Two-body tensor interactions in the nuclear matter response function

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Abstract. The inclusive scattering response of nuclear matter is studied in the regime of large momentum transfer q, and around the quasielastic peak. We review interaction corrections to free propagation as embodied in the impulse approximation. Calculations of the two-body and many-body corrections within an eikonal approach are presented. These use an approximated two-body density matrix which takes account of spin and isospin degrees of freedom. Both calculations give similar and sizable corrections at \( q = 550 \text{ MeV} \) and reproduce data extrapolated from finite nuclei; this indicates the relevance of two-body tensor contributions in this regime.

Resumen. La respuesta inclusiva de dispersión de la materia nuclear es estudiada en el régimen de alto momento transferido \( q \), y alrededor de la cima cuasi-elástica. Son descritas las correcciones de interacción a la propagación libre, la cual es implicada por la aproximación impulsiva. Son presentados cálculos de las correcciones de dos cuerpos y muchos cuerpos, hechos con un tratamiento eikonal. Estos utilizan una matriz de densidad aproximada de dos cuerpos que toma en consideración los grados de libertad espín e isospín. Ambos cálculos producen correcciones similares y considerables a \( q = 550 \text{ MeV} \) y asimismo reproducen datos experimentales extrapolados de núcleos finitos; esto indica la relevancia de correcciones de dos cuerpos en este régimen.

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1. Introduction

Inclusive scattering of electrons on nuclei provides an important source of information on the system probed [1]. In this type of collision the electron with initial momentum \( \vec{k}_i \), and energy \( \epsilon_i \), scatters from a nucleus emerging with final momentum and energy \( \vec{k}_f \) and \( \epsilon_f \), and, among the products of the reaction, only the electron is detected, thus giving to the system a momentum and energy transfer \( \vec{q} = \vec{k}_i - \vec{k}_f \) and \( \omega = \epsilon_i - \epsilon_f \), respectively. The amplitude involved in this process is represented in Fig. 1 where the system is excited from the ground state \( |\psi_0\rangle \) with ground state energy \( E_0 \) to a state \( |\psi_n\rangle \) with energy \( E_n = E_0 + \omega \). Given that the interaction between the projectile and the particles in the target is weak, it is possible to take only the first Born term describing the interaction between the electron and the nucleon. In this case the effects of the electron-nucleon interaction and the excitation of the system decouple [2] and the cross section in this order is given by the expression

\[
\frac{d\sigma}{d\Omega d\epsilon} = \frac{d\sigma}{d\Omega} S(q, \omega).
\]
\[
\frac{d\sigma}{d\Omega} \text{ is the elementary cross section between the electron and the nucleon (containing the form factor), and } S(q, \omega) \text{ is the response function which contains information on the system. In the following equalities we start from an expression for the response as a sum over the square of vertex amplitudes representing transitions to different excited eigenstates } |\Psi_n\rangle \text{ of the Hamiltonian with energy } E_n, \text{ corresponding to a sum over the square of diagrams represented in Fig. 1, and then we convert it [2] into an equivalent expression in terms of the Compton amplitude in the forward direction}
\]

\[
S_L(q, \omega) = \sum_n \sum_i \frac{1}{A} |\langle \Psi_0 | e^{-i\vec{q}\cdot\vec{r}_i} | \Psi_n \rangle|^2 \delta(\omega - (E_n - E_0))
\]

\[
= \sum_i A \frac{i}{2\pi A} \left\langle \Psi_0 | e^{-i\vec{q}\cdot\vec{r}_i} \left[ \frac{1}{\omega + E_0 - H + i\epsilon} - \frac{1}{\omega + E_0 - H - i\epsilon} \right] e^{i\vec{q}\cdot\vec{r}_j} | \Psi_0 \right\rangle
\]

\[
= -\frac{1}{\pi} \text{Im} \langle \Psi_0 | [1 + (A - 1)e^{i\vec{q}\cdot(\vec{r}_1 - \vec{r}_2)}]G(q, \omega + E_0)|\Psi_0\rangle,
\]

with

\[
G(q, \omega + E_0) = e^{-i\vec{q}\cdot\vec{r}_i} \frac{1}{\omega + E_0 - H + i\epsilon} e^{i\vec{q}\cdot\vec{r}_j}
\]

\[
= \frac{1}{\omega + E_0 - H - H'' + i\epsilon}
\]

