Inclusive $\pi d$ breakup reactions

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ABSTRACT. The inelastic reactions $d(\pi^+,\pi^+)X$, $d(\pi^-,\pi^0)X$, $d(\pi^+,p)X$, and $d(\pi^-,p)X$ are discussed within a relativistic three-body model. Single and double differential cross sections are calculated and compared with available data and predictions are made where no data exist.

RESUMEN. Discutimos las reacciones $d(\pi^+,\pi^+)X$, $d(\pi^-,\pi^0)X$, $d(\pi^+,p)X$, and $d(\pi^-,p)X$ usando un modelo relativista de tres cuerpos. Calculamos la sección diferencial y la sección diferencial doble y los resultados los comparamos con las medidas experimentales disponibles, también hacemos predicciones en una región en donde no hay medidas experimentales.

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1. INTRODUCTION

In the energy regime in which the production of two pions is negligible ($T_{LAB} < 300$ MeV in pion-deuteron scattering), the amplitudes of the processes $\pi d \rightarrow \pi d, \pi d \rightarrow \pi NN, \pi d \rightarrow NN, NN \rightarrow \pi d, NN \rightarrow NN$, and $NN \rightarrow \pi NN$, form a closed system. That means they obey coupled integral equations such that a) all the amplitudes corresponding to the same initial state are coupled together, and b) the kernels of the two sets of integral equations are the same [1]. For a complete description of the theoretical and experimental methods developed to explore the $\pi NN$ system, the reader is referred to Ref. [1]. In this talk we will concentrate on the second of these reactions for the particular case in which only one of the three final particles is observed [2,3].

2. THEORY

In order to derive the integral equations which are used in practice to describe theoretically the $\pi NN$ system one requires them to fulfill the following requirements:

a) They should satisfy 2- and 3-body unitarity,

b) They should be Lorentz invariant,

c) They should be reduced from a 4-vector to a 3-vector description,

d) The two-body amplitudes are assumed to be separable (isobar approximation),

e) Three-body forces are neglected.
Different sets of integral equations satisfying these requirements have been proposed by several groups [4-7]. These different formulations of the \( \pi NN \) system differ among each other in the detailed form of the integral equations although their final results when they are compared to the experimental data are very similar for all of them.

A. The relativistic Faddeev Equations

We define the amplitude \( T_i \) with \( i = 1, 2, 3 \) as the sum of all possible diagrams in which particles \( j \) and \( k \) interact first. If particles 2 and 3 (the two nucleons) are initially in a bound state (the deuteron) while particle 1 (the pion) is free, then the integral equations satisfied by the amplitudes \( T_i \) are

\[
T_i = t_i(1 - \delta_{ii}) + t_i G_j G_k T_j + t_i G_j G_k T_k,
\]

where \( t_i \) is the scattering amplitude of particles \( j \) and \( k \) (that is, the sum of diagrams in which particles \( j \) and \( k \) interact in all possible ways), and \( G_j \) and \( G_k \) are the propagators of particles \( j \) and \( k \) respectively.

The breakup operator is given by

\[
A = T_1 + T_2 + T_3,
\]

where the physical amplitude is obtained by taking the matrix elements of this operator between the initial state wave function consisting of a free pion times a \( np \) bound state and the final state wave function consisting of three free particles.

The impulse approximation corresponds to taking just the single scattering terms, that is,

\[
A^{\text{IA}} = t_2 + t_3
\]

since the amplitude \( T_1 \) does not contribute in first order.

The distorted-wave impulse approximation (DWIA) consists in taking the matrix elements of Eq. (3) between the initial state wave function and replacing the final plane waves of the two nucleons by a \( np \) distorted wave for the relative motion times a plane wave for the motion of the center of mass.

