Pauli blocking and transparency in elastic heavy ions scattering*

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ABSTRACT. For energies below $E/A \approx 100$ MeV a comparison of elastic heavy-ion phase shifts calculated by a double-Glauber approximation with those extracted from optical-model fits shows the Glauber approach to grossly overestimate the absorption out of the elastic channel, even though it produces plausible fits to the available angular distributions. We suggest that this overestimate, as a function of impact parameter, may provide a useful measure of the way in which Pauli blocking of NN collisions depends on the overlap density of the colliding nuclei.

RESUMEN. Para energías menores que $E/A \approx 100$ MeV, la comparación de los corrimientos de fase de iones pesados elásticos, calculados mediante una doble aproximación de Glauber, con los extraídos de ajustes de modelo óptico, muestra que la aproximación de Glauber sobreestima fuertemente la absorción del canal elástico, aún cuando produce ajustes plausibles a las distribuciones angulares disponibles. Sugerimos que esta sobreestimación, como función del parámetro de impacto puede dar lugar a una medida útil de la forma en la que el bloqueo de Pauli de colisiones NN depende del traslape de densidades de los núcleos que chocan.

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I. INTRODUCTION

A substantial body of elastic scattering data for light heavy-ion systems is now available at energies below about 100 MeV/nucleon [1,2]. The systems which have been most thoroughly studied are $^{12}{C} + ^{12}{C}$, $^{16}{O} + ^{12}{C}$ and $^{16}{O} + ^{16}{O}$, and perhaps the most striking result to emerge from 6-parameter optical potential fits to them is their surprising degree of transparency. Since the elastic optical $S$-matrix elements vary slowly with $l$ for small $l$, a useful measure of this transparency is $|S(l = 0)|$, which is typically as large as $10^{-2}$; this is large enough to support nuclear rainbows, and contrasts with values like $10^{-4}$ for heavier nuclei.

Double-Glauber calculations (which we shall refer to simply as Glauber calculations) have recently been performed for these systems [3,4], and we find them to produce $S$-matrix elements very much more absorptive than those resulting from optical fits. As an example, for $^{12}{C} + ^{12}{C}$ we find $|S_{Gl}(l = 0)|$ to be less than $10^{-7}$ at 85 MeV/A, and less than $10^{-20}$ at 30 MeV/A, very absorptive indeed. The cross sections produced by these $S$-matrix elements, on the other hand, yield more-or-less acceptable fits to the data at

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laboratory energies of 360, 608 and 1016 MeV, a result which, in view of their strong absorption, we find very surprising.

Our purpose here is to explore the differences between the optical and Glauber complex phase shifts for $^{12}\text{C} + ^{12}\text{C}$ and $^{16}\text{O} + ^{12}\text{C}$ at these three representative energies. We also compare them with a third set of phases, produced by the PIPS $S(\ell)$ parametrization of Cooper et al. [5], which employs about twice as many parameters as the optical potential, and so provides an even better fit to the experimental data.

The Glauber phases describe the nucleus-nucleus collision in terms of the multiple scattering of their composite nucleons, and are expressed directly in terms of the nucleon-nucleon (NN) scattering amplitude at the same relative velocity. Consequently, the fact that the "empirical" phase shifts of a phenomenological optical potential fit exhibit much greater transparency suggests that NN scattering within nuclei is immensely suppressed relative to that in free space. DeGiacomo et al. [6] encountered a similar (though less severe) suppression in their Glauber analysis of total cross sections for proton-nucleus scattering, and attributed it largely to Pauli blocking. Total cross sections, however, represent a sum over all impact parameters and are very insensitive to details of the interaction. In contrast, the elastic angular distributions now available provide far more detailed information on these interactions, and a comparison of their Glauber and optical partial-wave decompositions offers the possibility of examining the in-medium suppression of NN scattering \textit{partial wave by partial wave}, \textit{i.e.}, as a function of the nuclear densities which contribute to this suppression. Whether the suppression is due to Pauli blocking is not \textit{a priori} clear, but the fact that both the empirical suppression and the Fermi momentum increase with increasing nuclear density suggests that Pauli blocking may be playing an important role. Whatever its cause, the empirical optical phases (as well as those from the PIPS fits) provide unique evidence that the suppression of NN scattering is very large for head-on heavy-ion collisions, and decreases smoothly with increasing impact parameter, as we show below.

