The interaction of matter and radiation in stochastic electrodynamics

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Abstract. A nonperturbative version of stochastic electrodynamics has been recently developed, that leads to the formalism of quantum mechanics in Hilbert space. On the basis of the established connection between the two theories, we attempt in this paper to gain an understanding of the specific mechanism of interaction between the random radiation field and the charged particle; in particular, the linear response of the particle to certain field modes is disclosed. This property is exploited to rederive the Einstein coefficients and Planck's distribution, from the stochastic perspective.

PACS: 03.65.Bz; 05.40.+j; 32.80.-t

1. Introduction

Recently a nonperturbative version of stochastic electrodynamics (SED) for atomic systems (or bound systems in general) has been presented [1] that leads to the quantum mechanical description under certain circumstances, namely: a) in the asymptotic time limit, when the random electromagnetic field has taken control of the dynamics and the mechanical system has forgotten its initial conditions, and b) after a partial averaging is performed to go from the complete (field and particle) phase space over to the configuration (or momentum) space of the particle.

This new form of SED not only strengthens the fundamental hypothesis on the reality of the zero-point field, but also offers a basis for the elucidation of many of the controversial points that plague the physical interpretation of quantum mechanics [2]. However, the formal nature of the procedure used to retrieve the quantum equations tends to conceal much of the underlying physics. On the other side, a more detailed treatment is not feasible with the present tools, due to the complexity of the stochastic equations.

As a partial remedy to this situation, we propose here to take advantage of the quantum results obtained, in order to analyse, from the perspective of SED, certain aspects of the response of the mechanical system to the radiation field. As a result of this rather hybrid procedure, that has the advantage of simplifying considerably the mathematics—at the expense of the details of the dynamics—, the linear response of the mechanical system (in the quantum regime) to certain field modes is disclosed. With the help of this property it is shown that only the zero-point field with spectrum \( \propto \omega^3 \) guarantees detailed energy balance, and also the blackbody radiation law and Einstein's A and B coefficients are correctly derived without
resorting to a quantized field. Spontaneous emission as well as the nonexistence of spontaneous absorptions find a clear explanation within this approach.

2. The poissonian equations and their connection with quantum mechanics

We recall that in SED, a typical system consists of a charged particle (for instance, an atomic electron) plus the background radiation field. The minimum field that is always present is the zero-point radiation field with power spectrum

\[ S_0(\omega) = \frac{2\hbar \omega^3}{3\pi c^3} \]  

(2.1)
corresponding to an energy \( \frac{1}{2} \hbar \omega \) per normal mode. The vector potential in the Coulomb gauge is

\[ A = \sum_{n, \sigma} \left( \frac{2\pi \hbar c^2}{V \omega_n} \right)^{1/2} \hat{\epsilon}_{n\sigma} a_{n\sigma} \exp[-i(\omega_n t - k_n \cdot \mathbf{x})] + \text{c.c.} \]

where the volume \( V \) is eventually taken as infinite, \( \omega_n = ck_n, \sigma \) is the polarization index, and \( \hat{\epsilon}_{n\sigma}, k_n \) are orthogonal for every \( n \). The amplitudes \( a_\alpha \equiv a_{n\sigma} \) are complex, independent, stochastic variables, with the following statistical properties:

\[ \langle a_{n\sigma} \rangle = 0, \quad \langle a_{n\sigma} a_{n'\sigma'}^\ast \rangle = 0, \quad \langle a_{n\sigma} a_{n'\sigma'}^\ast \rangle = \frac{1}{2} \delta_{n n'} \delta_{\sigma \sigma'}. \]  

(2.2)
The Hamiltonian of the complete system is

\[ H_T = \frac{1}{2m} (\mathbf{p} - \frac{e}{c} \mathbf{A})^2 + V(\mathbf{x}) + H_R \]  

(2.3)
where \( H_R \) stands for the hamiltonian of the total radiation field:

\[ H_R = \frac{1}{2} \sum_{n, \sigma} (p_{n\sigma}^2 + \omega_n^2 q_{n\sigma}^2) = \sum_{n, \sigma} \hbar \omega_n b_{n\sigma}^\ast b_{n\sigma}. \]  

(2.4)
The mode amplitudes \( b_{n\sigma} = (2\hbar \omega_n)^{-1/2} (p_{n\sigma} - i\omega_n q_{n\sigma}) \) differ from \( a_{n\sigma} \) because they include the field radiated by the particle.

