Nuclear structure far from the valley of $\beta$-stability

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Exotic nuclei far from the line of beta-stability have gained considerable interest in recent years both on the experimental and on the theoretical side. New phenomena are expected such as dramatic changes in the nuclear shell structure. A reliable description of the spin-orbit term and its isospin dependence is therefore necessary. On the other hand the Fermi surface comes very close to the continuum limit in this region. A careful treatment of the continuum and its coupling to the states in the Fermi sea is essential. Therefore we report on investigations in the framework of relativistic Hartree-Bogoliubov (RHB) theory in the continuum for nuclei close to the drip lines. Depending on the shell structure at the continuum limit we find the formation of neutron skins and neutron halos.

Keywords: Exotic nuclei; neutron-halo; relativistic mean field; relativistic Hartree-Bogoliubov; spin-orbit splitting

Los núcleos exóticos lejanos al valle de estabilidad beta han ido ganando interés en años recientes tanto desde el punto de vista experimental como teórico. Se espera encontrar nuevos fenómenos tales como cambios drásticos en la estructura de capas del núcleo. Por eso es necesaria una descripción confiable del término de espín-orbita y su dependencia en el isospin. Por otro lado la superficie de Fermi queda muy próxima al límite continuo en esa región. Es esencial realizar un tratamiento cuidadoso del continuo y su acoplamiento con los estados en el mar de Fermi. Por ello aquí se presenta un reporte de investigaciones realizadas en el contexto de la teoría relativista Hartree-Bogoliubov (RHB) en el continuo para núcleos cercanos a las líneas de emisión de partículas. Según sea la estructura de capas en el límite continuo se encuentra la formación de pieles de neutrones o halos de neutrones.

Descriptores: Núcleos exóticos; halo de neutrones; campo medio relativista; Hartree-Bogoliubov relativista; separación espín-orbita


1. Introduction

New accelerators with radioactive beams allow the experimental study of nuclei far from stability. This allows us to extend our understanding of nuclear structure considerably because of the large isospin values in this region. Theoretical investigations predict considerable changes in the shell structure, which provide the basic understanding of many effects in nuclear structure.

In recent years Relativistic Mean-Field (RMF) models have been successfully applied in calculations of nuclear matter and properties of finite nuclei throughout the periodic table [1]. Using only six or seven parameters they allow a fully self-consistent description of many nuclear properties with high accuracy. As compared to other non-relativistic mean field approximations such as density dependent Hartree-Fock calculations these models have the advantage to provide a consistent description of the spin-orbit term and its isospin dependence.

An essential problem in the theoretical description of drip-line nuclei arises from the closeness of the Fermi level to the particle continuum: particle-hole and pair excitations reach the continuum. The coupling between bound states and the particle continuum has to be explicitly taken into account. The Relativistic Hartree Bogoliubov (RHB) theory [2, 3], which is a relativistic extension of the Hartree Fock Bogoliubov theory, provides a unified description of mean-field and pairing correlations. A fully self-consistent RHB theory in coordinate space [4, 5] correctly describes the coupling between bound and continuum states. The theory provides a framework for describing the nuclear many-body problem as a relativistic system of baryons and mesons not only in the valley of $\beta$-stability but also in regions with large neutron or proton excess even close to the drip-lines.

2. The relativistic Hartree-Bogoliubov model

In comparison with conventional non-relativistic approaches, relativistic models explicitly include mesonic degrees of freedom and describe the nucleons as Dirac particles. Nucleons interact in a relativistic covariant manner through the exchange of virtual mesons: the isoscalar scalar $\sigma$-meson, the isoscalar vector $\omega$-meson and the isovector vector $\rho$-meson. The model is based on the one boson exchange description of the nucleon-nucleon interaction. We start from the effective Lagrangian density [6]

$$\mathcal{L} = \bar{\psi}(i\gamma \cdot \partial - m) \psi + \frac{1}{2} (\partial \sigma)^2 - U(\sigma) - \frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu} + \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{4} R_{\mu \nu} R^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - g_\sigma \bar{\psi} \gamma^5 \psi \sigma \psi$$

- $g_\omega \bar{\psi} \gamma^\mu \omega \psi - g_\rho \bar{\psi} \gamma^\mu \rho \psi - c \bar{\psi} \gamma^\mu \psi \sigma A^\mu(1 - \gamma_5) \psi$,  