B. The spectator-on-mass-shell approximation

In order to see the explicit form of these equations, let us consider the second term in the r.h.s. of Eq. (1)

\[
I = \int d^4 k'_j \langle k_j k_k | t_i | k'_j k'_k \rangle G_j(k'_j) G_k(k'_k) T_j(k_i, k'_j, k'_k).
\]

The propagator \( G_j(k'_j) \) is written as (considering for the moment particle \( j \) as a scalar particle)

\[
G_j(k'_j) = \frac{1}{k'_j^2 - m_j^2 + i\epsilon} = \frac{1}{(k'_j 0 - \omega_j' + i\epsilon)(k'_j 0 + \omega_j' - i\epsilon)},
\]
where \( \omega_j' = \sqrt{k_j'^2 + m_j^2} \). If we now integrate over \( dk'_j \) by closing the contour from below and neglect other possible contributions from additional poles of the integrand we get

\[
I = 2\pi i \int \frac{dk'_j}{2\omega_j} \left< k_j k_k | t'_i | k'_j k'_k \right> G_k(k'_k) T_j(k_i, k'_j, k'_k).
\]

Thus, by this procedure we have eliminated the fourth component in the integration of Eq. (4). A consequence of this is that the particle with momentum \( k'_j \) which is the spectator has been put on the mass shell. Therefore, if we also put the external particles on the mass shell, we obtain three-vector integral equations in which all the spectator particles in the initial, final, and intermediate states are on the mass shell and only the exchanged particles are off the mass shell \[7,8\].

C. The separable or isobar approximation

In order to simplify the structure of the integral equations (1) one makes the further assumption that the two-body amplitudes \( t_i \) can be well represented by a sum of isobars so that they can be written as

\[
t_i(q_i) = \sum_{I j m P} \Gamma_{I j P} \left| \phi_m^j(q_i) \right> \tau_{I j P}(s_i) \left< \phi_m^j(q_i) \right| \Gamma_{I j P}^\dagger
\]

where \( q_i \) is the four-momentum of the isobar, \( s_i = q_i^2 \), and \( I, j, m, \) and \( P \), are the isospin, spin, helicity, and parity of each isobar. \( \Gamma_{I j P} \) is the vertex that couples the isobar to the particles \( j \) and \( k \) and \( \phi_m^j(q_i) \) are the isobar spinors, while \( \tau_{I j P}(s_i) \) is the propagator of the isobar.

D. The quasi-two-body equations

As a consequence of the isobar approximation the three-body integral equations (1) reduce to quasi-two-body integral equations describing all possible transitions from a particle and an isobar to any other particle and isobar. These integral equations are written explicitly as

\[
F_{i1}^{\alpha_1 \alpha_10}(\vec{k}_i, \vec{k}_{10}) = V_{i1}^{\alpha_1 \alpha_10}(\vec{k}_i, \vec{k}_{10})(1 - \delta_{i1})
\]

\[
+ \sum_{j \neq i} \sum_{\alpha_j} \int \frac{d\vec{k}_j}{2\omega_j} V_{ij}^{\alpha_1 \alpha_j}(\vec{k}_i, \vec{k}_j) \tau_{\alpha_j}(s_j) F_{j1}^{\alpha_j \alpha_10}(\vec{k}_j, \vec{k}_{10})
\]

where the quantum numbers are \( \alpha_i = \{I_{ij}, m_i, P_i\} \) corresponding to the isospin, spin, helicity, and parity of the isobar, and the transition potentials are given by

\[
V_{ij}^{\alpha_1 \alpha_j}(\vec{k}_i, \vec{k}_j) = \left< \phi_{m_i}^j | \Gamma_{\alpha_i}^\dagger | u_{\alpha_j}^j \right> G_k \left< u_{\alpha_i}^i | \Gamma_{\alpha_j} \phi_{m_j}^j \right>
\]
where $u_{ij}^s$ is the spinor of particle $j$, $\phi_{ij}^\alpha$ is the spinor of the quasiparticle $i$, and $G_k$ is the propagator of the exchanged particle. The transition potentials can be of two kinds; either a pion is exchanged to make a transition from a nucleon and a $\pi N$ isobar into another nucleon and $\pi N$ isobar, or a nucleon is exchanged to make a transition from a nucleon and a $\pi N$ isobar into a pion and a $NN$ isobar.

E. The breakup amplitude

The partial breakup amplitudes are constructed from the solutions of Eqs. (8) as

$$\langle \vec{p}_i \vec{k}_j; \alpha_i|T_i|k_{10}m_{10} \rangle = g_{\alpha_i}(\vec{p}_i) \tau_{\alpha_i}(s_i) F_{i1}^{\alpha_i\alpha_{10}}(k_i, k_{10})$$  \hspace{1cm} (10)

where $\vec{p}_i$ is the relative momentum of particles $j$ and $k$ measured in the two-body c.m. frame, and $\vec{k}_i$ is the momentum of particle $i$ measured in the three-body c.m. frame. $g_{\alpha_i}(\vec{p}_i)$ is the form factor that describes the dissociation of the isobar into particles $j$ and $k$, while $k_{10}$ and $m_{10}$ are the initial momentum of the pion and the initial helicity of the deuteron respectively. All three amplitudes $T_i$ with $i = 1, 2, 3$ as defined in Eq. (10), contribute to the breakup amplitude as shown by Eq. (2).

