Coherent pion production in charge-exchange reactions at intermediate energies

T. Udagawa
Department of Physics, University of Texas
Austin, Texas 78712

ABSTRACT. Recent studies are summarized of coherent pion production (CPP) in charge-exchange reactions at intermediate energies. The experimental data of CPP show a strong collective effect on the excited states in nuclei in the spin-longitudinal channels, from where the coherent pion dominantly comes. The observed collective effect and also other characteristic features of the coherent pion are shown to be well understood by theoretical calculations that take into account correlations due to the \( \pi \)-exchange interaction.


1. INTRODUCTION

One of the most important and widely publicized issues in the study of intermediate energy charge-exchange reactions is the shift of the peak position of the \( \Delta \) excitation spectra in nuclear targets [1] compared to the proton target [2]. This is shown in Fig. 1. As seen there, the \( \Delta \) peak in the proton target spectrum appears at \( \omega_L \approx 370 \text{ MeV} \), \( \omega_L \) being the energy transfer in the laboratory system. On the other hand, in the nuclear target (\( ^{12}\text{C} \)) the peak appears at \( \omega_L \approx 300 \text{ MeV} \), which is 70 MeV lower than the peak in the proton target spectrum. The peak position of \( \omega_L \approx 370 \text{ MeV} \) for the proton target can essentially be understood as the sum of the energy (294 MeV) needed to make the \( \Delta \) and the recoil energy (95 MeV) received by the \( \Delta \) in the excitation process. The small difference (\(-19 \text{ MeV} \)) between the summed energy (389 MeV) and the observed energy may be ascribed to the momentum transfer dependence of the transition amplitude involved. The downward energy shift of about 70 MeV in nuclear targets has been a problem for some time. Its quantitative explanation came from theoretical analyses made by us [3,4] and others [5].

In Ref. [3], the analyses of the inclusive data were carried out in an approach based on the isobar-hole model [6] and the DWIA. The model included all important nuclear medium effects such as those of the one-body mean-field and the two-body \( \Delta \)-particle-nucleon-hole (\( \Delta N^{-1} \)) interaction. The results showed that about 40 MeV out of the 70 MeV shift is due to the one-body mean field effects on the target nucleons and the \( \Delta \). The rest (30 MeV) is due to a nuclear correlation effect produced by the \( \Delta N^{-1} \) interaction.
in the medium. This interaction is strongly attractive in the spin-longitudinal channel due to the \( \pi \)-exchange interaction, leading to a lowering of the \( \Delta \) mass produced in the target. Other effects contributing to the shift come from an additional two-body interaction effect that causes the conversion of \( \Delta \) into \( N \) via the conversion process, \( \Delta + N \rightarrow N + N \). We treated this effect by adding an imaginary part to the real mean field of the \( \Delta \).

In Fig. 2, we reproduce our calculated 0-degree inclusive spectrum of the \( {}^{12}\text{C}(p,n) \) reaction at 800 MeV incident energy (full line), in comparison with the experimental data. We introduced there an arbitrary normalization factor of \( N = 1.3 \). With this factor, the calculated cross sections fit the data fairly well. Fig. 2 also shows contributions from the spin-longitudinal (LO) and spin-transverse (TR) channels separately. It is seen that the peak position of the spin-LO spectrum is lowered considerably in energy in comparison with the spin-TR spectrum or with the uncorrelated result that neglects the two-body \( \Delta N^{-1} \) interaction effects. The large energy shift seen in the spin-LO spectrum is due to the attractive \( \pi \)-exchange interaction. The peak of the uncorrelated spectrum appears at \( \omega \approx 320 \text{ MeV} \), somewhat lower than the 330 (\( \approx 370 - 40 \) MeV) that is expected from the binding energy effect on the \( \Delta \) and the \( N \). This small shift is due to the conversion (spreading) effect, which we took into account in terms of \( W_\Delta \).

The theoretical cross sections shown in Fig. 2 includes contributions from three different physical processes characterized by the mode of the subsequent decay of the \( \Delta \). The three different decay modes in question are the quasi-free decay \( \Delta \rightarrow \pi + N \), the decay due to the conversion (spreading) process \( \Delta + N \rightarrow N + N \), and the decay resulting from single pion production \( \Delta + N^{-1} \rightarrow \pi \). The corresponding physical processes are called the quasi-free (qf), conversion, and coherent pion production (CPP) processes, respectively.

