Resonance phenomena at high level density and quantum chaos

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ABSTRACT. The resonance phenomena in nuclear reactions are traced numerically as well as analytically from low level density to high level density in a model in which relevant and irrelevant modes are taken into account. At the transition from low to high level density, a redistribution takes place in the nucleus which changes relevant modes into irrelevant ones and vice versa. Further, regular motion goes over into chaotic motion accompanied by a new order with less degrees of freedom. The chaotic modes at high level density are irrelevant. They appear in the nuclear reaction cross section in the form of fluctuations. The regular modes, however, are relevant at low as well as at high level density. They can be seen in the nuclear reaction cross section as resonances or as intermediate structures.

RESUMEN. Los fenómenos de resonancia en reacciones nucleares son estudiados numéricamente y analíticamente desde una baja densidad a una alta densidad de niveles, con un modelo en el cual modos relevantes e irrelevantes son tomados en cuenta. En la transición de la baja a la alta densidad de niveles, una redistribución se lleva a cabo en el núcleo que cambia los modos relevantes a irrelevantes y vice versa. Además, el movimiento regular cambia a movimiento caótico acompañado por un nuevo orden con menos grados de libertad. Los modos caóticos en alta densidad de niveles son irrelevantes. Estos modos aparecen en la sección eficaz de la reacción nuclear en la forma de fluctuaciones. Los modos regulares, sin embargo, son relevantes tanto en baja como en alta densidad de niveles. Se puede verlos en la sección total de la reacción nuclear como resonancias o como estructuras intermedias.

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1. INTRODUCTION

Resonance phenomena in nuclear reactions are investigated for a long time. At low level density, there is a one-to-one correspondence between the states of the system and the resonances observed in the nuclear reaction cross section. Most nuclear states are investigated by using this identity. At high level density, however, fluctuations appear in the cross section instead of resonances, as a rule. The resonance states can be identified in the nuclear reaction cross section only in a very few cases. An example are the narrow neutron resonances appearing in a small energy region above the neutron decay threshold in heavy nuclei. Another example are the so-called doorway states which are collective modes coupled strongly to the decay channels. The underlying fine structure resonance states are treated successfully by using statistical methods.

The resonance phenomena at high level density are investigated properly in heavy ion
reactions. As a result, the experimental data made clear that a reaction theory has to be
developed which encompasses simultaneously both shorter and longer time scales [1]. This
result is surely connected with the phenomenon of collectivization and trapping which was
shown theoretically to appear at high level density or strong coupling strength [2]: At a
certain critical value of the level density, a dramatical redistribution inside the nucleus
takes place. A few coherent states are formed the widths of which are large in comparison
to the sum of the widths of all states. The remaining states become trapped. The number
of coherent states is equal to the number of open decay channels, while the number of
trapped modes (fine structure resonances) may be very large in realistic many-particle
systems.

The problem arises to trace the transition between the two different situations which
are observed in nuclear reactions at low and at high level density.

In the following, this transition will be traced numerically as well as analytically on
the basis of the continuum shell model. As a result, the redistribution taking place in the
nucleus is shown to be connected with a transition from regular motion to chaotic motion
of the nucleons accompanied by the formation of a new order. The number of degrees of
freedom is effectively reduced by the redistribution: some of them are frozen out. By this,
the relation between the direct reaction part, in which only a few (coherent) degrees of
freedom are excited, and the resonance reaction part in which a larger number of degrees
of freedom is excited, is changed. The cross section changes from a resonance picture to
a fluctuation picture.

2. THE CORRECTIONS TO THE SPECTROSCOPIC VALUES DUE TO THE COUPLING TO THE
CONTINUUM

In order to trace the transition from low to high level density numerically, the calculations
have to be performed in a function space which contains not only the relevant modes but
also the irrelevant ones. The point is that some of the relevant modes become irrelevant
and vice versa as a consequence of the transition.

The calculations at low level density are performed usually in the function space \( Q \)
of discrete states by neglecting the continuum (\( P \)-subspace) in which most of them are
embedded, altogether [3]. The correction terms to the spectroscopic values which arise
from the coupling between the two subspaces are described by the constraint operator

\[
Z = Q + G^{(+)}_P H_{PQ}
\]

which appears instead of the operator \( Q \) in the spectroscopic values if the coupling to the
continuum is taken into account in a straightforward manner [4]. Here, \( G^{(+)}_P \) is the Green
function for the motion of the particle in the continuum while \( H_{PQ} = PHQ \) describes the
coupling between the two subspaces and \( H = H_0 + V \).