F. The two-body input

In order to solve the integral equations (8), we use as input the pion-nucleon $S_{11}$, $S_{31}$, $P_{11}$, $P_{31}$, $P_{13}$, and $P_{33}$ channels and the nucleon-nucleon $^1S_0$ and $^3S_1-^3D_1$ channels which are represented by separable t-matrices as it is implicit in the isobar approximation (7).

For the initial-state wave function of the deuteron we considered two models, namely, the Paris [9] and the Bonn [10] deuteron wave functions.

3. RESULTS

Data on inclusive $\pi d$ breakup reactions are naturally divided into two sets, depending on whether a pion or a nucleon is detected. If a pion is detected, one can have the normal breakup reactions $d(\pi^+ , \pi^+) X$ where $X = pn$, or a charge exchange process $d(\pi^+, \pi^0) X$, where $X$ represents either two protons or two neutrons depending on the charge of the incident pion. If a nucleon is detected, one can have either a pure breakup reaction like $d(\pi^+, n) X$ and $d(\pi^-, p) X$, where $X = \pi^+ p$ and $X = \pi^- n$, respectively, or a mixture of breakup and charge exchange like $d(\pi^+, p) X$ and $d(\pi^-, n) X$, where in the first case $X = \pi^+ n$ or $X = \pi^0 p$, and in the second case $X = \pi^- p$ or $X = \pi^0 n$.

A. Pion detection results

The reaction $d(\pi^+, \pi^+) X$ was measured by Kahndaker et al. [11], for an incident pion energy of 96.5 MeV. We show in Fig. 1 the differential cross section $d\sigma/d\Omega$ of Kahndaker...
et al. and compare it with our theoretical results. The dashed line is the impulse approximation, while the solid line is the result of the full calculation using the Paris deuteron wave function [9]. The results of the full calculation describe the data reasonably well. The impulse approximation, on the other hand, disagrees strongly with the data, in particular, as one approaches the forward direction. The failure of the impulse approximation is a consequence of the fact that in that approximation the wave function of the two final nucleons is not orthogonal to the deuteron wave function. In order to understand these results, let us consider the simpler case of the distorted-wave impulse approximation. In that approximation we have that

\[
\langle \vec{k}_1 \vec{p} | A_{\text{DWA}} | \phi_d \vec{k}_{10} \rangle = \sum_{i=1,2,3} \langle \vec{k}_1 \chi_{\vec{p}} | t_i | \phi_d \vec{k}_{10} \rangle,
\]

where \(\vec{k}_{10}\) and \(\vec{k}_1\) are the initial and final momenta of the pion, \(\phi_d\) is the deuteron wave function, and \(\chi_{\vec{p}}\) is a nucleon-nucleon distorted wave. Since the pion-nucleon amplitude \(t_i\) varies slowly in momentum space as compared with the nucleon-nucleon wave functions \(\phi_d\) and \(\chi_{\vec{p}}\), the matrix elements in the r.h.s. of Eq. (11) can be factorized as

\[
\langle \vec{k}_1 \chi_{\vec{p}} | t_i | \phi_d \vec{k}_{10} \rangle \approx \langle \vec{k}_1 | t_i | \vec{k}_{10} \rangle \int d\vec{r} \chi^*_{\vec{p}}(\vec{r}) e^{i(\vec{k}_1 - \vec{k}_{10}) \cdot \vec{r}} \phi_d(\vec{r}),
\]

therefore, since the functions \(\chi_{\vec{p}}\) and \(\phi_d\) are orthogonal to each other, one must have that

\[
\langle \vec{k}_1 \vec{p} | A_{\text{DWA}} | \phi_d \vec{k}_{10} \rangle \rightarrow 0 \quad \text{if} \quad \vec{k}_1 \rightarrow \vec{k}_{10},
\]
so that the cross section must vanish in the forward direction $\vec{k}_1 = \vec{k}_{10}$. In the case of the impulse approximation [see Eq. (3)], the distorted wave $|\chi_p\rangle$ is replaced by a plane wave $|\vec{p}\rangle$ and therefore the effect of orthogonality is destroyed. We show in Fig. 2 the corresponding results for the double differential cross section $d^2\sigma/d\Omega dE_{\pi}$ for four different pion angles. Again as in the previous figure, the full calculation describes the data reasonably well, while the impulse approximation fails badly as one approaches the forward direction.

