Airy, Glauber and Pauli: what can we learn about in-medium NN-interactions from elastic heavy-ion scattering?*

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ABSTRACT. The $t_{pp}$ approximation to the optical potential for light-element nucleus-nucleus scattering for CM energies below ~ 100 MeV/nucleon is found to grossly overestimate the imaginary part of this potential, apparently because of neglect of Pauli suppression of in-medium NN scattering. The same difficulty is found to plague the double-Glauber approximation at these energies, since it is simply the eikonal approximation applied to the $t_{pp}$ potential. The fact that the problem persists even in the low-density tail of the potential suggests that major problems remain in our understanding of the in-medium NN interaction.

RESUMEN. La aproximación $t_{pp}$ al potencial óptico para la dispersión núcleo-núcleo en elementos ligeros a energías del CM por abajo de 100 MeV/nucleón sobre estima burdamente la parte imaginaria de dicho potencial, aparentemente por el descuido de la supresión de Pauli en la dispersión NN en el medio. La misma dificultad se encuentra en la aproximación “double-Glauber” a estas energías, puesto que es simplemente la aproximación eikonal aplicada al potencial $t_{pp}$. El hecho de que el problema persista aún en el extremo de baja densidad del potencial sugiere que queden problemas mayores en nuestro entendimiento de la interacción NN en el medio.

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1. INTRODUCTION

Elastic scattering is the simplest of all heavy-ion reactions, and has been studied, both experimentally and theoretically, for more than 30 years. The measured angular distributions, for $E/A < 100$ MeV, are unique in strong-interaction physics, in exhibiting unambiguous nuclear (attractive) rainbow phenomena. It is these rainbows which, following Goldberg’s suggestion [1,2], have permitted the determination of a large number of essentially-unique optical potentials for the lighter ($A < 25$) heavy ions.

This compendium of local phenomenological potentials provides a concise summary of an extensive body of experimental results, and also presents many-body theory with the challenge of understanding these potentials from first principles. Although some progress has been made in understanding their real parts in terms of the underlying nucleon-nucleon (NN) interaction [3], the same cannot be said for their imaginary parts. As an aid toward

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outlining the magnitude of the problem, we provide in Figs. 1 and 2 a summary of a few of the best-known light-ion potentials, $-\left[V(r) + iW(r)\right]$, and identify their prominent features. The most striking is the very different radial shapes of $V(r)$ and $W(r)$.

In particular, we note the three features:

1) Volume transparency: $V(0)/W(0)$ is typically 5 or more —and becomes as large as 24 for $^{16}\text{O} + ^{16}\text{O}$ at 145 MeV.

2) Far-surface transparency: In Woods-Saxon terms, the real surface thickness is from 30% to a factor of 2 larger than the imaginary surface thickness.
3) \( W(R) \approx V(R) \), for \( R \) near the surface of the potential.

Although these features apply to most of the light systems studied, recent data on \( ^{20}\text{Ne} + ^{12}\text{C} \) by Bohlen et al. [4] stand in striking contrast, for this particular system exhibits far stronger absorption. This suggests that there may be much greater variation from system to system than has so far been realized.

As we explain below, the main conclusion which seems to emerge from these potentials is that the Pauli principle must play a major role in determining \( W(r) \) [and perhaps \( V(r) \) ?] — and does so to an unexpected extent in the far surface of the interaction region.

These results have an important bearing on the use of the Glauber approximation to describe these systems, for Glauber turns out to be an approximation which violates all 3 of the above optical-potential features. Indeed, although some Glauber calculations performed at these energies appear to provide plausible fits to some experimental angular
distributions [5], we find that they do so with entirely incorrect phase shifts, and so must be viewed with considerable suspicion. This is discussed further in Section 4 below.

2. Weak internal absorption

Perhaps the simplest (but least correct) estimate of the depth of the imaginary part of the optical potential is that given by the single-scattering '\( t_{pp} \)' approximation (sometimes known as the ‘frivolous approximation’), which neglects Pauli blocking and proves to be interesting precisely because of its failure.

