The Production of Higgs Bosons and $Z^0$ Bosons in Heavy-Ion Colliders

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ABSTRACT. We investigate the formation of exotic particles in ultrarelativistic heavy-ion collisions. We calculate the impact-parameter dependence of the electromagnetic production of intermediate-mass Higgs bosons. For an assumed scalar Higgs mass of $M_H = 100$ GeV we obtain a reduced cross section $\sigma_H = 57$ Pb for a Pb-Pb collision at LHC energies with $\Gamma = 3500$. In addition we study a possible generation of magnetic monopoles in the superstrong electromagnetic field of relativistic heavy ions. Also hadronic reactions in central collisions of nuclei are considered. Theoretical results for the production cross section of top quarks, Higgs bosons and $Z^0$ bosons will be presented. Relevant quantities of the toponium are calculated within a nonrelativistic potential model. It is demonstrated that the multiplicity distribution of produced lepton pairs can be utilized to determine the impact parameter of the heavy-ion collision.

RESUMEN. Investiguamos la formación de partículas exóticas en colisiones ultrarelativistas de iones pesados. Calculamos la dependencia del parámetro de impacto de la producción electromagnetic de bosones de Higgs con masas intermedias. Para una masa supuesta del boson escalar de Higgs $M_H = 100$ GeV, obtenemos una sección eficaz reducida $\sigma_H = 57$ Pb para una colisión de Pb-Pb a energías del LHC con $\Gamma = 3500$. Adicionalmente estudiamos la posible generación de monopoles magnéticos en el campo electromagnetic superfuerte de iones pesados relativistas. También consideramos reacciones hadrónicas en colisiones centrales de núcleos. Presentamos resultados teóricos para la sección eficaz de producción de top quarks, bosones de Higgs y bosones $Z^0$. Calculamos cantidades relevantes del toponium dentro de un modelo de potencial non-relativista. Demostramos que la distribución de la multiplicidad de producción de pares de leptones se pueden utilizar para determinar el parámetro de impacto en la colisión de iones pesados.

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1. Introduction

Currently, the possibility of accelerating heavy ions with the high-energy colliders LHC (CERN) and SSC (Texas) is under discussion. The major motivation for the construction
of these colliders is the detection and investigation of top quarks and Higgs bosons, but in
the available energy region (3.5 TeV/u for LHC and 8 TeV/u for SSC; both for accelerating
208 Pb) other electroweak phenomena could also play an important role.

In this report we concentrate on the formation of Higgs bosons and $Z^0$-bosons in central
as well as in peripheral ultrarelativistic heavy-ion collisions. While peripheral collisions
have been investigated already in previous publications we present here first estimates for
the production cross sections of exotic particles in central nucleus-nucleus collisions, which
in the considered energy domain are dominated by parton interactions. Furthermore, we
evaluate the production rate for top-quark pairs in heavy-ion reactions. The possible
generation of magnetic monopoles in the superstrong electromagnetic field of heavy-ion
transits is briefly outlined. Finally we demonstrate that the high multiplicity of about
1000 created electron-positron pairs may be utilized to determine the impact parameter
of the collision.

2. HIGGS-BOSON PRODUCTION IN CENTRAL HEAVY-ION COLLISIONS

The search for the Higgs particle(s) predicted by the Standard Model (and its exten-
sions) is one of the topics in contemporary physics. Here we restrict ourselves to the
Minimal Standard Model where only one Higgs boson is involved. The Higgs-boson mass
enters the theory as a free parameter and is bounded from below by recent experimental
results to be larger than 50 GeV. An upper bound of about 1 TeV may be derived
from theoretical considerations such as triviality of the scalar field sector of the theory
and the loss of unitarity. The remaining energy range is generally divided into three
regions determined by collider capabilities and decay modes of the Higgs boson. We are
basically interested in the intermediate mass range ($m_Z \leq M_H \leq 2 m_W$), in which the
branching ratio of the Higgs decay into $b\bar{b}$ is large. However, in this energy range
only the proton colliders SSC and LHC will be available in the foreseeable future. But
also an option to collide heavy ions is under discussion. Central collisions presumably
produce an overwhelming hadronic background which will hide the $b\bar{b}$ signal for the Higgs
decay completely. Thus it was proposed [1 – 4] to create Higgs particles in the strong
electromagnetic fields of peripheral heavy-ion collisions. But recently it turned out [5, 6]
that the Higgs-boson production rate is too small and that the background processes
are not negligible. Since there exist calculations [7] for the Higgs-boson production in
proton-proton collisions which incorporate a chance for the detection of Higgs bosons
by their photon decay, we investigated the production rate of Higgs bosons in central
heavy-ion collisions at LHC energies.