Among these processes, the CPP process, in which the residual nucleus is left in the
ground state of the target, is an intriguing process. The process can be viewed as an elastic scattering of virtual pion emitted in the projectile charge-exchange process, e.g. the \((p,n)\) reaction process. After the scattering, the virtual pion turns into a real pion by giving the extra momentum to the target as a recoil momentum. Since all \(\Delta N^{-1}\) pairs excited in the scattering process contribute coherently, the process has been referred to as CPP. Much focus has recently been given to the process both experimentally \([7–10]\) and theoretically \([11]\). Because of the virtual (off-the-energy-shell) nature, the study of the process is expected to provide information of the off-the-energy-shell part of the \(\pi\)-nucleus interaction. In what follows, we summarize the results of the studies of CPP.

### 2. THEORETICAL METHOD

Let us first discuss very briefly the theoretical description of CPP. It is based on the isobar-hole model which has been successfully used in the description of pion-nucleus scattering \([6]\). We write the charge exchange reaction as \(A + a \rightarrow (B + \Delta) + b\), where \(A(B)\) is the target (residual hole) nucleus while \(a(b)\) is the projectile (ejectile). In the isobar-hole model the \(\Delta\) is assumed to move in a complex one-body potential \(U_\Delta = V_\Delta + iW_\Delta\). The imaginary part describes one of the important nuclear medium effects, i.e., the decay of the \(\Delta\) due to the conversion process. Further, the \(\Delta\) interacts with \(B\) via a two-body \(\Delta N^{-1}\) interaction \(V_{\Delta N,\Delta N}\) and is allowed to decay into \(\pi + N\). The latter decay of the \(\Delta\) is described by means of the energy dependent (free) decay width \(\Gamma_\Delta(\omega)\). The wave function
\[ |\psi\rangle = G|\rho\rangle = \frac{1}{E + iT_\Delta/2 - H_B - T_\Delta - U_\Delta - V_{N_\Delta,N_\Delta}}|\rho\rangle, \]

where \( E \) is the excitation energy of the \((B + \Delta)\) system, \( T_\Delta \) is the kinetic energy operator for the \( \Delta \), and \( H_B \) is the Hamiltonian of the hole nucleus \( B \).

\(|\rho\rangle \) in Eq. (1) is the doorway state defined as

\[ |\rho\rangle = (\chi^+_a \phi_b |t_{N_\Delta,N_\Delta}| \chi^+_a \phi_a \phi_A), \]

where \( \chi^+_a \) and \( \chi^+_b \) are the projectile distorted wave functions in the incident and exit channel, respectively, \( \phi_a \) and \( \phi_b \) are the intrinsic wave functions of \( a \) and \( b \), and \( \phi_A \) is the initial target wave function assumed to be of spin-parity \( I_A = 0^+ \). Further, \( t_{N_\Delta,N_\Delta} \) is the effective \( NN \rightarrow N\Delta \) transition operator, on which some detailed discussions will be given later. The round bra on the right hand side of Eq. (2) denotes integration with respect to the projectile coordinates only. As is clear, \( G \) in Eq. (1) serves as a propagator for the residual \( B + \Delta \) system.

The differential cross section for the coherent \( \pi^- \)-production reaction \( A(p, n\pi^+)A(\text{g.s}) \) may then be given as \( |4,11| \)

\[ \frac{d^3\sigma}{dE_{\pi}d\Omega_{\pi}d\Omega_{\pi}} = \frac{1}{(2\pi)^5 16\sqrt{\lambda(s,M_a^2,M_B^2)}} \left| (E_A + \omega) + \frac{E_{p_\pi}}{p_\pi} (p_b \cos \theta_{b\pi} - p_a \cos \theta_{a\pi}) \right| \]

\[ \times M_a M_b \sum |T_{fi}|^2, \]

where \( E_i = \sqrt{p_i^2 + M_i^2} \), \( p_i \) and \( M_i \) being the momentum and mass of the particle \( i \) (= \( a, b, A, \) or \( B \)). Further, \( T_{fi} \) is the invariant transition amplitude and can be given in terms of \(|\psi\rangle \) of Eq. (1) as

\[ T_{fi} = \sqrt{2E_A 2E_\pi} \phi(\vec{p}_\pi) \left( \varphi_A, \varphi(\vec{p}_\pi) \right) \frac{f_{\pi N\Delta} \vec{S} \cdot \vec{r}_\pi F(\kappa^2_\pi) T_{\mu}}{m_\pi} |\psi\rangle \sqrt{2E_A 2E_\pi}. \]