In Ref. [3] we have shown that under certain conditions, the above system leads to the following set of equations:

\[ \langle x_i; x_j \rangle = \langle p_i; p_j \rangle = 0, \quad \langle x_i; p_j \rangle = i\hbar \delta_{ij}. \]  

(2.5)
and
\[ i\hbar \dot{x}_i = \langle x_i; H \rangle - \frac{i\hbar}{mc} A_i \]  \hspace{1cm} (2.6a)
\[ i\hbar \dot{p}_i = \langle p_i; H \rangle + \frac{i\hbar}{mc} \frac{\partial A_j}{\partial x_i} , \]  \hspace{1cm} (2.6b)

where
\[ \sum_\alpha \left( \frac{\partial f}{\partial a_\alpha^*} \frac{\partial g}{\partial a_\alpha} - \frac{\partial g}{\partial a_\alpha^*} \frac{\partial f}{\partial a_\alpha} \right) = \langle f; g \rangle \]  \hspace{1cm} (2.7)

is the so-called poissonian of \( f \) and \( g \) (up to a numerical factor, it is the Poisson bracket of \( f \) and \( g \) with respect to the complete set \( \{a, a^*\} \)), and \( H \) is the mechanical hamiltonian: \( H = p^2/2m + V \). For the details of the derivation of Eqs. (2.5, 6) we refer the reader to the literature. Let us just briefly recall that in deriving them, it is assumed that due to the permanent action of the random field—including radiation reaction—, the mechanical system eventually forgets its initial conditions and thus, for times long compared with the relaxation times of the system, the Poisson brackets with respect to the initial canonical variables of the particle become negligible. Under such conditions we say that the system has reached the quantum regime.

As indicated by Eqs. (2.5), which are valid once the quantum regime has set up, even to zero order in \( e \) the particle variables depend explicitly on the random field amplitudes \( a_\alpha \); within this approximation Eqs. (2.6) reduce to
\[ i\hbar \dot{x}_i = \langle x_i; H \rangle, \quad i\hbar \dot{p}_i = \langle p_i; H \rangle, \]  \hspace{1cm} (2.8)

the remaining terms representing simply radiative corrections [3] (in the quantum regime). These equations exhibit the essential role played by the background field in connection with the dynamics of the particle. They evidently cannot be derived by a perturbative treatment; here we find an explanation for the failure of all previous (perturbative) treatments of the atomic problem in SED [4,5].

The Eqs. (2.5) and (2.8) can be further transformed by means of a statistical procedure into the sets of operator equations [1,6]
\[ [\hat{x}_i, \hat{x}_j] = [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = i\hbar \delta_{ij} \]  \hspace{1cm} (2.9)
and
\[ i\hbar \dot{\hat{x}}_i = [\hat{x}_i, \hat{H}], \quad i\hbar \dot{\hat{p}}_i = [\hat{p}_i, \hat{H}], \]  \hspace{1cm} (2.10)

which are clearly the Dirac quantization rules and the Heisenberg equations of quantum mechanics. The steps involved in this transition are basically the following:
a) introduction of the density in configuration space, $\rho(x)$, so that the value of an arbitrary function $F(x)$ in the $x$-representation is given by

$$F(x)\rho(x) = \int \delta(x - x(a))F(x(a))W(a) da,$$

where $W(a)$ is the probability distribution of the field amplitudes;

b) factorization of $\rho(x)$ into a pair of functions $\psi_+$ and $\psi_-$, such that the moments of $p$ can be represented in configuration space by means of the general expression

$$\langle p^r \rangle \equiv \int p^r(a)W(a) da = \int \psi_+ \hat{p}^r \psi_- dx; \quad (2.11)$$

c) the solution of Eq. (2.11) consistent with the poissonian conditions (2.5) turns out to be precisely the quantum mechanical rule $\hat{p} = -i\hbar \partial/\partial x$; simultaneously, all poissonians are transformed into the corresponding quantum commutators with dynamical variables represented by operators acting in Hilbert space;

d) alternatively, it is possible to introduce the complementary $p$-representation; the connection between the amplitudes $\psi$ in the $x$-representation and the corresponding ones in the $p$-representation turns out to be given by a Fourier transformation. Note that in a given representation, the corresponding variable is treated as independent whereas the canonically conjugate variable is represented by an operator. For further details the reader is referred to the cited literature.

