Vibrational degrees of freedom in deformed nuclei

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ABSTRACT. The existence of multi-phonon vibrations in deformed nuclei has been an open question in nuclear structure physics for over thirty years. We have recently found 17 examples of two-phonon vibrations of the $\gamma\gamma$ type in the deformed rare-earth region of nuclei. The excitation energies of the two-phonon vibrational states show a wide range of anharmonicities and not simply the expected dependencies on the one-phonon vibration. There is further evidence that a signature for characterization of these types of vibrational bands may be their deduced dynamic moments of inertia.

RESUMEN. La existencia de vibraciones de varios fonones en núcleos deformados ha sido, por mas de treinta años, una incógnita abierta en la física de la estuctura nuclear. Hemos encontrado recientemente 17 ejemplos de vibraciones de dos fonones del tipo $\gamma\gamma$ en núcleos deformados de la región de las tierras raras. La energía de excitación de estados vibracionales de dos fonones muestran un amplio intervalo de efectos anarmónicos y no simplemente las esperadas dependencias de vibraciones de un fonón. Existe evidencia adicional de que la marca para caracterizar este tipo de bandas rotacionales puede ser los momentos de inercia dinámicos deducidos.

PACS: 27.70.+q; 25.70.Gh; 21.60.Ev

The excitation spectra of many-body systems is traditionally described in terms of elementary excitation modes resulting from various fluctuations around an equilibrium shape [1]. The most important mode for the low-lying levels in the nuclear case for both spherical and deformed nuclei has been shown to be the quadrupole mode where each quantum of excitation is a vibrational phonon. In spherical nuclei, harmonic vibrational motion results in an excitation spectrum consisting of equally spaced degenerate phonon multiplets. Although exact harmonic motion has not been observed, there are numerous examples of nuclei exhibiting near-harmonic or anharmonic vibrational motion. In fact, one and two phonon excitations have been observed in tens of nuclei, as well as, a single isolated case of a set of three phonon excitations [2]. In the low-lying excited states of deformed nuclei, there are two types of quadrupole vibrations superimposed on the rotational states; $\beta$ and $\gamma$. The $\beta$ vibration has its angular momentum aligned along the symmetry axis, whereas, the $\gamma$ vibration breaks axial symmetry and has a projection of $K = 2$ on the symmetry axis. There are many deformed nuclei with known single $\beta$ and $\gamma$ bands. However, the existence of collective two-phonon vibrations of the type $\beta\beta$, ...
$\beta\gamma$, and $\gamma\gamma$ had been an open question [1,3] in nuclear structure physics for over three decades and therefore, the focus of a raging theoretical controversy.

The basic essence of the controversy was the non-observation of any two-phonon vibrational states at the expected excitation energies. That is at energies near twice the excitation energies of the single $\beta$ and $\gamma$ phonons.

The various theoretical approaches can be viewed through a unified perspective of attempts to solve the many-body nuclear problem through the inclusive treatment of collective and single particle degrees of freedom to varying degrees. The existence or nonexistence of collective two-phonon vibrations have a direct impact on our understanding of collectivity in nuclei in general, as well as, elucidating the exact character of the observed single phonon vibrations.

Nuclei in the $A = 150$ deformed region represent the most extensively studied region of nuclei. Yet, there was no information on the existence of two phonon vibrational states. We studied several nuclei in this region with the main aim of searching for multi-phonon vibrational bands. The $^{154,156}$Gd nuclei were studied by gamma-ray spectroscopy following the $(4\text{He},2\text{n})$ reaction on thin targets of $^{152,154}$Sm at the University of Notre Dame Nuclear Structure Laboratory using the University of Pittsburgh multi-detector array [4] of six compton suppressed Ge detectors.

The $^{162}$Dy nucleus was studied by a similar reaction at Lawrence Berkeley Laboratory using the HERA array. In each case, the measurements include angular distributions, gamma-ray singles, and gamma-gamma coincidences. Lifetimes were measured [5] in the $^{168}$Er nucleus using the GRID (gamma-ray induced doppler broadening) technique. This powerful new technique allows the measurements of lifetimes down to femtoseconds and it was adapted for use in heavy nuclei by H. Börner of the ILL.

Two-phonon $\gamma\gamma$ ($K = 4$) vibrations were found in each case. All of the $K = 4$ bands were previously known but misidentified in the literature as quasiparticle excitations. In each case, we were able to add a number of new transitions within the bands, thereby establishing the band structure.