We first calculated the total cross section for producing Higgs particles in nucleon-
nucleon collisions. For this reason we multiplied the elementary cross sections for the
processes $gg \rightarrow H$ and $q \bar{q} \rightarrow H$ with the corresponding gluon-distribution function
$G(x)$ and quark-distribution functions $q(x)$ and $\bar{q}(x)$ of a nucleon [11] and performed
the integration over the scaling variables

$$
\sigma_1 = \int dx \int dx' G(x)G(x') \sigma_{gg \rightarrow H}(x, x'),
$$

(1)
FIGURE 1. Total cross section for Higgs-boson production in central Pb–Pb collisions at 3.5 TeV/u. The gluon (g)– and quark (q) – fusion contribution is indicated by the full and dashed line, respectively.

The elementary cross sections read [7]

\[
\sigma_{gg\rightarrow H} = \frac{\alpha_s^2 M_H^2 G_F^2}{32 \pi} |A_q|^2 ,
\]

\[
\sigma_{q\bar{q}\rightarrow H} = \frac{\pi G_F}{3 \sqrt{2}} \left( 1 - 4 m_q^2 / M_H^2 \right)^{1/2} M_q^2 ,
\]

where \( G_F \) is the weak coupling constant and \( A_q \) is the amplitude of the quark loop in the triangle diagram for the process \( gg \rightarrow H \). The quark masses \( m_q \) were approximated in next-to-leading-log order.

In order to derive the total Higgs-boson production cross section \( \sigma_1 \) and \( \sigma_2 \) were multiplied with the total number of individual nucleon-nucleon reactions during the heavy-ion collision. This number was computed employing a simplified version of the RQMD model [8] which yields up to 1250 nucleon-nucleon collisions at LHC energies.

In Fig. 1 the total Higgs-boson production cross section is plotted versus the Higgs-boson mass assuming a Pb-Pb collision at LHC (\( E_{\text{ion}} = 3.5 \text{ TeV/u} \)). The full line corresponds to the gluon-fusion process which dominates the \( q\bar{q} \rightarrow H \) process (represented
The strong electromagnetic fields prevailing in ultrarelativistic heavy-ion collisions could give rise to the possible production of exotic particles, particularly Higgs bosons in the intermediate mass region $m_Z < m_H < 2m_W$. First calculations [1-5] lead to production cross sections in the 1 nb region for SSC energies. All previous calculations rely on the Weizsäcker-Williams or equivalent photon method leading to the total cross section

$$\sigma = \int d\omega_1 d\omega_2 n_A(\omega_1) n_B(\omega_2) \sigma_{\gamma\gamma \rightarrow H}^{}(\omega_1, \omega_2)$$

with the photon-distribution function $n(\omega)$ and the elementary cross section $\sigma_{\gamma\gamma \rightarrow H}$ for the production of a Higgs boson via two real photons.

Looking closer on the production of Higgs bosons, one might question whether the main contribution to the electromagnetic cross section really results from impact parameters being larger than twice the nuclear radius. Central collisions should be omitted because of
nuclear reactions, which would mask a clean experimental signal. The equivalent photon method which has been employed to calculate the production cross sections only yields total cross sections, because the wave front of the equivalent photons is by definition extended to infinity in transverse directions. In order to overcome this short-coming, we developed an impact-parameter dependent equivalent photon method for the Higgs-boson production [4,9]. The impact-parameter dependent cross section reads

\[ \frac{d^2 \sigma}{db^2} = \int d\omega_1 \int d\omega_2 \ n_{AB}(\omega_1, \omega_2, b) \sigma_{\gamma\gamma \rightarrow H}(\omega_1, \omega_2) \]  

with the two-photon distribution function \( n(\omega_1, \omega_2, b) \) of the two nuclei colliding at a distance \( b \). Introducing a sharp cut-off impact parameter, \( b_C = 2R \), and thus discarding central collisions, the reduced cross section

\[ \bar{\sigma} = \int d^2 b \frac{d^2 \sigma}{db^2} \theta(b - 2R) \]  

for the scalar Higgs-boson production in Pb–Pb collision was calculated. \( R \) denotes the nuclear radius. Fig. 2 displays the total (full curve) and the reduced (broken curve) cross section for the maximum LHC energy (lower curves) and SSC energy (upper curves). The reduced cross section for the scalar Higgs-boson production is only by a factor of 2.3(3.1) at LHC energies and 1.6(1.8) at SSC energies for a Higgs mass of 100(150) GeV smaller than the corresponding equivalent photon cross section.