Here \( \vec{S} \) \((\vec{S})\) and \( \vec{T} \) \((\vec{T})\) denote the spin and isospin transition operators, respectively, that change the spin and isospin of the \( \Delta \) \((N)\) to those of the \( N \) \((\Delta)\). \( f_{\pi N\Delta} \) is the pion-nucleon coupling constant, \( m_\pi \) is the pion mass, \( \kappa^2_\pi \) is the relative pion-nucleon momentum in the \( \Delta \) rest frame, \( F(\kappa^2_\pi) \) is the pion-nucleon form factor, and \( \phi(\vec{p}_\pi) \) is the plane wave pion wave function. Further in Eq. (3), \( \omega \) is the time component of the transferred four-momentum \((\omega, \vec{q})\) and \((E_{\pi}, \vec{p}_\pi)\) is the four-momentum of the emitted \( \pi \).

One of the most important and yet most unknown theoretical parameters involved in the above theoretical cross section is the transition operator \( t_{N_\Delta,N_\Delta} \). In the calculations performed so far \([3,4]\), we made the following simple ansatz:

\[ t_{N_\Delta,N_\Delta} = t'_{N_\Delta} J_{\pi N_\Delta} \left( \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t} \right)^2 \left( [\vec{\sigma}_1 \cdot \vec{q}] (\vec{S}_2 \cdot \vec{q}) + (\vec{\sigma}_1 \times \vec{q}) \cdot (\vec{S}_2 \times \vec{q}) \right) \vec{r}_1 \cdot \vec{T}_2 \]

Here \( \vec{S} \) \((\vec{S})\) and \( \vec{T} \) \((\vec{T})\) denote the spin and isospin transition operators, respectively, that change the spin and isospin of the \( \Delta \) \((N)\) to those of the \( N \) \((\Delta)\). \( f_{\pi N\Delta} \) is the pion-nucleon coupling constant, \( m_\pi \) is the pion mass, \( \kappa^2_\pi \) is the relative pion-nucleon momentum in the \( \Delta \) rest frame, \( F(\kappa^2_\pi) \) is the pion-nucleon form factor, and \( \phi(\vec{p}_\pi) \) is the plane wave pion wave function. Further in Eq. (3), \( \omega \) is the time component of the transferred four-momentum \((\omega, \vec{q})\) and \((E_{\pi}, \vec{p}_\pi)\) is the four-momentum of the emitted \( \pi \).

One of the most important and yet most unknown theoretical parameters involved in the above theoretical cross section is the transition operator \( t_{N_\Delta,N_\Delta} \). In the calculations performed so far \([3,4]\), we made the following simple ansatz:
with \( J_{\pi N\Delta} = 4\pi\hbar c f_{\pi NN} f_{\pi N\Delta} / m_{\pi}^2 \approx 800 \text{ MeV fm}^3 \), \( t'_{N\Delta} = 0.61 \), and \( \Lambda_\pi = 650 \text{ MeV} \). 

\( t \) is the momentum transfer defined by \( t = w^2 - q^2 \). The above operator is essentially of \( \delta \)-function type; the only momentum dependence comes from the vertex form factor \( ((\Lambda_\pi^2 - m_{\pi}^2)/(\Lambda_\pi^2 - t))^2 \). Further, \( t_{NNN, N\Delta} \) has a very simple spin structure; the strength of the spin-LO term (the first term in the bracket) is equal to that of the spin-TR term (the second term). Despite of its simple structure, the \( t_{NNN, N\Delta} \) operator of Eq. (5) allows for an explanation of the \( p(\vec{p}, \vec{\pi})\Delta^{++} \) data [12, 13]. This concerns not only the cross section, but also the spin observables [4].