Assuming a zero-range NN interaction (adequate for bombarding energies below 100 MeV/nucleon), \( t_{pp} \) gives

\[
V(\vec{r}) + iW(\vec{r}) = t_{NN} \int \rho_p(\vec{r}) \rho_T(\vec{r} - \vec{s}) d^3 \vec{s}. \tag{1}
\]

As an illustration which can be worked through analytically, we consider \( A_P = A_T = A \), with identical ‘square’ radial shapes of volume \( V \) for both target and projectile. Then \( \rho = A/V \), and at complete overlap (\( \vec{r} = 0 \)), the above integral becomes \( \int \rho^2 d^3 \vec{s} = A^2/V = A \rho \), so the depth of the imaginary potential is given in this approximation by

\[
W_0 = A(\text{Im}[t_{NN}])\rho = A[2\pi\hbar^2 \text{Im}[f_{NN}(0)]/(m/2)]\rho = A\hbar^2 k_{NN} \sigma_{NN}/m|\rho = \hbar^2 k \sigma_{NN} \rho/m, \tag{2}
\]

using the optical theorem and \( k = Ak_{NN} \), where \( \hbar k \) is the relative, or CM, AA momentum, \( \hbar k_{NN} \) is the same momentum for a nucleon pair, and \( m \) is the nucleon mass. Considering \( ^{12}\text{C} + ^{12}\text{C} \) at \( E_{\text{lab}} = 500 \text{ MeV} \) for definiteness, and using the empirical value \( V_0 \approx 200 \text{ MeV} \), the internal relative kinetic energy of the two nuclei is \( 250 + 200 = 450 \text{ MeV} \), giving \( k_{\text{CM}} = 11.5 \text{ F}^{-1} \). Using \( r_0 = 1.2 \text{ F} \) for the nuclear radial parameter gives \( \rho = .138 \text{ F}^{-3} \), and using the empirical \( \sigma_{NN} = 7.3 \text{ F}^2 \) at the corresponding \( E_{\text{lab}}^{\text{int}} = 75 \text{ MeV/nucleon} \) finally gives \( W_0 = 484 \text{ MeV} \). This is some 19 times deeper than the typical \( W_0 = 25 \text{ MeV} \) found empirically.

In other words, the \( t_{pp} \) approximation to the \( ^{12}\text{C} + ^{12}\text{C} \) optical potential at \( E_{\text{lab}}/A = 40 \text{ MeV} \) grossly overestimates its imaginary part, and predicts the scattering to be totally absorptive. This simple result is the essence of our message, and it is this which leads us to conclude, by comparison, that many \( (A_1 + A_2) \) systems, with \( A_1 \) or \( A_2 \) or both below 25, are unusually transparent in this energy range. Although the factor of 19 is specific to the square-well potential with equal geometries for \( V(r) \) and \( W(r) \), we find a similar result more realistically below.

It is useful to note that the estimate given by Eq. (2) can be obtained even more simply, by defining real and imaginary parts of the relative momentum for propagation of the projectile through the “infinite nuclear medium” represented by the target at complete overlap, by

\[
\frac{\hbar^2 (k + i\kappa)^2}{2\mu} = E_{\text{CM}} + V_0 + iW_0, \tag{3}
\]
with $\mu = M_1 M_2 / (M_1 + M_2) = Am / 2$ for $(A+A)$ scattering. Thus

$$V_0 = \frac{\hbar^2 (k^2 - \kappa^2)}{2\mu} \approx \frac{\hbar^2 k^2}{2\mu}, \quad (4)$$

$$W_0 = \frac{\hbar^2 k\kappa}{\mu}, \quad (5)$$

using the empirical result $k \gg \kappa$. Since the system propagates through the medium with the wave function $\psi(x) = e^{i(k+\kappa)x}$, it decays according to $|\psi|^2 = e^{-2\kappa x}$, from which

$$1/2\kappa = \lambda_{AA} = 1/\rho\sigma_{AA}, \quad (6)$$

with $\lambda_{AA}$ the mean free path for propagation through the medium, and $\sigma_{AA}$ the $(A+A)$ total cross section. However, since $k = AK_{NN}$, we must also have $\kappa = AK_{NN} = A/2\lambda_{NN}$, or

$$\lambda_{AA} = \frac{\lambda_{NN}}{A}, \quad (2.6)$$

a result which is also given by the Glauber approximation. It is central to explaining the failure of this simple-minded approach, for it says that the mean free path of a cluster of $A$ nucleons is smaller than that of a single nucleon by the factor $1/A$. This is directly due to the $t_{pp}$ assumption that these $A$ nucleons scatter independently. It also clearly requires that $\sigma_{AA} = A\sigma_{NN}$, empirically an overestimate by a factor of 2 or more.