The minimal supersymmetric extension of the standard model requires five Higgs bosons: two scalar, one pseudoscalar and two oppositely charged bosons. The reduction factors for the two scalar Higgs bosons are the same as for the standard Higgs boson. In the case of the pseudoscalar Higgs boson one has to take into consideration that the electric fields of the two photons have to be perpendicular. The reduced cross section for the pseudoscalar Higgs-boson production is only by a factor of 2.3(3.3) at the LHC energy and 1.5(1.7) at the SSC energy for a Higgs mass of 100(150) GeV smaller than the corresponding total equivalent photon cross section.

4. \( Z^0 \) AND DILEPTON PRODUCTION

This section deals with the evaluation of the production rate for neutral intermediate \( Z^0 \) bosons during an high-energy heavy-ion collision. This process can contribute to the background for Higgs-boson production and, in general, can serve for investigations of electroweak phenomena in heavy-ion collisions.

Our calculation comprises several steps:

- First the possible elementary processes that could contribute significantly were selected. Quarks (\( q \)), gluons (\( g \)) and photons (\( \gamma \)) as input particles were taken into account.
- The elementary total cross sections were calculated using Mandelstam variables.
- C. Kao [10] performed the calculation for the process \( gg \rightarrow Z^0Z^0 \), partially by incorporating a triangle graph with an intermediate Higgs boson.
In a next step the total nucleon–nucleon cross sections $\sigma_{n-n}$ were derived employing the parton model with the parton distribution functions of Duke and Owens [11].

The incoherence assumption underlying the parton model was also applied on the next higher level, i.e., the nucleus–nucleus collisions were treated as an incoherent sum over all binary nucleon–nucleon collisions which occur during an heavy–ion collision. To estimate the average number $n(b)$ of binary collisions during a nucleus–nucleus collision with an impact parameter $b$, again a simplified version of the RQMD model [8] was employed. The total cross sections for $^{208}\text{Pb}^{208}\text{Pb}$ collisions were calculated according to

$$\sigma_{\text{Pb-Pb}} = 2\pi \frac{\sigma_{n-n}}{\sigma_{\text{tot}}} \int_0^{2R} n(b) b \, db. \tag{8}$$

This expression gives the total integrated nuclear interaction area normalized to the area of the total hadronic interaction (i.e. the total hadronic cross section $\sigma_{\text{tot}}$), which yields the requested total heavy–ion cross section. $R$ denotes the nuclear radius.

Alternatively, we utilized the equivalent photon method to calculate the heavy–ion cross section for $Z^0$ production in peripheral collisions that proceed via the process $q\gamma \rightarrow qZ^0$, which only plays an important role for collisions of highly charged particles since the integrated photon spectrum rises quadratically with the charge number $Z$ of the collision partners.

The detection of produced $Z^0$ bosons is supposed to proceed via their decay into dileptons, which will lead to a detectable clear signal. Results of the latest LEP measurements [12] were used to fix the partial decay rate of the $Z^0$ boson to

$$\frac{\Gamma_{Z^0 \rightarrow \ell^+\ell^-}}{\Gamma_{\text{tot}}} = 3.35\%. \tag{9}$$

All results of our and Kao’s calculations for LHC energies are summarized in Table I. The respective columns indicate the nucleon–nucleon cross sections $\sigma_{n-n}$, the total heavy–ion cross sections $\sigma_{\text{Pb-Pb}}$, the numbers $N_{Z^0}$ of the produced $Z^0$ bosons during a one “year” ($= 10^7$ s) run of the LHC collider with an assumed luminosity of $\mathcal{L} = 10^{28}$ cm$^{-2}$s$^{-1}$ and the numbers $N_{\mu^+\mu^-}$ of the $Z^0$ bosons which could be detected via their decay into a dilepton pair.
Z\(^0\)–Production via LHC (\(\sqrt{s}/2 = 3.5 \text{ TeV/u, } \mathcal{L} = 10^{28} \text{ cm}^{-2} \text{s}^{-1}\))