where \(H\) is the Hamiltonian of a system of \(A\) identical particles. In the third equality of Eq. (2) we have written the response in terms of the momentum shifted Green's function in Eq. (3), where \(H'' = q \cdot p_1/m + q^2/(2m)\), and \(p_1\) is the momentum of the hit particle. This work describes the study of the incoherent response function in the asymptotic regime, that is, considering the first part of Eq. (2) (third equality) at large values of \(q\) and at values of \(\omega\) around \(\omega = q^2/2m\). The second coherent part expected to give a small contribution in this regime [2].
The expression for the response in terms of the Compton amplitude makes explicit the complication in the calculation of the response as it involves taking account of the propagation and interaction of all particles. However, in the asymptotic regime the scattering process description may be simplified by focusing on the hit particle, which is that scattered by the projectile and carries a distinctive larger momentum than other particles in the system. Indeed, in general a quasielastic peak is seen around values of $\omega = q^2/(2m)$ (kinematic conditions for a single particle taking most of the energy and momentum entering the system) and in the asymptotic regime the response approximately scales on a variable describing the initial momentum of the hit particle, around the quasielastic peak. These characteristics are exhibited by the impulse approximation (IA) for the response, given by

$$S_{IM}(q, \omega) = \frac{m}{q} \int \frac{d^3p}{(2\pi)^3} n(p) \delta(y - \hat{q} \cdot \hat{p}) = \frac{1}{(2\pi)^2} \frac{m}{q} \int_{|y|}^{\infty} dp \rho n(p)$$

which corresponds to neglecting $E_0 - H$ in $G(q, \omega + E_0)$ in Eq. (3), where $n(p)$ is the momentum distribution, and $y$ is the scaling function, $y = m\omega/q - q/2$. As all interactions have been neglected in this approximation, it describes propagation of the hit particle through a linear kinetic energy term in $H''$. A principal aim in this work has been to consider corrections to this approximation, accounting for further interactions of the hit particle. The work has centered on developing different expansions of the response in the asymptotic regime and also on actual calculations for the response for nuclear matter. In the following, we report on these subjects.

2. Perturbation expansions of the response

A starting point for considering the behavior of response in the asymptotic regime is the Gersch series [3] which is formally obtained by expanding in the momentum-shifted Green’s function in Eq. (3)

$$G(q, \omega + E_0) = G_E + G_E O_E G_E + G_E O_E G_E O_E G_E + \ldots,$$

in terms of

$$G_E = \frac{1}{\omega - q^2/2m - \hat{q} \cdot \hat{p}_1/m + i\epsilon},$$

and the operator $O_E = H - E_0$, and then taking expectation values of the operator over the ground state wave function. Explicitly, by construction the Gersch series has the IA as a zero order term (see Eqs. 2–6). Higher order corrections are given by expectation values of operators in orders of $1/v_q = m/q$. The series allows for $n$-particle contribution separation of the terms, implying only the $n$-particle density matrix is required for their calculation. This series is useless when large or diverging potentials produce large corrections. A standard procedure for these cases has been to consider the multiple-scattering series in which the interaction is accounted for by the finite $t$-matrix. However, the operators required in this expansion are all $A$-body operators. Thus, one faces the
problem of finding a partial $1/v_q$ expansion preferably having the IA as a zero order term which is finite for singular potentials, and whose corresponding operators can be classified according to the $n$-particle density matrix required for their evaluation.

In this context, (for details, see Ref. 4) alternative expansions of the response were proposed which are useful in the asymptotic regime but whose application could go beyond that. The starting point is to consider the momentum shifted Green's function appearing in the response (see Eq. 3), and to expand in orders of the ground state energy $E_0$, and in some part of the Hamiltonian, $\overline{H} = H - H'$, that is, we take a similar expansion to Eq. (5) but now with $O_H = \overline{H} - E_0$ and the corresponding Green's function. The Gersch series simply corresponds to the case $\overline{H} = H$. For the general case, the terms can be rearranged and resumed. This results in a perturbative series for the response $S(q, \omega) = \sum_n S^{(n)}(q, \omega)$, where each term $S^{(n)}(q, \omega)$ is of order $n$ in $\overline{H}$ and is given as an expectation value of some operators $B^{(n)}$

$$S^{(n)}(q, \omega) = \text{Re}(\Psi_0|B^{(n)}|\Psi_0)$$

The lowest order term $B^{(0)}$ is

$$B^{(0)} = \int d\lambda \frac{1}{\omega - \lambda - H' - H'' + i\epsilon} \left[ \frac{1}{-\lambda - H' + i\epsilon} - \frac{1}{-\lambda - H' - i\epsilon} \right]$$

where $H' = H - \overline{H}$. $B^{(0)}$ is composed of two terms, one of which contains the operator $H''$. This feature persists also in higher order terms of the response $S(q, \omega)$ which are given by expectation values of

$$B^{(n)} = \int d\lambda \frac{1}{\omega - \lambda - H' - H'' + i\epsilon} \left[ \overline{H}, B^{(n-1)} \right] \delta(\lambda + H').$$

In fact, we have associated the rhs and lhs operators in Eqs. (8) and (9) respectively with initial (ISE) and final state effects (FSE) contributions. This identification is in accordance with a previous assignment [5] in which the ISE contribution comes from the binding energy $-E_0$ [in the one-particle case, see Eq. (2)].