2. EXPERIMENTAL DATA AND THE DOUBLE-GLAUBER APPROXIMATION

The three angular distributions we have studied so far are $^{12}\text{C} + ^{12}\text{C}$ at laboratory energies of 360 [7] and 1016 [9] MeV and $^{16}\text{O} + ^{12}\text{C}$ at 608 MeV [8]. Fig. 1 shows the experimental data, together with the Glauber [4] and Brandan [1] optical model fits, and Table I provides Brandan's optical parameters. We have not included the PIPS fits, since they essentially pass through the data points. The Glauber (\textit{i.e.}, no-parameter) fits are clearly not as good as the 6-parameter optical fits, but we find it surprising that the two agree as well as they do, in view of the great differences between their $S$-matrix elements. The Glauber fits are best at forward angles, where they only sample the far surface region of the nucleus-nucleus interaction. At 608 MeV, however, the Glauber fit even persists beyond the nearside/farside crossover, and includes the farside-dominant region. On the other hand, we find it very surprising that the fit at the higher 1016 MeV energy is distinctly worse than that at 360 or 608 MeV, since the Glauber approximation is expected to improve with increasing energy.
FIGURE 1. Experimental angular distributions compared to optical-model fits and Glauber calculations.
Table I. Optical Model Parameters from Brandan, Ref. [1].

<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$V_0$ (MeV)</th>
<th>$R_V$ (Fm)</th>
<th>$a$ (Fm)</th>
<th>$W_0$ (MeV)</th>
<th>$R_W$ (Fm)</th>
<th>$a_W$ (Fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>360</td>
<td>175</td>
<td>2.87</td>
<td>0.894</td>
<td>25</td>
<td>4.87</td>
<td>0.645</td>
</tr>
<tr>
<td>608</td>
<td>175</td>
<td>3.15</td>
<td>0.975</td>
<td>24.7</td>
<td>5.21</td>
<td>0.662</td>
</tr>
<tr>
<td>1016</td>
<td>120</td>
<td>3.01</td>
<td>0.879</td>
<td>25</td>
<td>4.41</td>
<td>0.862</td>
</tr>
</tbody>
</table>

![Figure 2. Sketch of the colliding nuclei to define integration variables.](image)

Before examining the Glauber phase shifts, we briefly recall the essence of the "optical limit" of the Glauber formalism, which is the form employed in Refs. [3] and [4].

Figure 2 shows the geometry of the spherical target and projectile nuclei, containing $A_T$ and $A_P$ nucleons, colliding at the center-to-center impact parameter $b$. The only computationally manageable version of the Glauber description for this many-body collision is that known as its "optical limit", in which (among other approximations) the ground-state wave functions of target and projectile are taken as simple products of a single single-particle wave function—i.e., all nucleons are put into the same single-particle state, in violation of the Pauli principle.

As Glauber explains [10,3,4], the end result of this optical limit is that the Glauber approximation to the (complex) phase shift at impact parameter $b$ is given, in terms of the NN scattering amplitude, by the usual form of the Glauber integral,

$$
\delta_{\text{GI}}(b) = -\frac{k_{\text{cm}}}{4E_{\text{cm}}} \int_{-\infty}^{\infty} V_{DF}(b,z) \, dz,
$$

in which $V_{DF}$ is the "double-folding" potential in its zero-range [4] version,

$$
V_{DF}(r) = -\frac{1}{2} \hbar \omega T_{\text{NN}}^{\alpha + i} A_P A_T \int \rho_T(r_T) \rho_P(r_T - r) \, d^3r_T.
$$

