis of the use of one field only and it is obviously incomplete, see also Ref. [6]. But, it is interesting to note that the spin structure was proved there to be the same, regardless we consider the two-Dirac-fermion interaction or the two-Weinberg($S=1$)-boson interaction. However, the denominator slightly differs ($1/\vec{\Delta}^2 \to 1/2m(\Delta_0 - m)$) in the cited papers [7] from the fermion-fermion case, where $\Delta_0, \vec{\Delta}$ is the momentum-transger 4-vector in the Lobachevsky space. More accurate considerations of the fermion-boson and boson-boson interactions in the framework of the Weinberg theory has been reported elsewhere [8]. So, the conclusion of this Section is: one can construct an analog of the Feynman-Dyson propagator for the $2(2S+1)$ model and, hence, a 'local' theory provided that the Weinberg states are quadrupled ($S=1$ case).

2. The self/anti-self charge conjugate construction in the $(1/2,0)\oplus(0,1/2)$ representation

The first formulations with doubling solutions of the Dirac equations have been presented in Refs. [9], and [10]. The group-theoretical basis for such doubling has been given in the papers by Gelfand, Tsetlin and Sokolik [12], who first presented the theory later called 'the Bargmann-Wightman-Wigner-type quantum field theory'. M. Markov wrote long ago *two* Dirac equations with the opposite signs at the mass term [9][iv]:

$$\left[i\gamma^\mu\partial_\mu - m\right]\Psi_1(x) = 0\,, \tag{26}$$

$$\left[i\gamma^\mu\partial_\mu + m\right]\Psi_2(x) = 0\,, \tag{27}$$

where γ^μ are the Dirac matrices. Of course, these two equations are equivalent each other on the free level since we are convinced that the relative intrinsic parity has physical significance only. In fact, he studied all properties of this relativistic quantum model while he did not know yet the quantum field theory in 1937. Next, he added and subtracted these equations. As a result the equations are

$$i\gamma^\mu\partial_\mu\varphi(x) - m\chi(x) = 0\,, \tag{28}$$

$$i\gamma^\mu\partial_\mu\chi(x) - m\varphi(x) = 0\,. \tag{29}$$

Thus, $\varphi-$ and $\chi-$ solutions can be presented as some superpositions of the Dirac 4-spinors $u-$ and $v-$. These equations, of course, can be identified with the equations for the Majorana-like $\lambda-$ and $\rho-$ spinors, which we presented in

Ref. [13, 14]^v. The four-component Majorana-like spinors are defined as

$$\lambda(\mathbf{p}) = \begin{pmatrix} \vartheta\Theta\phi_L^*(\mathbf{p}) \\ \phi_L(\mathbf{p}) \end{pmatrix}, \tag{30}$$

$$\Theta_{[1/2]} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{31}$$

They become eigenspinors of the charge conjugation operator S_c with eigenvalues ± 1 if the phase ϑ is set to $\pm i$:

$$S_c\ \lambda(\mathbf{p})\Big|_{\vartheta=\pm i} = \pm\lambda(\mathbf{p})\Big|_{\vartheta=\pm i}. \tag{32}$$

In a similar way one can construct $\rho-$ spinors on using ϕ_R. The dynamical equations are:

$$i\gamma^\mu\partial_\mu\lambda^S(x) - m\rho^A(x) = 0\,, \tag{33}$$

$$i\gamma^\mu\partial_\mu\rho^A(x) - m\lambda^S(x) = 0\,, \tag{34}$$

$$i\gamma^\mu\partial_\mu\lambda^A(x) + m\rho^S(x) = 0\,, \tag{35}$$

$$i\gamma^\mu\partial_\mu\rho^S(x) + m\lambda^A(x) = 0\,. \tag{36}$$

None of them can be regarded as the Dirac equation. However, they can be written in the 8-component form as follows:

$$[i\Gamma^\mu\partial_\mu - m]\,\Psi_{(+)}(x) = 0\,, \tag{37}$$

$$[i\Gamma^\mu\partial_\mu + m]\,\Psi_{(-)}(x) = 0\,, \tag{38}$$

with

$$\Psi_{(+)}(x) = \begin{pmatrix} \rho^A(x) \\ \lambda^S(x) \end{pmatrix},$$

$$\Psi_{(-)}(x) = \begin{pmatrix} \rho^S(x) \\ \lambda^A(x) \end{pmatrix}, \quad \Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix}. \tag{39}$$

It is easy to find the corresponding projection operators, and the Feynman-Dyson-Stueckelberg propagator.