3. The linear response

Once it is established that in the quantum regime the SED system is statistically described by quantum mechanics, we can try to use this knowledge to increase our understanding of the dynamical behaviour of the particle. This is especially important since the transition from the poissonian equations to the quantum mechanical ones has been rather a mathematical exercise; we would like to pay now closer attention to the physics.

A first general conclusion can be drawn that will be relevant to our analysis. From Eqs. (2.10) it follows that the matrix elements of $x$ between stationary states satisfy the differential equation (we consider one dimension, for simplicity)

$$\ddot{x}_{\alpha\beta} + \omega^2_{\alpha\beta}x_{\alpha\beta} = 0 \quad (3.1)$$

with $\omega_{\alpha\beta}$ given by Bohr's formula

$$\omega_{\alpha\beta} = \frac{E_{\alpha} - E_{\beta}}{\hbar}, \quad (3.2)$$
which means that \( x \) behaves as a set of independent oscillators of frequencies \( \omega_{\alpha \beta} \) and amplitudes which we shall denote by \( \tilde{x}_{\alpha \beta} \) and which are determined by solving the complete set of equations. Translated to the level of description of SED, since the independent elementary oscillators are the field amplitudes \( a_{\alpha \beta}(t) \) that oscillate with frequency \( \omega_{\alpha \beta} \), this statistical behaviour is represented by

\[
x^{(0)}(t) = \sum_{\beta} \tilde{x}_{\beta \alpha}(t) + \text{c.c.}
\]  

where we have omitted for simplicity the index \( \alpha \) referring to the state under consideration; below we shall comment on the superscript \((0)\).

Eq. (3.3) is somewhat unexpected; it is convenient to remark, however, that in writing it down we are merely translating the already established quantum behaviour into a language more appropriate to the stochastic description, without further hypotheses. As shown below, Eq. (3.3) is remarkably useful to get results that refer not only to quantum mechanical systems, but even to quantum electrodynamics.

According to Eq. (3.3) the SED system in equilibrium responds selectively to certain field modes — those whose frequencies are precisely the frequencies of the possible transitions — and is virtually insensitive to the rest of the spectrum. This resonant response to the field is linear, although of course the amplitudes \( \tilde{x}_{\alpha \beta} \) that characterize the response to a given mode obey in general nonlinear equations (in \( \tilde{x} \)).

It is convenient to notice that introduction of Eq. (3.3) and \( p^{(0)} = mx^{(0)} \) into the last of Eqs. (2.5) leads to

\[
\sum_{\alpha} \omega_{\alpha} |\tilde{x}_{\alpha}|^2 = \frac{\hbar}{2m},
\]

which is just the Thomas-Reiche-Kuhn sum rule of quantum mechanics. In fact, it is easy to see that the only function \( x(a(t)) \) — up to terms containing \( |a|^2 \), which is constant and hence dynamically uninteresting — that solves Eqs. (2.5) is precisely the linear expression (3.3), which shows that the basic poissonians (2.5) are very strong conditions indeed — no less than the Dirac quantization rules. From (3.3) it also follows that the dispersions of \( x \) and \( p \) are given by

\[
\sigma_x^2 = \sum |\tilde{x}_{\alpha}|^2, \quad \sigma_p^2 = m^2 \sum \omega_{\alpha}^2 |\tilde{x}_{\alpha}|^2,
\]

whence \( \sigma_x^2 \sigma_p^2 \geq \hbar^2/4 \).

For the poissonian equations of motion themselves, (2.6), things are quite more complicated; some possibilities are discussed in Ref. [6]. On the other hand, the example to be worked out below shows that with the present theory we can go beyond quantum mechanics and obtain results that normally require the formalism of second quantization.
4. Detailed balance in the quantum regime

Let us investigate the problem of absorption and emission of radiation by an atom in an external electromagnetic field. From the Abraham–Lorentz equation of motion for the particle, that can be derived from the complete Hamiltonian (2.3),

\[ m\ddot{x} = F(x) + m\tau\dot{x} + eE + \frac{e}{c}x \times B \]

it follows that the average energy of the particle varies according to

\[ \frac{d\langle H \rangle}{dt} = m\tau \langle \dot{x} \cdot \ddot{x} \rangle + e\langle \dot{x} \cdot E \rangle. \quad (4.1) \]