<table>
<thead>
<tr>
<th>Process</th>
<th>(\sigma_{\text{n-n}} \text{ [nb]})</th>
<th>(\sigma_{\text{Pb-Pb}} \text{ [nb]})</th>
<th>(N_{Z^0})</th>
<th>(N_{\mu^+\mu^-})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q\bar{q} \rightarrow Z^0 \rightarrow l^+l^-)</td>
<td>(1.0 \cdot 10^{+0})</td>
<td>(1.7 \cdot 10^{+3})</td>
<td>169393</td>
<td>169393</td>
</tr>
<tr>
<td>(q\bar{q} \rightarrow Z^0Z^0)</td>
<td>(3.0 \cdot 10^{-3})</td>
<td>(5.0 \cdot 10^{0})</td>
<td>497</td>
<td>17</td>
</tr>
<tr>
<td>(q\bar{q} \rightarrow Z^0\gamma)</td>
<td>(5.3 \cdot 10^{+3})</td>
<td>(8.7 \cdot 10^{+3})</td>
<td>867435</td>
<td>30360</td>
</tr>
<tr>
<td>(q\gamma \rightarrow qZ^0)</td>
<td>(m_\gamma = 10^{-1} \text{ GeV})</td>
<td>(1.6 \cdot 10^{+0})</td>
<td>(2.6 \cdot 10^{+3})</td>
<td>264820</td>
</tr>
<tr>
<td>(m_\gamma = 10^{+0} \text{ GeV})</td>
<td>(6.3 \cdot 10^{+1})</td>
<td>(1.0 \cdot 10^{+3})</td>
<td>104806</td>
<td>3511</td>
</tr>
<tr>
<td>(m_\gamma = 10^{+1} \text{ GeV})</td>
<td>(1.3 \cdot 10^{+1})</td>
<td>(2.1 \cdot 10^{+2})</td>
<td>21116</td>
<td>707</td>
</tr>
<tr>
<td>(m_\gamma = 10^{+2} \text{ GeV})</td>
<td>(6.3 \cdot 10^{+3})</td>
<td>(1.0 \cdot 10^{+1})</td>
<td>1038</td>
<td>35</td>
</tr>
<tr>
<td>(gg \rightarrow Z^0Z^0)</td>
<td>(q\gamma \rightarrow qZ^0)</td>
<td>(m_\gamma = 10^{-1} \text{ GeV})</td>
<td>(6.4 \cdot 10^{+3})</td>
<td>641300</td>
</tr>
</tbody>
</table>

Table I. Results for Z\(^0\)-production in Pb–Pb collisions at LHC energies.

The results in Table I demonstrate that the dominant process for Z\(^0\) production is the resonant process \(q\bar{q} \rightarrow Z^0 \rightarrow l^+l^-\). The contribution of non-resonant processes is smaller by orders of magnitude, depending on the masses of the additionally produced particles (cf. \(q\bar{q} \rightarrow Z^0 + X\)). One should emphasize that the process \(q\gamma \rightarrow qZ^0\), which occurs also in peripheral collisions, achieves the second highest yield of all non-resonant processes. This is a direct consequence of the involved high nuclear charges and will be negligible in pure hadronic collisions. The investigated energy range is too low for a comparable Z\(^0\) production via gluon–gluon fusion \(gg \rightarrow Z^0Z^0\). According to Ref. [10] this will change at still higher energies.

To resume our results we can state that there will be a sufficient production rate of Z\(^0\) bosons in these high–energy heavy–ion colliders which yields an important background for the search of top quarks and Higgs bosons, but it allows additionally for further investigations of electroweak interactions during ultrarelativistic heavy–ion collisions.

5. Top-quark production

The ultimate verification of the Standard Model requires the existence of the top-quark. Current experiments did not find a clear signal for a produced top-quark below a mass of \(m_t < 89 \text{ GeV}\) [13]. In addition it is relatively uncertain whether the top-quark can be detected in these experiments, because an extraordinarily high integrated luminosity is necessary to identify a heavy top-quark. Likewise the planned energetically boosted LEP2 with its 100 GeV on 100 GeV \(e^+e^-\)–beam will not provide the possibility for top-quark creation with a mass above \(m_t > 100 \text{ GeV}\). Unfortunately, theoretical predictions prefer a top mass of about \(m_t \approx 150 \text{ GeV}\) [14]. For this reason it will be almost impossible to
create top-quarks with existing colliders, including the Tevatron. We considered top-quark and toponium creation in ultrarelativistic heavy-ion collisions at the LHC and the SSC [15]. Here we present results for free top-quark creation in the two scenarios of central and peripheral heavy-ion collisions.