(2.1)
Here $v$ is the relative velocity between the two nuclei, $\sigma_T^{NN}$ is an $I$-spin-averaged NN total cross section at this relative velocity, $\alpha = \text{Re}[f^{NN}] / \text{Im}[f^{NN}]$, evaluated at $0^\circ$ and the same velocity, and $\rho_p$ and $\rho_T$ are normalized to unity. By the optical theorem,

$$\frac{1}{2} \hbar v \sigma_T^{NN}(\alpha + i) = t^{NN}(0^\circ) = 2\pi \frac{\hbar^2}{m} f^{NN}(0^\circ),$$

(2.3)

where $f^{NN}(0)$ is the forward NN scattering amplitude (actually assumed to be independent of angle), and $t^{NN}(0)$ is the corresponding $t$-matrix, showing that $V_{DF}$ is of the customary "$t_{pp}$" form.

Equations (2.1) and (2.2), however, immediately "show where the trouble lies" in this Glauber approximation, for they imply that

$$\frac{\text{Re}[\delta_G]}{\text{Im}[\delta_G]} = \alpha = -\frac{\text{Re}[f^{NN}]}{\text{Im}[f^{NN}]},$$

(2.4)

(independently of $b$). In the 30–100 MeV/A energy range considered here, $\alpha$ is near unity [4], whereas, as we show below, this ratio at $\ell = 0$, for both the empirical optical potential and PIPS phases, is closer to 20 at 30 MeV/A and 5–10 at 85 MeV/A. Furthermore, it depends on $\ell = kb$, which the Glauber phase-ratio does not. Thus it is immediately clear that the Glauber approach must greatly overestimate the imaginary part of the nucleus-nucleus phases shifts, i.e., that it produces far too much absorption, relative to the best-fit optical or PIPS phases.

Once we recognize that this Glauber phase shift is merely the eikonal approximation for the double-folding potential, with both real and imaginary parts of the potential taken from $t_{pp}$, this overestimate of the absorption comes as no surprise, It has long been recognized [11] that, although the real part of this optical potential is reasonably well-approximated by double folding or $t_{pp}$, its imaginary part comes from the "dynamic polarization potential" of the Feshbach form,

$$\Delta V \equiv V \frac{1}{E - H_{QQ} + i\epsilon} V,$$

(2.5)

which can clearly be very different from the simple folding result. This gross overestimate of $\text{Im}[V]$ by the simple $t_{pp}$ approach has been recognized for some time [11], and has lead to the use of a purely phenomenological imaginary potential in conjunction with the real double-folding potential for fits to heavy-ion data. It is this experience which makes even the limited success of the simple Glauber approach so surprising.

Before proceeding to further details, we address a minor point connected with the Coulomb potential. In optical model calculations it is customary to assume the Coulomb interaction between the nuclei to result from two spherical charge distributions of uniform density (or of a point charge and a single uniform sphere of radius $R_C$, generally taken to be the same as the radius $R$ of the real nuclear potential). The difference between this Coulomb potential and that of a point charge of the same magnitude vanishes for $r > R_C$, and is normally included with $U_{opt}$ in computing the "nuclear" phase shifts.
The Glauber phase shifts calculated by Lenzi et al. [4], whose method we follow here, assume the presence of point charges only, and neglect this correction for extended charges. As a check of its significance, we found that, in an optical model calculation for $^{12}$C+$^{12}$C at 360 MeV, reducing $R_C$ to 0.01 Fm had no visible influence on the angular distribution forward of the near/far crossover, and produced only about a 10% change in $d\sigma/d\Omega$ at 40°. Its effect on the phase shifts was barely visible.

Even a point Coulomb field, however, has a significant effect on the classical trajectory outside the range of the strong interaction. Lenzi et al. included a rough approximation to this coulomb trajectory effect by doing their straight-line eikonal integral at a “local” impact parameter $b'$, which they included in the partial wave sum at the smaller asymptotic impact parameter $b$, by using $\ell$-conservation to require that

$$\ell = kb = k'b',$$

with $k'(b')$ the Coulomb-reduced momentum at the separation $b'$ between the two point charges. This gives

$$b' = \left[\eta + (\eta^2 + \ell^2)^{1/2}\right]/k,$$

(3.1)
a $b' \leftrightarrow \ell$ correlation we shall find useful below.