Since \( \tau = 2e^2/3mc^3 \), the first term on the right hand side is at least of order \( e^2 \), whereas the second one may be of order \( e \); hence, in equilibrium the contribution of lowest order in \( e \) from this second term should vanish. Writing \( x \) as a series in powers of \( e \), \( x = x(0) + ex(1) + \cdots \), we must have in equilibrium \( \langle \dot{x}(0) \cdot E \rangle = 0 \) and Eq. (4.1) gives then to second order:

\[ \frac{d\langle H \rangle}{dt} = m\tau \langle \dot{x}(0) \cdot \ddot{x}(0) \rangle + e^2\langle \dot{x}(1) \cdot E \rangle. \quad (4.2) \]

Note that \( x \) given by Eq. (3.3) is indeed of zero order in \( e \) (this is the meaning of the superscript (0) that we added to it) and moreover, it satisfies the above condition \( \langle \dot{x}(0) \cdot E \rangle = 0 \), as a direct calculation shows. We shall take \( E \) in the dipole approximation, \( E = E(t) \). Now, to use Eq. (4.2) we need to construct an expression for \( x^{(1)} \). With this purpose consider a general dynamical equation of the form

\[ \ddot{x}_i = A_i + eB_i + \cdots \]

where \( A_i \) and \( B_i \) do not depend on \( e \). Then the first-order term is a solution of the equation \( \ddot{x}_i^{(1)} = (\partial A_i/\partial x_j)x_j^{(1)} + B_i \), with \( \partial A_i/\partial x_j \) calculated at \( x^{(0)} \). Writing it as

\[ x_i^{(1)} = \int_{-\infty}^{t} G_{ij}(t,t')B_j(t')dt', \]

the Green function must satisfy \( \ddot{G}_{ij} = (\partial A_i/\partial x_k)G_{kj} \) with \( G_{ij}(t,t) = 0 \) and \( G_{ij}(t,t')|_{t'=t} = \delta_{ij} \). The solution is

\[ G_{ij}(t,t') = \left. \frac{\partial x_i(t)}{\partial p_j(t')} \right|_0 \]
when \( p = m \dot{x} \), as can be confirmed by direct substitution. Thus, \( x_i^{(1)} \) becomes

\[
x_i^{(1)} = e \int_{-\infty}^{t} \frac{\partial x_i(t)}{\partial p_j(t')} \left| B_j(t') \right| dt'.
\] (4.3)

Note that this solution does not depend explicitly on \( A_i \). Observing that

\[
\frac{\partial x_i(t)}{\partial p_j(t')} = - [x_i(t), x_j(t')] = \frac{i}{\hbar} (x_i(t); x_j(t')),
\]

and taking \( B_i = E_i \), we get by substitution of Eq. (3.3)

\[
\frac{\partial x_i^{(0)}(t)}{\partial p_j^{(0)}(t')} = \frac{2}{\hbar} \delta_{ij} \sum_\alpha |\tilde{x}_\alpha^{(i)}|^2 \sin \omega_\alpha(t - t')
\]

whence

\[
x_i^{(1)} = \frac{2e}{\hbar} \sum_\alpha |\tilde{x}_\alpha^{(i)}|^2 \int_0^\infty E_i(t - t') \sin \omega_\alpha t' dt',
\]

and the absorption term of Eq. (4.2) becomes

\[
\frac{\pi e^2}{\hbar} \sum_\alpha \omega_\alpha |\tilde{x}_\alpha^{(i)}|^2 S(\omega_\alpha),
\]

where \( S(\omega) \) is the power spectrum of the field, defined through

\[
\langle E_i(t')E_j(t) \rangle = \delta_{ij} \int_0^\infty S(\omega) \cos \omega(t - t') d\omega.
\] (4.4)

From Eqs. (2.2) and (3.3) one obtains for the radiation term:

\[
-m \tau \sum_\alpha \omega_\alpha^4 |\tilde{x}_\alpha^{(i)}|^2
\]

and the energy rate equation becomes thus

\[
\frac{d\langle H \rangle}{dt} = \sum_\alpha \left[ \frac{\pi e^2}{\hbar} \omega_\alpha S(\omega_\alpha) - m \tau \omega_\alpha^4 \right] |\tilde{x}_\alpha|^2.
\] (4.5)