Using $\kappa = \frac{1}{2}\rho A\sigma_{NN}$ and $\mu = \frac{1}{2}Am$, Eq. (5) reduces directly to the $t_{pp}$ result, Eq. (2). In their recent textbook, Siemens and Jensen [6] note that exactly this argument, applied to proton-nucleus scattering, predicts a mean free path for protons in nuclei which is 10 times shorter than that observed —i.e., a $W_0$ ten times larger than observed. He notes further that Jeukenne, Lejeune and Mahaux [7], using the Brueckner-Hartree-Fock approach to scattering in an infinite medium, together with a local-density approximation for finite nuclei, were successful in attributing this empirical suppression of NN scattering (long mean free path) to effects of the Pauli principle. It thus seems highly likely that the corresponding suppressions in the heavy-ion case, at energies per nucleon not far above the Fermi energy, are likewise a manifestation of Pauli blocking. However, the recent Berlin result, that some systems are substantially more absorptive than others, provides a warning that properties of individual nuclei (like low-lying level densities) can also play an important role, thus requiring something beyond the simple local-density approximation.

3. HOW ACCURATELY ARE OPTICAL POTENTIALS KNOWN?

The recent Bohlen data on the strongly-absorbing system $^{20}\text{Ne} + ^{12}\text{C}$ [4] raises the important point that empirical potentials can be determined accurately only for weakly absorbing systems —for the obvious reason that if no flux is permitted to enter a strongly-absorbing region, it cannot carry out any information about the interior of that region.
We should remark at the outset that there is currently considerable discussion [8] of the accuracy with which optical potentials can be extracted from elastic scattering data by employing potential parametrizations with unusually large numbers of parameters. We restrict ourselves here to the family of 6-parameter Woods-Saxon shapes, since they are by far the most widely available, and the phenomena we wish to investigate have been found to be unchanged by enlarging the number of parameters employed.

Figure 3 displays the Bohlen data for both $^{20}$Ne + $^{12}$C at 390 MeV [4] and for the weakly-absorbing $^{16}$O + $^{16}$O at 350 MeV [9]. The obvious difference between them is the pronounced minimum in the farside component [10] of the cross section at about 44°, well beyond the near-far crossover. This is an Airy minimum of the nuclear rainbow exhibited by this system, and it provides the most direct evidence of its transparency, for this minimum is the result of an interference between the inner (deeply-penetrating) and outer (surface) components of the farside amplitude. Since the surface component is determined by the Fraunhofer or near/far interference oscillations of the angular distribution, the far/far interference which yields the Airy minimum produces direct information on the inner farside amplitude, and hence on the interior region of the potential: it is the rainbow phenomenon which permits the determination of unique 6-parameter optical potentials.

Actually, the situation is slightly more complex than this, for even the "transparent" nuclear systems have sufficient absorption that deep Airy minima do not occur at all scattering energies. This is illustrated by Fig. 4, taken from a study [11] of $^{12}$C + $^{12}$C data over the 70–130 MeV energy range. The curves give the farside cross sections only, and the dashed lines trace the motion in angle of the various Airy minima, showing that each of

\[ \text{FIGURE 3. Elastic angular distributions measured by Bohlen et al for the weakly-absorbing system} \]
\[ ^{16}\text{O} + ^{16}\text{O} (+) \text{ at } E_{\text{CM}} = 225 \text{ MeV} \text{ [9] and the strongly-absorbing system } ^{20}\text{Ne} + ^{12}\text{C} (\circ) \text{ (reduced by 1/10) at } E_{\text{CM}} = 244 \text{ MeV} \text{ [4]. The 44° minimum in the top curve is an Airy minimum of a nuclear rainbow.} \]
Figure 4. Farside cross sections from optical potentials [11] which fit elastic $^{12}\text{C}+^{12}\text{C}$ scattering at the indicated laboratory energies. All minima are Airy minima, each of which is deep enough to have a substantial effect on the full cross section only over a limited energy range.

them has an energy at which it is maximally deep, yet may be almost invisibly shallow at other energies. For this reason, unique optical potentials generally cannot be determined from an angular distribution at a single energy: even if the system is transparent, its Airy minima may disappear at some energies, making the system appear to be strongly absorbing. This is exactly the present situation with $^{16}\text{O}+^{12}\text{C}$, for example. At the energies currently available, including a quite extensive angular distribution at 608 MeV [12], no Airy minimum has been seen. In spite of this, a weak-absorption potential has been
found [13] in addition to a variety of strong-absorption ones. This, in addition to the fact that $^{16}\text{O} + ^{16}\text{O}$ and $^{12}\text{C} + ^{12}\text{C}$ are found to be weakly-absorbing, raises the strong probability that $^{16}\text{O} + ^{12}\text{C}$ is actually weakly absorbing and will exhibit Airy minima at other, as yet unexplored, energies. The Bohlen data for $^{20}\text{Ne} + ^{12}\text{C}$ at 390 MeV, on the other hand, appear to be genuinely strongly absorbing, since it has so far proved impossible to find a weak-absorption potential which will fit this angular distribution.