A suitable choice of the operator $\overline{H}$ is implied by the physics of the problem. We now consider the resulting expansions for two choices which are relevant in the asymptotic regime:

2.1. EIKONAL FORMULA

The momentum shift $q$ in the response produces a linear term in the momentum operator (contained in $H''$) in terms of which the response can be expanded, together with the potential energy. One then chooses $H' = V_1$, where $V_1 = \sum_{j \neq 1} v_{1j}$ is the potential energy of the hit particle and the other particles. By making this substitution into Eqs. (7) and (8) one can obtain the explicit formula evaluated in coordinate space.
\[ S_{\text{eik}}(q, \omega) = \text{Re} \frac{1}{2\pi v_\omega^2} \int dz_1 dz_2' d^3 b_1 \rho_\lambda(b_1, z_1; b_1, z_2') \exp[iy(z_1' - z_1)] \]

\[ + \sum_{n=2}^{A} \frac{1}{(n-1)!} dz_1 dz_2' d^3 b_1 d^3 r_2 \ldots d^3 r_n \rho_n(b_1, z_1; b_1, z_2' \ldots r_n; b_1, z_2' \ldots r_n) \exp[iy(z_1' - z_1)] \]

\[ \times \prod_{i=2}^{n} \{ \exp \left[ -i \int_{z_1}^{z_1'} d\zeta_1 v(b_1 - b_i, \zeta_1 - z_1) + \frac{i}{v_\omega} (z_1' - z_1) v(b_1 - b_i, z_1 - z_1) - 1 \} \}

\]

(This formula has been expanded in terms of different \( n \)-body contributions but the operator whose expectation value is taken can actually be written in a very similar form to the usual eikonal Green's function [4,6] with \( V_1 \) as potential, and here one must correct with the expected ISE term). As the potential enters the propagator, and is therefore fully summed, the terms in this expansion are finite for singular potentials.

2.2. TWO-BODY CONTRIBUTION

The complete two-body contribution may be isolated and obtained with the choice \( H' = K + V_1 \), and by expanding each of the corresponding Green's functions in Eq. (8) in a multiple scattering series [7]. By examining the resulting expanded terms as well as others from higher order terms \( S^{(n)}(q, \omega) \), one sees that only three of these terms (coming from \( S^{(0)} \)) correspond to a two-body contribution. The resulting expression is

\[ S_{V12}(q, \omega) = -\text{Re} \frac{1}{4\pi} (A - 1) \int d\lambda \langle \Psi_0 | G_{02}^\lambda t_{02}^\lambda G_{02}^\dagger [G_{02} - G_{02}^\dagger] \]

\[ + G_{02}^\dagger [G_{02} t_{02} G_{02} - G_{02}^\dagger t_{02} G_{02}^\dagger] + G_{02}^\dagger t_{02}^\dagger G_{02}^\dagger G_{02} t_{02} G_{02} - G_{02}^\dagger t_{02}^\dagger G_{02}^\dagger G_{02} t_{02} G_{02} - \{cc\} | \Psi_0 \rangle, \]

where the \( G \)'s are two-body propagators (depending on \( \lambda \)) of the hit and spectator particles while the \( t \)'s are the full, undressed interaction between the hit particle and the spectator, and the superscript \( q \) denotes a shift in momentum [4]. These terms may be pictured as in Fig. 2, with the ovals representing the interaction \( t \), and each line propagation of the particle. Each term accounts respectively for FSE, ISE and both FSE and ISE of the hit particle and a spectator. Quadratic terms in the momentum of the hit particle and the spectator's with which it interacts are thus accounted. This term may improve over the two-body linear contribution in Eq. (10).

3. THE NUCLEAR MATTER RESPONSE FUNCTION

A calculation of the response requires some knowledge of the wave-function of the system. In particular, as we were interested in the effects of two-body corrections one major concern in performing such a calculation was therefore to construct a model for the two-particle density matrix \( \rho_2 \). For nuclear matter, the calculation is made more complicated with the presence of the additional degrees of freedom spin and isospin. Simple models
can be constructed for the required off-diagonal $\rho_2$ using the two-pair correlation function as well as the nuclear matter one-particle density matrix $\rho_1$ for which there is information from numerical calculations. For example [3], in the scalar case the model (using translational invariance)

$$\rho_2(r, 0; r + x\hat{x}, 0) = \rho \rho_1(0; x) \sqrt{g(r) g(|r + x|)}$$

reproduces the correct diagonal form for $\rho_2$, partially accounts for short distance behaviour and mimics a Hartree approximation which gives a correct long distance behaviour. In order to extend this model to the nuclear case a basis $(STM_r)$ for the representation of the two-particle density matrix was proposed. This extension can take advantage of the numerically calculated spin-isospin correlation functions [8]. Ad-hoc assumptions taken from scalar cases, can similarly be used to construct models for the off-diagonal parts. The basis is given in terms of the total two-particle isospin ($T$), the total two-particle spin ($S$) and the component of spin along the particles’ linking vector ($M_r$). The basis exhibits the information contained in the correlation functions in terms of probabilities, which allows to construct a model for the density matrix in a way that extends the scalar model of Eq. (12).