The term \( G^{(+)}_P H_{PQ} \) in the constraint operator \( Z \) describes the feedback from the con-
tinuum of decay channels on the properties of the bound states. It contains the difference
between a system the accessible phase space of which is restricted by boundary conditions,
and a system for which no such restrictions exist. The feedback term becomes important
at high level density. It causes the redistribution taking place in the nucleus as soon as
the resonances start to overlap.

The correction terms to the spectroscopic values are energy dependent. The effective
Hamiltonian in the $Q$-subspace

$$H_{QQ}^{\text{eff}} = H_{QQ} + H_{QP}G_P^{(+)}H_{PQ}$$  \hspace{1cm} (2)

is non-Hermitean. Its eigenvalues are complex and give not only the energies $\tilde{E}_R$ of the
resonance states but also their total widths $\tilde{\Gamma}_R$. The wavefunction

$$\tilde{\Omega}_R = \tilde{\Phi}_R + \tilde{\omega}_R$$  \hspace{1cm} (3)

of the state $R$ contains the correction term

$$\tilde{\omega}_R = G_P^{(+)}H_{PQ}\tilde{\Phi}_R$$  \hspace{1cm} (4)

where $\tilde{\Phi}_R$ is an eigenfunction of $H_{QQ}^{\text{eff}}$. The wavefunction $\tilde{\Omega}_R$ has the true asymptotic
behaviour. The partial width amplitude is defined by

$$\tilde{\gamma}_{Rc} = (2\pi)^{1/2} < \tilde{\Omega}_R|V|\chi_E^c >$$  \hspace{1cm} (5)

where $\chi_E^c$ is the unperturbed wavefunction of the channel $c$ and $V$ is the residual inter-
action. It contains a correction term arising from $\tilde{\omega}_R$ [2].

In the continuum shell model, the total width $\tilde{\Gamma}_R$ and the partial widths $|\tilde{\gamma}_{Rc}^2|$ are
calculated independently from each other. It follows

$$\tilde{\Gamma}_R = \sum_c |\tilde{\gamma}_{Rc}^2|$$  \hspace{1cm} (6)

in a nontrivial manner for isolated resonance states. Generally, the total width is smaller
than the sum of the partial widths [2],

$$\tilde{\Gamma}_R \leq \sum_c |\tilde{\gamma}_{Rc}^2|.$$  \hspace{1cm} (7)

The widths $\tilde{\Gamma}_R$ of the states embedded in the continuum of decay channels follow
immediately from the eigenvalues $\tilde{E}_R = \bar{E}_R - \frac{i}{2}\tilde{\Gamma}_R$ of the Hamiltonian $H_{QQ}^{\text{eff}}$, Eq. (2).
They are almost independent of energy in an energy region characteristic of the system,
and it holds [2]

$$\sum_R \tilde{\Gamma}_R = \text{const}$$  \hspace{1cm} (8)

at a certain energy $E$ of the system for the simple reason that the sum of the eigenvalues
is equal to the sum of the diagonal elements. Physically, the relation (8) expresses the
conservation law for the quantum fluctuations between the system and the continuum of decay channels in which the system is embedded.

**Table I.** Resonances in the nuclear system with one open decay channel

<table>
<thead>
<tr>
<th>Low level density</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling to the continuum</td>
<td>weakly</td>
</tr>
<tr>
<td>Resonance parameters of ( N ) resonances</td>
<td>( E_R, \Gamma_R, \gamma_Rc ) energy independent</td>
</tr>
<tr>
<td>Background</td>
<td>small:</td>
</tr>
<tr>
<td></td>
<td><em>almost no feedback from the open decay channel</em></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High level density</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling to the continuum</td>
<td>strongly</td>
</tr>
<tr>
<td>Resonance parameters of ( N - 1 ) resonances (fine structure resonances)</td>
<td>( E_R, \Gamma_R, \gamma_Rc ) energy independent</td>
</tr>
<tr>
<td>Background</td>
<td>large: one broad mode</td>
</tr>
<tr>
<td></td>
<td><em>feedback from the open decay channel</em></td>
</tr>
</tbody>
</table>