3. ANALYSIS OF THE GLAUBER PARTIAL-WAVE DECOMPOSITIONS

3.a. Phase shift ratios

The Glauber phases, even in this simple optical limit, possess an inherent interest, for the following reason. If the two nuclei pass each other at very large impact parameter, only the outer tails of their two densities will interact. But the nucleons in these very low-density regions of the target and projectile are very close to being free particles, so all “medium effects” (like Pauli blocking, Fermi averaging and off-shell scattering) should disappear, and the Glauber approximation (in terms of on-shell NN collisions) should become valid. This suggests that the Glauber phases, if correctly calculated in this region, should provide a limit to which the “true” nucleus-nucleus phase shifts should tend, and that, consequently, deviations from them at smaller impact parameters should provide a direct measure of “medium effects” due to the increasing density of the bound nuclear matter.

The simplest prediction which follows from this observation is that the ratio $\text{Re}[\delta]/\text{Im}[\delta]$ for the “true” nucleus-nucleus phase shifts at energy $E_{\text{lab}}$ should tend, for large $\ell$, to the nucleon-nucleon $\alpha$ of Eq. (2.3), evaluated at $E_{\text{lab}}/A_p$; in the energy range considered here, $\alpha$ is near unity. The best approximations we have to the “true” phase shifts are those resulting from optical or PIPS fits, and Fig. 3 shows their real-to-imaginary ratios at the three energies considered here. As a further check, the optical phases are of course calculated from a potential, and at large $\ell$ are small enough to be given accurately by the Born approximation, which should make them linear in the potential strengths. this suggests that $V(r)/W(r)$ for the optical potential itself should approach $\alpha \approx 1$ in its
Figure 3. Ratio of Real to Imaginary parts of phases shifts and of $U_{opt}(r) = V(r) + iW(r)$. The ratio for $U_{opt}$ is plotted vs. $L$ by using the distance-of-closest-approach correlation of Eq. (3.1).
surface region, so on Fig. 3 we have included this ratio as well, using Eq. (3.1) to translate $r \equiv b'$ into $\ell$.

As the figure shows, $V_{\text{opt}}/W_{\text{opt}}$ varies with $r$, as follows from the fact that empirically $V$ and $W$ have different shapes. Their ratios are indeed seen to approach unity at $\ell$-values near their respective surface $\ell$'s; they rise beyond that because $a_R > a_I$ and $V_0 > W_0$, in terms of the Woods-Saxon surface parameters needed to fit the data.

The optical phase shift ratios are indeed seen to equal the corresponding potential ratios at surface $\ell$-values, where both ratios exceed the free-nucleon value of unity by only 30%. At smaller $\ell$'s the Born approximation fails, and at larger $\ell$'s the phase-shift ratio is somewhat larger than the potential ratio, presumably because $b'(\ell)$ represents the distance of closest approach, and $\delta(\ell)$ picks up contributions from larger $r$'s, where $V/W$ is larger. The PIPS ratio, where available, is significantly larger yet, for reasons unclear to us.

3.b. The phase shifts themselves

Figure 4 compares the real parts of the Glauber, optical and PIPS phase shifts. The Glauber phase exceeds the optical by a factor between 1.5 and 2 at small $\ell$; their differences must obviously decrease as both tend to zero at large $\ell$.

For $\text{Im}[\delta(\ell)]$, on the other hand, the ratio of Glauber to optical is far larger. As Fig. 5 shows, at $\ell = 0$ it varies between 7 and 13 for the cases considered.