This equation shows not only that equilibrium can be attained between the atom and radiation, but also that detailed balance holds if and only if the power spectrum is \( S(\omega) = 2\hbar \omega^3 / \pi e^2 \) (with \( \omega > 0 \)); this is precisely the spectrum of the zero-point
field, Eq. (2.1). The failure of all previous attempts to determine $S(\omega)$—which have led to the Rayleigh-Jeans spectrum instead [5]—is due to the use of a perturbative treatment of the dynamics, in which $x^{(0)}$ is the solution of the classical equations of motion instead of the poissonian equations.

5. Spontaneous and induced electromagnetic transitions [7]

Let us now investigate the energy rate in presence of a more general radiation field. With this purpose we use Eq. (2.1) to transform the last term of Eq. (4.5), thus obtaining

$$\frac{d(H)}{dt} = \sum_\alpha \frac{\pi e^2}{h} \omega_\alpha |\text{sign}(\omega_\alpha) S(\omega_\alpha) - S_0(\omega_\alpha)| |\bar{x}_\alpha|^2.$$  

In principle $\omega_\alpha$ may be positive or negative. By adding an index to the coefficients $\bar{x}$ to indicate the sign of the corresponding $\omega_\alpha$, we can rewrite this equation in the expanded form

$$\frac{d(H)}{dt} = \sum_\alpha \frac{4\pi^2 e^2}{3h} \omega_\alpha [\rho(\omega_\alpha) - \rho_0(\omega_\alpha)] |\bar{x}_\alpha^{(+)}|^2 - \sum_\alpha \frac{4\pi^2 e^2}{3h} |\omega_\alpha| [\rho(\omega_\alpha) + \rho_0(\omega_\alpha)] |\bar{x}_\alpha^{(-)}|^2. \quad (5.1)$$

in terms of the spectral energy density of the field, $\rho(\omega) = 3S(\omega)/4\pi$. The first term on the right hand side is different from zero when the field is excited ($\rho > \rho_0$); it represents an energy gain for the particle due to the external field. The second term is always negative: it represents an energy loss that occurs for $\omega_\alpha < 0$, even in absence of external field. Absorption of energy takes the particle to a state of higher energy, and this kind of process is represented by the first term in Eq. (5.1); the processes of emission are represented by the second term. Hence, for absorption to occur one must have $\rho > \rho_0$ and $|\bar{x}_\alpha^{(+)}| \neq 0$ for some $\omega_\alpha$; in particular, no spontaneous absorptions occur. On the other hand, if no $|\bar{x}_\alpha^{(-)}|$ is different from zero (i.e., if all $\omega_\alpha$ are positive), no emission can occur; the system is then in its lowest energy state. Detailed balance holds when both particle and field are in their ground state. Of course these important results are well known in quantum theory, but it is significant that we recover them from SED. In particular we see that according to the present theory no self-ionization of the $H$ atom occurs, as opposed to the more classical calculations of the standard version of SED (see, e.g. Refs. [5, 9 and 10]).

Since $|\bar{x}_\alpha^{(+)}|$ and $|\bar{x}_\alpha^{(-)}|$ refer to upward and downward transitions, respectively, one can write Eq. (5.1) in terms of energy absorption and emission rates (omitting
the index $\alpha$):

$$\frac{d(H)}{dt} = W_{\text{abs}} - W_{\text{emi}}, \quad (5.2)$$

with

$$W_{\text{abs}} = \frac{4\pi^2e^2}{3h} \sum \omega (\rho(\omega) - \rho_0(\omega)) |\tilde{\omega}(\omega)|^2, \quad (5.3a)$$

$$W_{\text{emi}} = \frac{4\pi^2e^2}{3h} \sum \omega (\rho(\omega) + \rho_0(\omega)) |\tilde{\omega}(\omega)|^2. \quad (5.3b)$$