The differential cross section of the charge-exchange reaction $d(\pi^+, \pi^+)X$ where $X = nn$ has been measured by Moinester et al. [12] for an incident pion energy of 164.1 MeV. We show these results in Fig. 3 where we compare them with the predictions of the impulse approximation with (dashed line) and without (dotted line) Pauli effects as well as with the predictions of the full calculation (solid line) using the Paris deuteron wave function. One should keep in mind that the condition of orthogonality has no effect here, since the final two neutrons are not allowed to be in the $^3S_1-^3D_1$ state. The fact that the cross section tends to vanish in the forward direction already of the level of the impulse approximation is a direct consequence of the Pauli principle, since the two neutrons are identical particles.
B. Nucleon detection results

We have calculated the predictions of our model for the reactions $d(\pi^-, p)X$ and $d(\pi^+, p)X$ at $T_\pi = 256$ MeV. We show in Figs. 4 and 5 our predictions for these differential cross sections. The main feature of these results is that while the $\pi^-$ cross section stays relatively flat, the $\pi^+$ cross section falls down by almost two orders of magnitude as one goes to large angles. No data is as yet available to check this prediction of our model. We will discuss next the meaning of these results.

The reaction $d(\pi^-, p)X$ is dominated by the single-scattering term $\pi^- n \rightarrow \pi^- n$ with the proton acting as spectator, since the contribution of the $\pi^- p \rightarrow p\pi^-$ process comes into the amplitude reduced by a factor of 1/3 due to isospin (assuming delta isobar dominance). Thus, the protons that are detected are mostly spectator protons which always tend to be produced with zero momentum and therefore they give rise to an isotropic angular distribution. The reaction $d(\pi^+, p)X$ is dominated by the single-scattering term $\pi^+ p \rightarrow p\pi^+$ with the neutron as spectator since the term $\pi^+ n \rightarrow \pi^+ n$ with the proton as spectator enters into the amplitude reduced by a factor of 1/3. Thus, the protons observed here are essentially those knocked out by the pion and the differential cross section follows the general shape of the free $\pi^+ p \rightarrow p\pi^+$ cross section. Since in the laboratory system the free $\pi^+ p \rightarrow p\pi^+$ cross section is zero for $\theta > 90^\circ$, this explains the dramatic fall off of the $d(\pi^+, p)X$ cross section in the backward hemisphere that is observed in Fig. 5.

The reactions $d(\pi^-, p)X$ and $d(\pi^+, p)X$ have been measured at an angle $\theta_p = 90^\circ$ as a function of the energy of the incident pion by Arvieux et al. [13], considering several fixed proton momenta. We show the data for the reaction $d(\pi^-, p)X$ in Fig. 6 and compare...
FIGURE 4. Differential cross section of the reaction \(d(\pi^-,p)X\) at 256 MeV. The dashed line is the result of the impulse approximation, the dot-dashed line is the result of the distorted-wave impulse approximation, the solid line is the result of the full calculation with the Paris deuteron wave function, and the dotted line is the result of the full calculation with the Bonn deuteron wave function.

FIGURE 5. Differential cross section of the reaction \(d(\pi^+,p)X\) at 256 MeV. The labeling of the curves is as in Fig. 4.
FIGURE 6. Double differential cross section of the reaction $d(\pi^-, p)X$ at an outgoing proton angle of 90° for three values of the proton momentum (in MeV/c), as a function of the incident energy of the pion. The labeling of the curves is as in Fig. 4. The experimental points are from Ref. [13].

them with the predictions of our model. As one sees, the results are considerably sensitive to the deuteron wave function being used, and the predictions of the full calculation with the Bonn deuteron wave function are the ones that describe the data better. The results of the $d(\pi^+, p)X$ reaction are qualitatively similar to those of Fig. 6.

REFERENCES