After reducing the problem of describing a central ultrarelativistic heavy-ion collision to an incoherent sum over single nucleon-nucleon collisions we have to compute the total cross section for top-quark pair creation in hadronic collisions. This is accomplished in the framework of the parton model: the nucleon is treated as a swarm of non-interacting partons (valence quarks, gluons and sea quarks). A top-quark pair is produced through gluon-gluon fusion and quark-antiquark annihilation. The hadronic cross section results from an integral over the parton distributions of the two impinging nucleons and the associated elementary cross section

\[
\sigma_{\text{had}}(S) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \sigma_{ij}(x_1, x_2, S),
\]

where \( S \) is the squared invariant energy of the colliding nucleons, \( Q^2 \) is the four-momentum transfer, and \( i \) and \( j \) distinguish between the different partons. As an example the elementary cross section for top-quark pair creation via quark-antiquark annihilation reads

\[
\sigma(q\bar{q} \rightarrow t\bar{t}) = \frac{2}{9} \frac{4\pi\alpha_s^2}{3s} \sqrt{\frac{s - 4m_t^2}{s - 4m_q^2}} \left(1 + \frac{2m_t^2}{s}\right) \left(1 + \frac{2m_q^2}{s}\right)
\]

with \( \alpha_s \) the running coupling of the strong interaction and \( s \) the squared invariant energy of the parton subprocess.

In order to evaluate total cross sections for top-quark creation via the photon-gluon fusion subprocess in peripheral heavy-ion collisions, the equivalent photon method is utilized. In this parton-like interpretation of the electromagnetic field of a relativistic moving charge the total nuclear cross section is similar to Eq. (10). It is an integral over the effective gluon distribution of the nucleus, the photon distribution and the elementary cross section for the photon-gluon fusion subprocess. As the effective gluon distribution of the nucleus we used

\[
g_{\text{eff}}(x, Q^2) = A^{0.9} g(x, Q^2).
\]

\( g(x, Q^2) \) is the gluon distribution of a nucleon and \( A^{0.9} \) is an experimentally deduced factor that takes into account the nuclear shadowing effect. In Fig. 3 the results for top-quark pair creation in central and peripheral heavy-ion collisions are plotted as function of the top-quark mass \( m_t \).

In comparing the displayed data it results that central collisions at future collider energies represent a top-quark factory, but it appears to be difficult whether the decay products
of the top-quark can unambiguously be isolated from the enormous hadronic background. Peripheral collisions with their reduced creation rate are an interesting alternative, since the hadronic background is drastically suppressed.

6. ELECTROMAGNETIC MONOPOLE-ANTIPOLE CREATION

In addition to the fundamental question of mass creation the quantization of electric charge is of definite interest. Dirac demonstrated that the charge quantization can be understood by assuming the existence of magnetic monopoles. However, no magnetic monopoles have been detected yet in nature. This may lead to the conclusion that either Diracs proposition is wrong and magnetic monopoles are non-existent or that magnetic monopoles like quarks are produced with their total magnetic charge coupled to zero. One can also provide the simple explanation that magnetic monopoles have not yet been found because their masses are too large in order to be generated with existing colliders.

Recently, a lower mass limit of 120 GeV for magnetic monopoles has been derived [16]. This energy threshold for creating monopole-antipole pairs exceeds the capability of present $e^+e^-$ colliders. We studied the photoproduction of magnetic monopoles in the strong electromagnetic fields occurring in heavy-ion collisions [17-19]. This process could take place even for an impact parameter of the heavy ions larger than twice the nuclear radius.
The elementary photoproduction cross section for magnetic monopoles has the same structure as the photoproduction cross section for electrically charged particles

\[
\sigma_{\gamma\gamma \to m\bar{m}}(s) = \frac{4\pi \alpha_m^2}{2s} \left[(3 - \beta^4) \ln \frac{1 + \beta}{1 - \beta} - 2\beta(2 - \beta^2)\right],
\]

with \(\beta = (1 - 4m_m/s)^{1/2}\). \(m_m\) denotes the unknown mass of the magnetic monopole and \(s\) is the ordinary Mandelstam variable. One should note that this expression involves the coupling constant \(\alpha_m = 1/4\alpha = 34.26\) rather than the electromagnetic coupling constant \(\alpha = 1/137.04\). Therefore the cross section for monopole production exceeds the cross section for electrically charged particles of the same mass by seven orders of magnitude!