Other important theoretical parameters besides the \( t_{NNN, N\Delta} \) operator in the calculations are the one-body \( \Delta \)-nucleus optical potential \( U_\Delta \) and the two-body \( \Delta N^{-1} \) interaction \( V_{N\Delta, N\Delta} \). We use for \( U_\Delta \) a complex Woods-Saxon potential determined from the \( \Delta \)-nucleus scattering analyses [6]. For \( V_{N\Delta, N\Delta} \), use is made of the \( \pi + p + g' \) model. The details of the values of the parameters involved will be found in Ref. [4].

3. COHERENT PION PRODUCTION

A great deal of effort has recently been devoted to measure CPP cross sections at various laboratories such as KEK [7], SATURNE [8, 9], and LAMPF [10]. We first concern the pion angle integrated coincidence cross section \( \sigma(\omega_L) \) as a function of \( \omega_L = E_a - E_b \);

\[
\sigma(\omega_L) = \int \frac{d^3 \sigma}{d\Omega_b d\omega_L d\Omega_\pi} d\Omega_\pi. \tag{6}
\]

Note that \( \omega_L \approx E_\pi \). Therefore, \( \sigma(\omega_L) \) is nothing but the energy spectrum of the coherent pion or the total cross section of the virtual pion scattering as a function of the incident energy \( E_\pi \).

In Fig. 3 we first present \( \sigma(\omega_L) \) of the \( ^{12}\text{C}(p, n\pi^+)^{12}\text{C}(\text{g.s.}) \) reaction taken at KEK [7]. The incident energy of the reaction is \( E = 822 \text{ MeV} \). Unfortunately, the experimental cross section shown there includes not only contributions from real CPP events, but also from false events, such as those of the \( ^{12}\text{C}(p, n\pi^+n) \) and \( ^{12}\text{C}(p, n\pi^+p) \) reactions where the neutron or proton escape the experimental detection [7]. With such data, it is difficult to make an unambiguous test of the theoretical calculation. Nevertheless, we show in Fig. 1 our calculated cross section by dotted line. As seen, the calculated cross section somewhat underestimates the data, and also the peak position of \( E_\pi = 230 \text{ MeV} \) is much lower than the experimental one. This discrepancy may, however, be understood as arising from the false events included in the data as discussed above. In Fig. 3 we also present among others the contribution from the \( ^{12}\text{C}(p, n\pi^+n) \) reaction (dashed curve). The calculated cross section of this reaction has its peak at much higher excitation energy \( (\omega_L \sim 350 \text{ MeV}) \) than the cross section of the CPP process. This additional contribution, if added, leads to an improvement of the fit to the data (full curve). Further improvements will be obtained if one adds the contributions from the other processes (or if one subtracts the false events from the data).

It is remarkable that the peak position, \( \omega_L = 230 \text{ MeV} \), of the calculated coherent pion energy spectrum is even more lower than the peak of the spin-LO spectrum shown in Fig. 1. This is due to the strong collective effects due to the \( \pi \)-exchange interaction.
COHERENT PION PRODUCTION IN CHARGE-EXCHANGE...

\[ ^{12}\text{C} (p, n) \]

\[ T_p = 822 \text{ MeV} \]

\[ \theta_\theta = 0^\circ \]

**FIGURE 3.** Comparison of the calculated pion coincidence spectrum for the \(^{12}\text{C}(p, n\pi^+)\) reaction at \(E = 822\) MeV with the experimental data [7].

\[ ^{12}\text{C} (^{3}\text{He, t\pi^+}) \]

\[ T_\text{He} = 2 \text{ GeV} \]

**FIGURE 4.** Comparison of the calculated pion coincidence spectrum for the \(^{12}\text{C}(^{3}\text{He, t\pi^+})\) reaction at \(E = 2\) GeV with the experimental data [9].