A schematic level scheme of $^{154}$Gd is shown in Figure 1; the inband transitions of 141.18 and 265.88 keV are an order of magnitude stronger than the transitions connecting the band to the single-phonon $\gamma$ band, and several orders of magnitude stronger than the transition to the ground state band. These results are based on relative $B(E2)$ ratios, however, the situation is similar in the neighboring $^{156}$Gd nucleus. The ($K = 4$) $4^+$ level in $^{156}$Gd has a measured lifetime of 190 ps [6]. The absolute $B(E2)$ values show that the transitions from the two-phonon $\gamma\gamma$ band to single-phonon $\gamma$ band are just as collective as the $\gamma$ to ground state transitions. This observed collectivity of the two-phonon to one-phonon transitions is also repeated in $^{168}$Er. Our results for $^{162}$Dy are also based on relative $B(E2)$ values. The emerging scenario is clearly one pointing to the existence of collective two-phonon vibrations in all of these deformed nuclei. There are, however, profound differences in these nuclei. The energy ratios of the excitation energies of the two-phonon vibrational bands with respect to the one-phonon vibrations are quite different. This ratio of energies is 2.50 for $^{168}$Er, 1.73 for $^{162}$Dy, 1.65 for $^{154}$Gd, and 1.31 for $^{156}$Gd.

We expanded our search to include a re-examination of existing data on all the deformed nuclei in the $A = 150$ region. The resulting systematics is shown in Figure 2 where the wide range of observed anharmonicities are from a minimum of 1.23 to a maximum of
2.89. This is a direct indication that the excitation energies of the two-phonon vibrational bands do not simply depend on the energy of the single $\gamma$ phonon.

We have further discovered the surprising fact that the one and two-phonon vibrational bands have identical dynamical moments of inertia. The E2 transition strengths of the first $2^+$ state to the ground state $0^+$ are typically several hundred spu, whereas, the g...
Figure 3. Top: Plot of gamma ray energies for transitions of $\Delta J = 2$ as a function of spin for the ground state, the $\gamma_1$ and the $\gamma\gamma$ bands in $^{154}$Gd. Bottom: The alignment of the $\gamma$ band with the $\gamma\gamma$ band.

band (2$^+$) to ground state transitions are 5–10 spu. This is an indication of the fraction of nucleons that are involved in the total rotational motion of the nucleus versus the vibrational motion. Therefore intuitively, one would expect that a two-phonon vibration may involve twice the number of nucleons. This should be observable in terms of differences in the moments of inertia of the two bands. What we find instead is a set of rigid and identical dynamic moments of inertia for each band [7].

The dynamic moment of inertia is commonly deduced from plots of differences in gamma-ray transition energies that are plotted against rotational frequency. The slope of such a plot is directly proportional to the dynamic moment of inertia. This relationship is shown in Equation 1 below for $J$ to $J - 2$ transitions. In the case of $J$ to $J - 1$ transitions, the moment of inertia differs by a factor of 2.

$$I^{(2)} = \left( \frac{d^2E}{dJ^2} \right)^{-1} \approx \left[ \frac{d}{dJ} \left( \frac{\Delta E}{\Delta J} \right) \right]^{-1} = \left[ \frac{d}{dJ} \left( \frac{E_\gamma}{2\hbar} \right) \right]^{-1} = 2\hbar \left( \frac{dJ}{dE_\gamma} \right).$$  \hspace{1cm} (1)
We were able to determine the dynamic moment of inertia for only a few cases due to the unavailability of enough known transitions and levels in each band. However, we suspect that the identical dynamic moments of inertia may be a signature of one and two-phonon vibrational bands. Work is still in progress on this topic. Some examples are shown in Figs. 3 and 4 for $^{154}$Gd and $^{168}$Er respectively. The lower portion of each figure shows the alignment of the two bands. Alignments have been used by the high-spin community studying superdeformed bands as a way of determining spins. In the low lying bands of the nucleus, the spins are well-known and perhaps there are clues to the structure of the nucleus from the deduced alignments in these two-phonon bands.

In conclusion, we have found 17 examples of $K = 4$ two-phonon $\gamma \gamma$ bands in the $A = 150$ region. The expected energy dependencies of the two-phonon bands is not a constant factor of two but a whole range of different values that seem to depend on both the single $\beta$ and $\gamma$ bands rather than on the $\gamma$ bands alone. Furthermore, we suspect that identical dynamic moments of inertia are signatures of two-phonon and one-phonon vibrational bands.

![Figure 4](image-url). Top: Gamma ray energies for transitions of $\Delta J = 1$ as a function of spin for the $\gamma$ and the $\gamma \gamma$ bands in $^{168}$Er. Bottom: the alignment of the $\gamma$ band with the $\gamma \gamma$ band.
This work was supported by the National Science Foundation under contract No. PHY-90-06246.

REFERENCES