We obtained the total cross section for producing monopole-antipole pairs in heavy-ion collisions by multiplying the elementary cross section with the photon distributions and integrating over the photon energies

\[
\sigma = \int d\omega \, n(\omega) \int d\omega' \, n'(\omega') \, \sigma_{\gamma\gamma \to m\bar{m}}
\]

The result is plotted in Fig. 4 versus the monopole mass for Pb-Pb collisions at the two collider energies 3.5 TeV/u (LHC) and 8 TeV/u (SSC). For monopole masses near
the threshold up to 1 TeV the cross sections are measurable quantities. Thus we propose to employ the electromagnetic production in heavy-ion collisions for verifying the existence of monopoles in this mass range. Similarly one can also generate the axion [20] and supersymmetric particles [9] in the superstrong electromagnetic field of heavy-ion encounters.

7. Multiplicity Distribution of Direct Electron-Positron Pairs

We investigate the direct creation of electron-positron pairs out of the electromagnetic fields in ultrarelativistic heavy-ion transits at future heavy-ion facilities. At these ultrarelativistic energies, even when the nuclei do not touch, direct pair creation from the electromagnetic fields is abundant. Within the framework of perturbation theory the pair-creation rate even violates unitarity. This is evident from the \( \ln^3 \gamma \) rise of the perturbative total cross section which violates the Froissart unitarity bound that prescribes an asymptotical \( \ln^2 \gamma \) rise. In addition, it has also been demonstrated [21] that the pair creation probability at a fixed small impact parameter exceeds unity already at collider energies as low as 10 GeV/u. This might indicate that perturbation theory overestimates the real cross sections, but it can also imply that the production of multiple pairs becomes dominant over single-pair production.

To describe multiple pair creation non-perturbatively, we employ a many-particle theory of non-interacting electrons in an external field. Both positive and negative energy states are available for the electrons, and the QED vacuum is thus described by a state in which all negative energy states are filled. Each electron obeys the Dirac equation in the given external field. It can be lifted into the positive energy continuum where it is interpreted as a real electron. Though it is not a priori evident that this theory describes QED correctly it can be shown that the \( n \)-point functions (vacuum expectation values) of this Dirac sea theory (which can be computed from the time evolution of the single-particle Dirac equation) coincide with those calculated from ordinary QED. The latter can be derived explicitly and non-perturbatively from the path integral of non-radiative QED in an external field [22] in terms of single-particle amplitudes by a method of quantization in a time-dependent basis.

As it has thus been established that non-radiative external-field QED can be modeled by a Dirac sea theory, both pair creation multiplicities and probabilities are computed from the single-particle scattering matrix \( a_{qp} \) of the Dirac equation. The number of pairs in a state \( p \) is then given by [23]

\[
\bar{n}_p = \sum_{q<0} |a_{qp}|^2.
\]

(15)

\( a_{qp} \) is the Dirac single-particle scattering matrix, while the amplitude to create one pair in the states \( q \) and \( p \) is approximately
where the factor \(< 0|S|0 >\) denotes the amplitude that the vacuum remains unchanged, i.e. no pairs are created. Continuing along these lines, the general probability to create \(n\) pairs is, if interferences are neglected, determined by a Poisson distribution

\[
P_n = e^{-\bar{n}} \frac{\bar{n}^n}{n!}
\]

as was to be suspected from the idea that the pairs are created independently from each other. The exponential factor in front of the expression is in this approximation the vacuum-to-vacuum amplitude \(< 0|S|0 >\) which must be assumed to be unity if perturbation theory is employed. Therefore, if \(\bar{n}\) is large the probability exceeds unity by the neglect of this factor alone, even if the perturbatively evaluated Dirac scattering matrix is still correct. Since this factor does not appear in the expression for the pair multiplicity, this quantity can be computed safely in perturbation theory (as there is a strong suspicion that the single-particle amplitudes are perturbative due to the very short time of interaction). As it turns out, the multiplicity in lowest order is identical to the expression for the perturbative single-pair creation probability \(P_1^{(pert)}\) in the same order:

\[
\bar{n} = \sum_{p>0} \sum_{q<0} |a_{sp}|^2 = P_1^{(pert)}
\]
pair multiplicities feasible which may be important for other electromagnetic processes. Also at RHIC energies (100 GeV/u) we obtained about 1500 $e^+e^-$-pairs for almost central U–U collisions. Beam loss from electron capture may be significant as total cross sections can be taken from well-known Weizsäcker-Williams and other perturbative calculations which give about 500 kbarn for U+U at 3.5 TeV/u.

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