You may say that all this is just related to the spin-parity basis rotation (unitary transformations). In the previous papers the connection with the Dirac spinors has been found [14, 15]. For instance,

$$\begin{pmatrix} \lambda^S_\uparrow(\mathbf{p}) \\ \lambda^S_\downarrow(\mathbf{p}) \\ \lambda^A_\uparrow(\mathbf{p}) \\ \lambda^A_\downarrow(\mathbf{p}) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & i & -1 & i \\ -i & 1 & -i & -1 \\ 1 & -i & -1 & -i \\ i & 1 & i & -1 \end{pmatrix} \times \begin{pmatrix} u_{+1/2}(\mathbf{p}) \\ u_{-1/2}(\mathbf{p}) \\ v_{+1/2}(\mathbf{p}) \\ v_{-1/2}(\mathbf{p}) \end{pmatrix}, \tag{40}$$

provided that the 4-spinors have the same physical dimension. Thus, we can see that the two 4-spinor systems are connected by the unitary transformations, and this represents itself the rotation of the spin-parity basis. However, it is usually assumed that the $\lambda-$ and $\rho-$ spinors describe the neutral particles, meanwhile $u-$ and $v-$ spinors describe the charged particles. Kirchbach [15] found the amplitudes for neutrinoless double beta decay ($00\nu\beta$) in this scheme. It is obvious from (40) that there are some additional terms comparing with the standard formulation.

One can also re-write the above equations into the two-component forms. Thus, one obtains the Feynman-Gell-Mann equations [16]. As Markov wrote himself, he was expecting "new physics" from these equations.

Barut and Ziino [10] proposed yet another model. They considered the γ^5 operator as the operator of the charge conjugation. Thus, the charge-conjugated Dirac equation has a different sign comparing with the ordinary formulation:

$$[i\gamma^\mu\partial_\mu + m]\Psi^c_{BZ} = 0\,, \tag{41}$$

and the so-defined charge conjugation applies to the whole system, fermion + electromagnetic field, $e \to -e$ in the covariant derivative. The superpositions of the Ψ_{BZ} and Ψ^c_{BZ} give us the 'doubled Dirac equation', as the equations for $\lambda-$ and $\rho-$ spinors. The concept of the doubling of the Fock space has been developed in the Ziino works (cf. [4, 12]) in the framework of the quantum field theory. In their case the self/anti-self charge conjugate states are simultaneously the eigenstates of the chirality. It is interesting to note that for the Majorana-like field operators ($a_\eta(\mathbf{p}) = b_\eta(\mathbf{p})$) we have

$$\left[\nu^{^{ML}}(x^\mu) + \mathcal{C}\nu^{^{ML\,\dagger}}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p} \tag{42}$$

$$\sum_\eta \left[\begin{pmatrix} i\Theta\phi^{*\,\eta}_{_L}(\mathbf{p}) \\ 0 \end{pmatrix} a_\eta(\mathbf{p})e^{-ip\cdot x} + \begin{pmatrix} 0 \\ \phi^\eta_L(\mathbf{p}) \end{pmatrix} a^\dagger_\eta(\mathbf{p})e^{ip\cdot x}\right], \tag{43}$$

$$\left[\nu^{^{ML}}(x^\mu) - \mathcal{C}\nu^{^{ML\,\dagger}}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p} \tag{44}$$

$$\sum_\eta \left[\begin{pmatrix} 0 \\ \phi^\eta_{_L}(\mathbf{p}) \end{pmatrix} a_\eta(\mathbf{p})e^{-ip\cdot x} + \begin{pmatrix} -i\Theta\phi^{*\,\eta}_{_L}(\mathbf{p}) \\ 0 \end{pmatrix} a^\dagger_\eta(\mathbf{p})e^{ip\cdot x}\right] \tag{45}$$

which naturally lead to the Ziino-Barut scheme of massive chiral fields, Ref. [10].

3. The controversy

I cite Ahluwalia *et al.*, Ref. [11] ^{vi}: *"To study the locality structure of the fields* $\Lambda(x)$ *and* $\lambda(x)$, *we observe that field momenta are*

$$\Pi(x) = \frac{\partial\mathcal{L}^\Lambda}{\partial\dot\Lambda} = \frac{\partial}{\partial t}\,\overset{\neg}{\Lambda}\,(x), \tag{46}$$

and similarly $\pi(x) = (\partial/\partial t) \stackrel{\neg}{\lambda} (x)$. The calculational details for the two fields now differ significantly. We begin with the evaluation of the equal time anticommutator for $\Lambda(x)$ and its conjugate momentum

$$\{\Lambda(\mathbf{x},t),\ \Pi(\mathbf{x}',t)\} = i\int \frac{d^3p}{(2\pi)^3}\frac{1}{2m}e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')} \times \underbrace{\sum_\alpha \left[\xi_\alpha(\mathbf{p}) \stackrel{\neg}{\xi}_\alpha (\mathbf{p}) - \zeta_\alpha(-\mathbf{p}) \stackrel{\neg}{\zeta}_\alpha (-\mathbf{p})\right]}_{=2m[I+\mathcal{G}(\mathbf{p})]}.$$