4. THE 'GLAUBER POTENTIAL'

The only form in which the 'double-Glauber' approximation has been applied to nucleus-nucleus scattering [5] is the high-energy limit known as the 'optical limit' [14], in which, among other things, the Pauli principle is neglected completely, both in the initial state (all nucleons of a nucleus are put into the same single-particle state) and in the final state (a struck nucleon is allowed to recoil into any state, occupied or not). Although neglecting the
FIGURE 6. Imaginary phase shifts for the cross sections shown in Fig. 5. The PIPS curves are from an \( \ell \)-space parametrization of the phase shifts.

final-state ('Pauli blocking') constraint should be reasonable if the incoming CM energy per nucleon is well above the Fermi energy (~35 MeV), that limit has not been reached in most of the double-Glauber applications made to date. In spite of this, Fig. 5 from Ref. [15] shows that some of these applications have yielded fairly reasonable fits to the data — though it is worrisome that the worst fit occurs at the highest energy (1016 MeV, or a CM energy of 43 MeV/nucleon).

Serious concern over this approach arises when one recognizes [15] that the phase shifts given by this optical limit of the Glauber approximation are in fact nothing more than the eikonal approximation applied to the \( t_{pp} \) potential, so that the ‘Glauber potential’ is just the \( t_{pp} \) potential. (The eikonal approximation itself is empirically found to be very reliable at the energies under discussion.) It suffers, therefore, from the shortcomings of that potential, and in particular is guaranteed to greatly overestimate its imaginary part for those systems known to be weakly absorbing. This can be seen explicitly in Fig. 6, also from Ref. [15], which compares the imaginary part of the Glauber phase shifts for the 3 cases of Fig. 5 with the empirical optical-model weak-absorption phases, showing that
FIGURE 7. Bohlen data (+) and double-Glauber calculation (dashes) for $^{16}\text{O} + ^{16}\text{O}$ at 350 MeV.

Glauber overestimates them by as much as 25 radians at $\ell = 0$ (and so underestimates $|S_{\ell}|$ by a factor of some $10^{10}$).

At first sight it might seem impossible that two sets of phase shifts which differ so drastically could produce angular distributions as similar as those of Fig. 5. However, we note that none of these angular distributions exhibit Airy minima, and that these are consequently energies at which both weakly-absorbing and strongly-absorbing optical potentials have been found. Apparently Glauber has modest success at these energies simply because its strong-absorption phase shifts to some extent mock up the analogous ones of the optical-potential searches: the secret is in the potential ambiguities.

For instance, the $^{16}\text{O} + ^{16}\text{O}$ data at 350 MeV [9], with its prominent Airy minimum, admits only a weak-absorption potential, and as Fig. 7 shows, Glauber fails completely in this case, even though the energy is quite comparable to those of Fig. 5.

5. CONCLUSIONS

In summary, at CM energies of less than 100 MeV or so per nucleon, the $t_{pp}$ approximation to nucleus-nucleus elastic scattering fails primarily by grossly overestimating the absorption found empirically for such systems as $^{12}\text{C} + ^{12}\text{C}$, $^{16}\text{O} + ^{16}\text{O}$, and $^{16}\text{O} + ^{12}\text{C}$. Comparison with the microscopic understanding of optical potentials for proton-nucleus scattering [7] strongly suggests that this failing arises largely from neglect of Pauli suppression of in-medium nucleon-nucleon scattering.

This has serious consequences for the application of the double-Glauber approximation to these systems in its optical-limit form, since the phase shifts it produces are just those
given by the eikonal approximation to the $t_{pp}$ potential. In addition, recent data [4] on $^{20}$Ne + $^{12}$C show that the imaginary part of the optical potential varies greatly, at the energies under discussion, from one system to another. This implies a dependence on properties specific to each nucleus (e.g., density of low-lying levels), which are entirely missing from the Glauber approximation.

A curious aspect of all this is the remarkable surface transparency found in many empirical optical potentials for these systems: the Woods-Saxon surface-thickness parameters for their real parts are found to be as much as a factor of 2 larger than those of their imaginary parts. This is particularly surprising, since in the low density matter of the nuclear surface, the Fermi energy (in the local-density approximation) should be correspondingly small, thus reducing Pauli blocking and permitting much stronger absorption, even at these low energies. This is currently a mystery, especially since modifications of empirical potentials which fit the data, to make their real and imaginary tails approximately equal (the $t_{pp}$ prediction at these energies) totally destroys the fit. Apparently much remains to be understood about the in-medium NN interaction.

References