The difference $\rho_e = \rho - \rho_0$ represents the part of the spectral density that is due to an external field (it may or may not contain contributions due to correlations with the vacuum field; a more detailed discussion is given in Ref. [8]). In terms of it Eqs. (5.3) become

$$W_{\text{abs}} = \sum \frac{4\pi^2e^2}{3h} \omega \rho_e (|\tilde{\omega}(\omega)|^2), \quad (5.4a)$$

$$W_{\text{emi}} = \sum \frac{4\pi^2e^2}{3h} \omega \rho_e |\tilde{\omega}(\omega)|^2 + \sum \frac{8\pi^2e^2}{3h} \omega \rho_0 |\tilde{\omega}(\omega)|^2. \quad (5.4b)$$

To make contact with more familiar concepts we introduce the Einstein coefficients, $A(\alpha)$ and $A(\beta)$ being the probabilities per unit time of spontaneous absorption and emission, respectively, and $B(\alpha)$ and $B(\beta)$ the corresponding probabilities for the transitions induced by the external field. Thus,

$$W_{\text{abs}} = \sum \Delta E \left( B(\alpha) \rho_e + A(\alpha) \right), \quad (5.5)$$

$$W_{\text{emi}} = \sum \Delta E \left( B(\beta) \rho_e + A(\beta) \right).$$

Comparing with Eqs. (5.4) we get the following equalities:

$$A(\alpha) = 0, \quad A(\beta) = \frac{8\pi^2e^2 \omega}{3h \Delta E} \rho_0 |\tilde{\omega}(\omega)|^2, \quad (5.6)$$

$$B(\alpha) = \frac{4\pi^2e^2 \omega}{3h \Delta E} |\tilde{\omega}(\alpha)|^2, \quad B(\beta) = \frac{4\pi^2e^2 \omega}{3h \Delta E} |\tilde{\omega}(\beta)|^2.$$ 

With $\Delta E$ given by Eq. (3.2), these equations agree with the corresponding formulas of QED.

Note from Eqs. (5.3) that for $\rho = \rho_0$, while the terms due to the vacuum field and to radiation reaction cancel each other in $W_{\text{abs}}$, they contribute with an equal amount to $W_{\text{emi}}$. Here we have a physical explanation for the long known (but
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poorly understood) factor 2 that appears in the above formula $A^{(-)} = 2\rho_0 B^{(-)}$. In QED, the question of whether the spontaneous transitions are due to the action of the vacuum field or to radiation reaction has been a matter of long discussions (a review on this subject is given in Ref. [12]; a discussion of the physical meaning of the quantities $\rho + \rho_0$ and $\rho - \rho_0$ may be found in Ref. [8]).

It is illustrative to rewrite the detailed-balance condition $W_{\text{abs}}(\omega) = W_{\text{emi}}(\omega)$ as a condition on the power spectrum:

$$S(\omega) = \frac{m \tau \hbar}{\pi e^2} |\omega|^3 \left( \frac{|\hat{x}^{(+)}|^2 + |\hat{x}^{(-)}|^2}{|\hat{x}^{(+)}|^2 - |\hat{x}^{(-)}|^2} \right).$$

Here $|\hat{x}^{(+)}|^2$ and $|\hat{x}^{(-)}|^2$ refer to upward and downward transitions from the same state $\alpha$ and with the same (absolute) frequency $\omega$. When the system is in an excited state, generally $\hat{x}^{(-)} \neq 0$ and $\hat{x}^{(+)} = 0$ for at least one frequency, which means that no power spectrum exists that can maintain a single excited atomic state in detailed balance with the radiation field. An important exception to this rule is the harmonic oscillator (because $\hat{x}^{(-)}$ and $\hat{x}^{(+)}$ are different from zero for the same $\omega = \omega_0$): its $n^{\text{th}}$ excited state is in equilibrium with a field of power spectrum

$$S = S_0(\omega) \frac{\left| \hat{x}^{(+)}_{n,n+1} \right|^2 + \left| \hat{x}^{(-)}_{n,n-1} \right|^2}{\left| \hat{x}^{(+)}_{n,n+1} \right|^2 - \left| \hat{x}^{(-)}_{n,n-1} \right|^2} = S_0(\omega)(1 + 2n).$$

This expression corresponds to what is usually called a field state of $n$ photons (of frequency $\omega = \omega_0$).