Explicitly, in the calculations we shortly describe we used the model for the two particle density matrix

$$\rho_{2NM}(r, 0; r + x\hat{x}, 0) = \rho \rho_1(0; x) \sum_{STM_r} \sqrt{g_{STM_r}(r) g_{STM_r}(|r + x|)} P_{STM_r}$$
Figure 3. Total response function $R_{NM}(q,y)$ with form factor at $q = 550$ MeV, calculated from the two-body response correction with $\phi(q,y) = (q/m)S(q,y)$, and with many-body formula based on Eq. (10) (solid). Corresponding corrections with an expansion based on a different scaling variable are given (dot-dashed and dots, respectively) (see Ref. 12 for details). The “X” are results from correlated basis functions calculations [13]. The pointed squares are data from $^{56}$Fe experiments [11].

We note that the diagonal expression for $\rho_{2NM}$ gives a two-pair correlation function for each of the $STM_t$ states which are positive definite (translational invariance properties for infinite matter and relative coordinates are used, and $P_{STM_t}$ is a projection operator) [12]. We used the first (most important) six components of the A14 interaction [10] which include local tensor correlations, and which is the one used to find the correlations $g_{STM_t}$ and $\rho_1$.

The calculations of the response followed two approaches. In one we calculated the two-body contribution correction to the IA as in Eq. (11) but in the eikonal approximation which corresponds to taking the $n = 2$ in Eq. (10). In the other one we used an approximate many-body sum as in Eq. (10). By using an approximation for the $n$-body density matrix in terms of $\rho_2$ and $\rho_1$ the sum over $\rho_n$ contributions can be done exactly [3, 4]. If the interaction did not contain a tensor component the calculation could be straightforwardly done by separating contributions orthogonal in the $STM_t$ basis since then the momentum and position operators commute with the spin and isospin operators. When a tensor force is considered, this separation is no longer possible and the problem of calculating the response, even within an eikonal approach requires further approximation. At this point we have set aside this problem by neglecting some of the mixing terms, after checking numerically that their contribution is small.

In Fig. 3 we show the results of calculation of the two-body correction to the impulse approximation, as given in Eq. (11), with the $t$'s in the eikonal approximation [Eq. (10)]. We see that the contribution shows oscillations, which we attribute to the finite inte-
gration interval used in the computer. It is noteworthy however that similar behavior is seen in data from nuclei [11], and it is tempting therefore to attribute this behavior to the finite number of nucleons and finite size of the system. Another interesting feature of all calculations (at finite $n$) is the discontinuity in the response, which is produced by the Fermi discontinuity in the momentum distribution, and even persists as off-shell components are considered as by Eq. (7).

In Fig. 3 we also show the many-body response calculation based on Eq. (10). A form factor was taken into account (see Ref. 12 for details). We note that the Fermi discontinuity disappears in the many-body case. This behavior can be understood as a consequence of the participation of all particles [14]. Both calculations are compared with another calculation [13] which uses correlated basis functions and with data from $^{56}$Fe experiments [11]. It is noteworthy the similarity between the many-body and two-body correction, which points at the relevance of the latter correction.

4. CONCLUSIONS AND OUTLOOK

In this work we have studied the nuclear matter response function in the large momentum transfer regime. We have reviewed relevant corrections to the impulse approximation which describes free propagations of the hit particle. We have shown calculations of the two-body and many-body interaction corrections and we have found a sizable contribution for the two-body part. The size of this correction was related to the participation of tensor correlations through the use of the $STM_r$ basis. In order to do this calculation the eikonal Green's function used required further approximation, as a tensor component interaction does not lead to a closed-form. Having in mind the eikonal formula in configuration space, the approximation consisted of taking the diagonal expression for the $STM_r$ components and extrapolating in each non-diagonal corresponding eikonal term. We have checked numerically that mixing among the different components is indeed small. However, in our aim of further formalizing this result, we have lately investigated the eikonal formula for tensor forces [15], for which an explicit approximation has been found. The need of checking this result has brought us to finding that an exact solution for the Schrödinger equation can be found for the case of a tensor centrifugal interaction [16]. We hope that we shall be able to go back to the response problem with these new tools.

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