3. The transition from low to high level density

Every redistribution taking place in the nuclear \( A \)-particle system occurs under the condition (8). Indeed, the widths of \( A \) states increase, at high level density, at the cost of the remaining \( N - A \) ones where \( A \) is the number of open decay channels. The \( A \) fast modes take almost the whole sum of the widths,

\[
\sum_{R=1}^{A} \tilde{\Gamma}_R \approx \sum_{R=1}^{N} \tilde{\Gamma}_R \tag{9}
\]

while

\[
\sum_{R=1}^{N-A} \tilde{\Gamma}_R \approx 0 \tag{10}
\]

for the widths of the remaining \( N - A \) states. Altogether, the properties of the short-lived states are “channelized” by the open decay channels while the remaining states are trapped. The trapped modes correspond to narrow resonances coupled weakly to the continuum while the broad modes resemble the background (Table I). This behaviour has been proved analytically [7]. In other words, the direct reaction part increases at the cost
of the resonance reaction part at high level density, i.e. a dynamical localization sets in [5].

The redistribution (9) and (10) of the widths which takes place at the transition from low to high level density in nuclei is in full agreement with our knowledge about the properties of nuclei. In the standard methods developed for their description, the relevant part of the function space changes from the $Q$-subspace of discrete states at low level density to the $P'$-subspace of open decay channels at high level density (Table II, where $\xi_E^c$ are coupled-channel wavefunctions obeying \( (H_{PP'} - E)\xi_E^c = 0 \)). This is exact the change which is obtained in the numerical calculations in tracing the transition from low to high level density [2].

**Table II. Standard methods for the description of nuclei**

**Low level density**

<table>
<thead>
<tr>
<th>Effective Hamiltonian</th>
<th>( H'<em>{QQ} = H</em>{QQ} + H_{QP}G^{(+)}<em>P H</em>{PQ} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavefunction</td>
<td>( \tilde{\Omega}_R^{(+)} = \tilde{\Phi}<em>R^{(+)} + G_P^{(+)} H</em>{PQ} \tilde{\Phi}_R^{(+)} )</td>
</tr>
<tr>
<td>Relevant part of the function space</td>
<td>$Q$-subspace: function space of a standard nuclear structure calculation (all nucleons occupy bound or quasibound single particle states inside the nucleus)</td>
</tr>
<tr>
<td>Remaining part of the function space</td>
<td>$P$-subspace: continuum of decay channels (open and closed decay channels and quasibound single-particle states at large radii)</td>
</tr>
<tr>
<td>Model calculation</td>
<td>( P + Q = 1 )</td>
</tr>
</tbody>
</table>

**High level density**

| Effective Hamiltonian | \( H'_{PP'} = H_{PP'} + 
\quad H_{P'Q'} (E - H_{Q'Q'})^{-1} H_{Q'P'} \) |
|-----------------------|--------------------------------------------------|
| Wavefunction          | \( \Psi_E^{c(+)} = \xi_E^c + 
\quad (1 + G_P^{(+)} H_{P'Q'}) (E - H_{Q'Q'})^{-1} H_{Q'P'} \xi_E^c \) |
| Relevant part of the function space | $P'$-subspace: function space of a standard coupled-channel calculation (open decay channels) |
| Remaining part of the function space | $Q'$-subspace: “traps” in the continuum (compound-nucleus states and closed decay channels) |
| Model calculation     | \( P' + Q' = 1 \) |
4. Fluctuations in the Cross Section and Quantum Chaos

In [5], the narrow neutron resonances are suggested to be trapped modes. Their properties are used in order to characterize quantum chaos (Table III). According to this suggestion, quantum chaos is caused by dynamical localization taking place in a quantum system at high level density. It has properties very similar to those of chaos in classical systems. It reacts, e.g., very sensitively to small changes in the initial conditions [6]. The wavefunctions are strongly mixed.