6. The Planck distribution

Consider now an ensemble of atoms in different states, in equilibrium with a given radiation field. Let the average population of state $\alpha$ be $N_\alpha$ and so on; the master equation for $N_\alpha$ is

$$\frac{dN_\alpha}{dt} = \sum_\beta [N_\beta P_{\beta \rightarrow \alpha} - N_\alpha P_{\alpha \rightarrow \beta}]$$

where $P_{\alpha \rightarrow \beta}$ is the probability per unit time of a transition $\alpha \rightarrow \beta$. Under the assumption of detailed equilibrium this equation should be satisfied term by term (assuming non-degenerate transition frequencies), hence $N_\beta P_{\beta \rightarrow \alpha} = N_\alpha P_{\alpha \rightarrow \beta}$. In our previous notation this equation reads

$$N_\beta W_{\text{emi}}(\beta \rightarrow \alpha) = N_\alpha W_{\text{abs}}(\alpha \rightarrow \beta) \quad (6.1)$$
because \( W(\alpha \to \beta) = |\Delta E_{\alpha\beta}| \rho_{\alpha\to\beta} \). Introducing Eqs. (5.5) and (5.6) we obtain a general formula for the spectrum of the field that is in equilibrium with the populations \( N_\alpha \) and \( N_\beta \):

\[
\rho = \rho_0 + \rho_e = \rho_0 \frac{N_\alpha + N_\beta}{N_\alpha - N_\beta}.
\]

In particular, in thermodynamic equilibrium the atomic populations follow the Maxwell-Boltzmann distribution, so that \( \frac{N_\alpha}{N_\beta} = \exp(\beta E_\beta - \beta E_\alpha) \) with \( \beta = \frac{1}{kT} \), and the last equation gives the Planck distribution

\[
\rho = \frac{\hbar \omega^3}{\pi^2 c^3} \left[ \frac{1}{2} + \frac{1}{e^{\beta \hbar \omega} - 1} \right]
\]

with \( \omega = |\omega_{\beta\alpha}| = |E_\beta - E_\alpha|/\hbar \). For other works concerning the derivation of Planck's law from SED, see the literature cited in Ref. [5] and more recent work in Refs. [8, 13 and 14].

In the above calculation the Planck distribution followed from the demand of conservation of energy during transitions. In his original paper, [15] Einstein showed that the same results for the \( A \) and \( B \) coefficients and for \( \rho(\omega, T) \) follow from the demand of conservation of momentum, if \( \hbar \omega/c \) is the momentum exchanged with the field in a definite but arbitrary direction. (For more recent work on the subject see Refs. [12 and 16].) A treatment similar to the above one [6] shows that there is a stepwise exchange of angular momentum, and leads to the same results for \( A, B \) and \( \rho \), though expressed in more detail. The general conclusion is that Planck's law is consistent with the (detailed) statistical balance of energy, linear momentum and angular momentum during atomic transitions.

Note that according to our discussion, in every transition only one mode of the radiation field is involved, and this mode absorbs or delivers the whole of the exchanged energy. This picture of a transition process as an elementary act of interaction contains the essence of the quantum description of the matter-field interaction, even though it is made in terms of continuous quantities. This approach helps us gain some understanding on the mechanism of matter quantization. Imagine an atomic system in its ground state, which is the stationary state in equilibrium with the zero-point field. Now, for an electronic excitation to occur it must be induced by an external field of a frequency \( \omega_{\alpha\alpha} \), and this excitation takes the system to a new state with precisely an additional energy \( \hbar \omega_{\alpha\alpha} \). The quantum states are the stationary states in the so-called quantum regime, that are mutually connected by means of such transitions.

Dedicatory

This work is a tribute to our friend and colleague Leopoldo García-Colín.
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References

7. A brief account of the material contained in this and the next section is given in M. Alcubierre, A.M. Cetto, L. de la Peña and N.S. Lozano, “Detailed balance and radiative transitions according to the new approach to stochastic electrodynamics”, preprint IFUNAM (1988).

Resumen. Recientemente ha sido desarrollada una versión no perturbativa de la electrodinámica estocástica que conduce al formalismo de la mecánica cuántica en el espacio de Hilbert. Con base en la conexión ya establecida entre estas dos teorías, se intenta en el presente trabajo avanzar en la comprensión del mecanismo específico de interacción entre el campo estocástico de radiación y la partícula cargada; en particular, se pone en evidencia la respuesta lineal de la partícula a ciertos modos del campo. Esta propiedad se explota para derivar los coeficientes de Einstein y la distribución de Planck, desde la perspectiva estocástica.