The term containing $\mathcal{G}(\mathbf{p})$ vanishes only when $\mathbf{x}-\mathbf{x}'$ lies along the z_e axis (see Eq. (24) [therein], and discussion of this integral in Ref. [17])

$$\mathbf{x}-\mathbf{x}' \textit{ along } z_e: \quad \{\Lambda(\mathbf{x},t),\ \Pi(\mathbf{x}',t)\} = i\delta^3(\mathbf{x}-\mathbf{x}')I. \tag{47}$$

The anticommutators for the particle/antiparticle annihilation and creation operators suffice to yield the remaining locality conditions,

$$\{\Lambda(\mathbf{x},t),\ \Lambda(\mathbf{x}',t)\} = O, \quad \{\Pi(\mathbf{x},t),\ \Pi(\mathbf{x}',t)\} = O. \tag{48}$$

The set of anticommutators contained in Eqs. (47) and (48) establish that $\Lambda(x)$ becomes local along the z_e axis. For this reason we call z_e as the dark axis of locality."

Next, I cite Rodrigues *et al.*, Ref. [18]: "*We have shown through explicitly and detailed calculation that the integral of $\mathcal{G}(\mathbf{p})$ appearing in Eq. (42) of [11] is null for $\mathbf{x}-\mathbf{x}'$ lying in three orthonormal spatial directions in the rest frame of an arbitrary inertial frame $\mathbf{e}_0 = \partial/\partial t$.*

This shows that the existence of elko spinor fields does not implies in any breakdown of locality concerning the anticommutator of $\{\Lambda(\mathbf{x},t), \Pi(\mathbf{x}',t\}$ and moreover does not implies in any preferred spacelike direction field in Minkowski spacetime."

Who is right? In 2013 W. Rodrigues [19] changed a bit his opinion. He wrote: "*When $\Delta_z \neq 0$, $\hat{\mathcal{G}}(\mathbf{x}-\mathbf{x}')$ is null the anticommutator is* local *and thus there exists in the elko theory as constructed in [11] an infinity number of "locality directions". On the other hand $\hat{\mathcal{G}}(\mathbf{x}-\mathbf{x}')$ is a distribution with support in $\Delta_z = 0$. So, the directions $\boldsymbol{\Delta} = (\Delta_x, \Delta_y, 0)$ are nonlocal in each arbitrary inertial reference frame $\mathbf{e}_0$ chosen to evaluate $\hat{\mathcal{G}}(\mathbf{x}-\mathbf{x}')$*", thus accepting the Ahluwalia *et al.* viewpoint. See the cited papers for the notation.

Meanwhile, I suggest to use the 8-component (or 16-component) formalism (see the Sec. 2) in similarity with the 12-component formalism of the Sec. 1. If we calculate

$$\begin{aligned} S_F^{(+,-)}(x_2,x_1) &= \int\frac{d^3p}{(2\pi)^3}\frac{m}{E_p}\left[\theta(t_2-t_1)\, a\ \Psi^\sigma_\pm(p)\overline{\Psi}^\sigma_\pm(p)e^{-ip\cdot x} + \theta(t_1-t_2)\, b\ \Psi^\sigma_\mp(p)\overline{\Psi}^\sigma_\mp(p)e^{ip\cdot x}\right] \\ &= \int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\frac{(\hat{p}\pm m)}{p^2-m^2+i\epsilon}, \end{aligned} \tag{49}$$

we easily come to the result that the corresponding Feynman-Dyson propagator gives the local theory in the sense:

$$\sum_\pm [i\Gamma_\mu\partial_2^\mu \mp m]S_F^{(+,-)}(x_2-x_1) = \delta^{(4)}(x_2-x_1). \tag{50}$$

However, physics should choose only one correct formalism. It is not clear, why two correct mathematical formalisms lead to different physical results? First of all, we should check, whether this possible non-locality in the propagators has influence on the physical observables such as the scattering amplitudes, the energy spectra and the decay widths. If not, we may find some unexpected symmetries in relativistic quantum mechanics/field theory. This is the task for future publications. However, it is already obvious if we would not enlarge the number of components in the fields (in the propagator) we would not be able to obtain the formally causal propagators for higher spins and/or for the neutral particles.