<table>
<thead>
<tr>
<th>Table III. Properties of quantum chaos</th>
</tr>
</thead>
<tbody>
<tr>
<td>— The spectral distribution shows level repulsion</td>
</tr>
<tr>
<td>— The system is stable and long-lived</td>
</tr>
<tr>
<td>— Small changes in the initial conditions create large changes in the final products</td>
</tr>
<tr>
<td>— The effective number of degrees of freedom is larger than the number of constants of motion</td>
</tr>
<tr>
<td>— The entropy is large due to clustering of levels which lie extremely dense</td>
</tr>
<tr>
<td>— The characteristic value is the number $N = \frac{\Delta E}{D}$ of resonances in an energy region $\Delta E$ typical for the system</td>
</tr>
</tbody>
</table>

in all many-particle configurations and the chaotic motion is stable as a whole. The entropy is large.

According to the theoretical investigations [2,7,8], the trapped modes (narrow resonances) are enveloped by broad single-particle resonances ("doorway states"). Nevertheless, the spectral distribution of the narrow resonances can be investigated directly in the one-channel case. An example are the narrow neutron resonances. In the many-channel case, the cross section shows fluctuations instead of narrow resonances [2,9]. However, the nucleus can register small changes in energy also in this case, since the fluctuation picture is caused by many dense lying narrow resonances.

The calculations show the transition from the resonance picture of the nuclear reaction cross section at low level density to the fluctuation picture at high level density (Fig. 1 with two open decay channels [2]). It is a transition from a regular motion of the nucleons to a chaotic motion accompanied by the formation of a new order [2].

For the nucleus, we get the picture which is illustrated in Table IV. It corresponds to our experience and to the models worked out for describing the (relevant) properties of the nuclei at low and at high level density.
FIGURE 1. Transition from the resonance picture of the nuclear reaction cross section at low level density to the fluctuation picture at high level density.
Table IV. Nucleus with regular and chaotic motion of the nucleons

### Regular motion

- low level density: $N \approx \Lambda$
- isolated resonance states
- nuclear spectroscopy:
  - ground state domain, isobaric analog resonances, Gamow-Teller-resonances, giant multipole resonances, high spin states
- standard nuclear structure theory:
  - The regular motion is expected to prevail up to high excitation energies
  - in contrast to our knowledge on the compound nucleus formation in reactions
- isolated resonances in the cross section (also in the many-channel case)
- constraint operator: $Z \approx Q$

### Chaotic motion together with a few coherent ("channelized") modes

- high level density: $N \gg \Lambda$
- clustering of resonance states (formation of intermediate structures)
- nuclear reactions:
  - compound nuclei, heavy ion reactions
- standard nuclear reaction theory:
  - The chaotic motion is expected to prevail all the way down to the ground state domain - in contrast to our knowledge on the spectroscopic properties of nuclei
- fluctuations in the cross section (in the many-channel case)
- constraint operator: $Z \approx G_P^{(+)} H_{PQ}$

5. Summary

In this talk, the resonance behaviour of the nuclear reaction cross section is traced from low level density to high level density by taking into account relevant as well as irrelevant modes. Summarizing the results, it can be stated that quantum chaos may be formed in
many-particle systems at sufficiently strong coupling in trapped modes as a consequence of the high level density which increases, as a function of energy, much faster than the number Λ of open decay channels. The number of degrees of freedom which correspond to the motion of the nucleons in the potential is much larger than the number of degrees of freedom belonging to the open decay channels. By trapping, most of them become ineffective. That means, quantum chaos is screened, to a great deal. It appears in irrelevant modes. Nevertheless, its existence can be tested in high-resolution experiments because the timescales of the fast and slow modes are very well separated from each other. It is possible, e.g., to prove whether small perturbations in the initial conditions are amplified and lead to large differences in the final results.

Only in the one-channel case, the spectral distribution at high level density can be investigated directly from the cross section without time discrimination since narrow resonances are observed. In most cases, more than one decay channel are open and the cross section shows fluctuations instead of isolated resonances. If the fluctuation picture is caused, indeed, by trapped modes, the system can register small changes in the energy also in this case and it is possible to investigate the question whether there is a sensitive dependence on initial conditions or not. In other words, it is possible to prove whether small changes in the initial energy lead to large changes of the total kinetic energy of the outgoing particles. The result of such an investigation testifies directly the existence of chaotic motion in the many-particle system investigated.

Acknowledgment

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References