Somewhat better data that can be used for a more unambiguous test of the calculations have been taken at SATURNE [9] for the \(^{12}\text{C}(^{3}\text{He, t\pi^+})^{12}\text{C}(\text{g.s.})\) reaction at \(E = 2\) GeV. In Fig. 4 we present the measured \(\sigma(\omega_L)\) as a function of \(\omega_L\). The data include coincident events with tritons emitted into the angle range between 2.5° and 3.5°. Because of this choice of the triton emission angle, the contribution of the false events from the
The calculated angular distribution is strongly forward peaked. This forward peaked angular distribution is very much the same as that of the $\pi$-nucleus elastic scattering. This similarity is not surprising if one recalls that the CPP process can be viewed as a kind of elastic scattering process, in which an initially off-mass shell pion is converted into an on-mass shell pion when the pion is emitted from the target. This conversion is possible since the target nucleus as a whole can pick up the extra momentum needed to lift the pion on its mass shell. In the $^{12}$C($^3$He,$t\pi^+$)$^{12}$C(g.s.) reaction the recoil momentum amounts to $\Delta q \approx 0.5$ fm$^{-1}$ at $\omega_L = 250$ MeV corresponding to a recoil energy of $\Delta \omega_L \leq 1$ MeV for the $^{12}$C nucleus. In Fig. 5, the calculated angular distribution is compared with the experimental data [9]. The calculated angular distribution agrees with experiment very well.

In Fig. 5, the contributions from the spin-LO and spin-TR channels are also shown separately. The shape of the LO angular distribution is strongly forward peaked, which means that most of the pions are emitted into the direction of the momentum transfer $\vec{q}$. This forward peaked character is partially explained in terms of the spin structure of the excitation ($\vec{S} \cdot \vec{q}$) and de-excitation ($\vec{S} \cdot \vec{k}_\pi$) operators involved in the pion production.
through the spin-LO channel. The product of these two operators gives rise to an factor \( qk_\pi \cos \theta_\pi \) in the resultant transition amplitude and thus a factor \( \cos^2 \theta_\pi \) in the cross section. This factor peaks at \( \theta_\pi = 0^\circ \). As seen, however, both the calculated and the experimental angular distributions are much more strongly forward peaked than the \( \cos^2 \theta_\pi \) factor predicts. An important additional angular dependent factor comes, however, from the radial overlap integral involved in Eq. (4). This overlap integral becomes the larger, the smaller the scattering angle \( \theta_\pi \). This is due to the dependence of the integral on the recoil momentum \( |\vec{q} - \vec{p}_\pi| \) transferred to the target in the coherent \( \pi \) decay process. This recoil momentum is smallest for \( \vec{q} \) parallel \( \vec{p}_\pi \), making the overlap integral largest for \( \theta_\pi = 0^\circ \).

It is also remarkable that the TR angular distribution has a characteristic shape with a minimum at \( \theta_\pi = 0^\circ \) and a maximum at \( \theta_\pi \approx 30^\circ \). This shape is very similar to that of angular distributions observed in pion-photoproduction \((\gamma, \pi)\) reactions [14-17] and pion-electroproduction [18,19]. The spin structure of the excitation \((\vec{S} \times \vec{q})\) and de-excitation \((\vec{S}_\pi \cdot \vec{k}_\pi)\) operators involved in \((\gamma, \pi)\) reactions is exactly the same as that of the TR excitation of the nucleus by \((p, n)\) reactions. The product of both operators makes the angular distribution proportional to \( |\vec{q} \times \vec{k}_\pi|^2 = (qk_\pi \sin \theta_\pi)^2 \). This product vanishes for \( \theta_\pi = 0^\circ \) and peaks for \( \theta_\pi = 90^\circ \). As discussed above, there is, however, the additional effect coming from the overlap integral, which makes the resultant angular distribution more forward peaked. Note that the magnitude of the TR cross section is small in comparison to that of the LO cross section. This is again an effect of the overlap integral which decreases rapidly with increasing momentum transfer.

4. Concluding remarks

Recent studies have been summarized of coherent pion production in charge-exchange reactions, which may be viewed as an elastic scattering of a virtual pion from the target. The data are expected to provide information of the off-the-energy-shell part of the \( \pi \)-exchange interaction. The data available are still not of good quality, but reveals a strong collective effect (pionic mode), which can be ascribed to the strong attractive character of the off-the-energy-shell part of the \( \pi \)-exchange interaction. It has been shown that the observed collective characters are well described by the present theory [4].

This work is supported in part by the U.S. Department of Energy under Contract DE-FG03-93ER40785.

References