Note added. The dilemma of the (non)local propagators for the spin $S=1$ has also been analized in [20] within the Duffin-Kemmer-Petiau (DKP) formalism or the Dirac-Kähler formalism [21]. However, the propagators given in [20] are those in the generalized Duffin-Kemmer-Petiau formalism, in fact. They are not in the Weinberg-Tucker-Hammer formalism. Moreover, the problem of the massless limit was not discussed in the DKP formalism, which is non-trivial (like that of the Proca formalism [22]).

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i. In the cited paper $\Delta_1(x) \equiv i\,[\Delta_+(x)+\Delta_+(-x)]$ and $\Delta(x) \equiv \Delta_+(x) - \Delta_+(-x)$ have been used. $i\Delta_+(x) \equiv (1/(2\pi)^3)\int(d^3p/2E_p)\exp(ip\cdot x)$ is the particle Green function.

ii. They also coincide with the Ahluwalia *et al.* ones within a unitary transformation [11].

iii. We use the Euclidean metrics in this Section due to many original papers on the Weinberg $2(2S+1)$ theory use it. This is in order the reader to have possibility to compare the formulas. In the next Section we turn to the pseudoEuclidean metrics on using simple correspondence rules.

iv. I turn to the pseudo-Euclidean metric because it is more usable in the recent literature.

v. Of course, the signs at the mass terms depend on, how do we associate the positive- or negative- frequency solutions with λ and ρ.

vi. The notation should be compared with the cited papers.

1. C. Itzykson and J.-B. Zuber, *Quantum Field Theory.* (McGraw-Hill Book Co. New York, 1980).
2. S. Weinberg, *Phys. Rev.* **B133** (1964) 1318.
3. D. V. Ahluwalia and D. J. Ernst, *Phys. Rev.* **C45** (1992) 3010.
4. V. V. Dvoeglazov, *Int. J. Theor. Phys.* **37** (1998) 1915.
5. A. O. Barut, I. Muzinich and D. Williams, *Phys. Rev.* **130** (1963) 442.
6. R. H. Tucker and C. L. Hammer, *Phys. Rev.* **D3** (1971) 2448.
7. V. V. Dvoeglazov, *Int. J. Theor. Phys.* **35** (1996) 115.
8. V. V. Dvoeglazov, *J. Phys. Conf. Ser.* **128** (2008) 012002.
9. M. Markov, *ZhETF* **7** (1937) 579; ibid. 603; *Nucl. Phys.* **55** (1964) 130.
10. A. Barut and G. Ziino, *Mod. Phys. Lett.* **A8** (1993) 1011; G. Ziino, *Int. J. Mod. Phys.* **A11** (1996) 2081.
11. D. V. Ahluwalia and D. J. Ernst, *Int. J. Mod. Phys.* E**2**, (1993); D. V. Ahluwalia *et al.*, *Phys. Lett.* **B687** (2010) 248; idem. *Phys. Rev.* **D83** (2011) 065017.
12. I. M. Gelfand and M. L. Tsetlin, *ZhETF* **31** (1956) 1107; G. A. Sokolik, *ZhETF* **33** (1957) 1515.
13. D. V. Ahluwalia, *Int. J. Mod. Phys.* **A11** (1996) 1855.
14. V. V. Dvoeglazov, *Int J. Theor. Phys.* **34** (1995) 2467; *Nuovo Cim.* **B111** (1996) 483; ibid. **A108** (1995) 1467; *Mod. Phys. Lett.* **A12** (1997) 2741.
15. M. Kirchbach, C. Compean and L. Noriega, *Eur. Phys. J.* A**22** (2004) 149.
16. R. P. Feynman and M. Gell-Mann, *Phys. Rev.* **109** (1958) 193.
17. D. V. Ahluwalia-Khalilova and D. Grumiller, *Phys. Rev.* **D72** (2005) 067701; *JCAP* **0507** (2005) 012.
18. W. Rodrigues, Jr. *et al.*, *Phys. Rev.* **D86** (2012) 128501; Erratum: ibid. **D88** (2013) 129901; ArXiV: 1210.7207v4.
19. W. Rodrigues, Jr. *et al.*, *Int. J. Theor. Phys.* **53** (2014) 4381, see Appendix B therein.
20. S. I. Kruglov, in *Einstein and Hilbert: Dark Matter.* (Nova Science Pubs, NY, USA, 2012) pp. 107-121, ArXiV: 0912.3716.
21. S. I. Kruglov, *Int. J. Theor. Phys.* **41** (2002) 653, ArXiV:hep-th/0110251.
22. A. V. Hradyskyi and Yu. P. Stepanovskiy, *Ukr. J. Phys.* **63** (2018) 584.