Recalling that

$$|S| = e^{-2\text{Im}[\delta]},$$

(3.2)

it is clear that the Glauber approximation grossly over-estimates the absorption out of the elastic channel. At 360 MeV, e.g., $|S_{\text{opt}}(\ell = 0)| \approx e^{-4} \approx .02$, whereas $|S_{\text{GI}}(0)| \approx e^{-53} \approx 10^{-23}$, giving some indication of the Glauber overestimate of the absorption. Of course, if the optical fit itself required a fairly black interior, say $|S(0)| \approx 10^{-4}$, then making it still blacker would have little effect, but this is not the case for these light nuclei. $|S_{\text{opt}}(0)| \approx .02$ is sufficiently transparent to even permit mild Airy minima in the optical farside cross section, as Fig. 6 shows. It is this which makes the moderate success of the Glauber calculation so surprising.

Finally, Fig. 7 provides a summary of the situation, by displaying the (natural) logarithms of the “absorption suppression ratio”, $|S_{\text{GI}}/S_{\text{opt}}|$; to remove the effect of the bombarding energy on the horizontal axis, they are plotted vs. $b'(\ell)$ rather than vs. $\ell$, again using Eq. (3.1) to estimate a (Coulomb) distance of closest approach for each $\ell$.

It is this ratio which provides the most direct measure of the extent to which the imaginary part of the double-folding potential, Eq. (2.2), must be reduced in order to obtain “realistic” phase shifts and a reasonable fit to the data. The ratio goes essentially to unity at large $b'$ and increasing overlap density. It is exactly this variation with density which may for the first time provide a direct measure of the dependence of the medium effects on nuclear-matter density, and do so in the context of finite nuclei.
FIGURE 4. Comparison of Real parts of Glauber, optical and PIPS phase shifts, in units of radians.
Figure 5. Comparison of Imaginary parts of Glauber, optical and PIPS phase shifts, in units of radians.
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\[ V = 175.00, R = 2.87, A = 0.894 \]
\[ VI = 25.00, RI = 4.87, AI = 0.645 \]
\[ K+AI = 4.64 \]

**FIGURE 6**. Nearsides/farside decomposition of the optical-model angular distribution for $^{12}\text{C}+^{12}\text{C}$ at 360 MeV.

**FIGURE 7**. Natural logarithm of $|S_{\text{Gl}}/S_{\text{opt}}|$, which provides a measure of the Glauber overestimate of absorption, relative to that found by an optical-model fit. The x-axis is the distance-of-closest-approach, $b'(\ell)$, defined by Eq. (3.1).
4. Summary

At sufficiently high bombarding energy, heavy-ion scattering must be describable in terms of unimpeded, on-shell nucleon-nucleon scattering. Even before this limit is reached for all NN collisions in the target and projectile, it is expected to hold for very peripherical heavy-ion collisions, in which only nucleons in the low-density tails of the two nuclei collide.

We do indeed find that Glauber phase shifts for the heavy-ion collisions examined here approach the empirical phase shifts obtained from phenomenological optical-model and PIPS fits to the data, at large \( \ell \)-values. At more interior \( \ell \)'s, the real parts of the Glauber phases continue to approximate the corresponding optical/PIPS phases rather well, but in contrast the imaginary parts of the Glauber phases grossly overestimate the amount of absorption of flux from the elastic channel. This indicates that the scattering of internal nucleons, in the region of normal nuclear matter density, is greatly suppressed in elastic scattering. We suggest that the difference between the imaginary parts of Glauber and empirical optical phase shifts, as function of \( \ell \), provides a direct measure of the magnitude of this suppression. If the suppression is due to Pauli blocking, this provides an estimate of the dependence of Pauli blocking on the density of the nuclear medium.

Regarding the Glauber fits to the angular distributions, we note that in all 3 cases the Fraunhofer oscillations of the calculations extend to larger angles than those of the data, in a manner which suggests that the main failing of the calculation is an overestimate of the nearside amplitudes. This is a plausible result of overly strong absorption but the remarkably accurate prediction of the farsides at 360 and 608 MeV remain a mystery.

Acknowledgements

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References