(1)

where $U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} g_2 \sigma_3^2 + \frac{1}{4} g_3 \sigma^4$ is a non-linear potential for the $\sigma$-meson [7], which takes into account in the density dependence of the parameters in a phenomenological way.
The lowest order of the quantum field theory is the mean-field approximation. The Dirac equation reads

\[-i\alpha \cdot \nabla + \beta (m + g_\sigma \sigma) + g_\omega \omega^0 + g_\rho \tau_3 \rho^0(\sigma) + e \frac{(1 - \tau_3)}{2} A^0(\sigma)] \psi_t = \varepsilon_t \psi_t. \tag{2}\]

The effective mass \(m^*(r)\) is defined as

\[m^*(r) = m + g_\sigma \sigma(r)\tag{3}\]

and the potential \(V(r)\) as

\[V(r) = g_\omega \omega^0(r) + g_\rho \tau_3 \rho^0(\sigma) + e \frac{(1 - \tau_3)}{2} A^0(\sigma). \tag{4}\]

In order to describe ground-state properties of spherical open-shell nuclei, pairing correlations have to be taken into account. For nuclei close to the \(\beta\)-stability line, pairing has been included in the relativistic mean-field model in the form of a simple BCS approximation [8]. However, for nuclei far from stability the BCS model presents only a poor approximation. In particular, in drip-line nuclei the Fermi level is found close to the particle continuum. The lowest particle-hole or particle-particle modes are often embedded in the continuum, and the coupling between bound and continuum states has to be taken into account explicitly. The BCS model does not provide a correct description of the scattering of nucleonic pairs from bound states to the positive energy continuum. It leads to an unbound system, because levels in the continuum are partially occupied. Including the system in a box of finite size leads to unreliable predictions for nuclear radii depending on the size of this box. In the non-relativistic case, a unified description of mean-field and pairing correlations is obtained in the framework of the Hartree-Fock-Bogoliubov (HFB) theory in coordinate space [9]. It has been extended to relativistic Hartree Bogoliubov (RHB) theory [2, 3], with the following RHB-equations:

\[
\begin{pmatrix}
\hat{h} - \lambda
-\Delta
\end{pmatrix}
\begin{pmatrix}
U_k
V_k
\end{pmatrix}
= E_k
\begin{pmatrix}
U_k
V_k
\end{pmatrix},
\tag{5}\]

where \(\hat{h}\) is the Dirac operator given in Eq. (2) and \(\Delta\) is the pairing field. HFB-theory, being a variational approximation, results in a violation of basic symmetries of the nuclear system, among which the most important is the non conservation of the number of particles. In order that the expectation value of the particle number operator in the ground state equals the number of nucleons, Eqs. (5) contain a chemical potential \(\lambda\) which has to be determined by the particle number subsidiary condition. The column vectors denote the quasi-particle wave functions, and \(E_k\) are the quasi-particle energies.

In the coordinate space representation of the pairing field \(\Delta\) is

\[\Delta_{ab}(\mathbf{r}, \mathbf{r}') = \frac{1}{2} \sum_{c,d} V_{abcd}(\mathbf{r}, \mathbf{r}') \kappa_{cd}(\mathbf{r}, \mathbf{r}'). \tag{6}\]

where \(a, b, c, d\) denote all quantum numbers, apart from the coordinate \(r\), that specify the single-nucleon states. \(V_{abcd}(\mathbf{r}, \mathbf{r}')\) are matrix elements of a general two-body pairing interaction, and the pairing tensor is defined as

\[\kappa_{cd}(\mathbf{r}, \mathbf{r}') = \sum_{E_k > 0} U_{ck}(\mathbf{r}) V_{dk}(\mathbf{r}'). \tag{7}\]

3. Surface diffuseness and spin-orbit reduction at the drip-line

The spin-orbit interaction plays a central role in the physics of nuclear structure. It is rooted in the basis of the nuclear shell model, where its inclusion is essential in order to reproduce the experimentally established magic numbers. In non-relativistic models based on the mean field approximation, the spin-orbit potential is included in a phenomenological way. Of course such an ansatz introduces an additional parameter, the strength of the spin-orbit interaction. The value of this parameter is usually adjusted to the experimental spin-orbit splittings in spherical nuclei, for example \(^{16}\)O. On the other hand, in the relativistic framework the nucleons are described as Dirac spinors. This means that in the relativistic description of the nuclear many-body problem, the spin-orbit interaction arises naturally from the Dirac-Lorentz structure of the effective Lagrangian. No additional strength parameter is necessary, and relativistic models reproduce the empirical spin-orbit splittings.

Many properties of nuclei along the line of beta stability have been successfully described in the framework of models based on the mean-field approximation. Conventional non-relativistic models that include density dependent interactions with finite range (Gogny) or zero-range (Skyrme) forces, have been extensively used to describe the structure of stable nuclei. More recently, it has been shown that models based on the relativistic mean-field theory [6] provide an elegant and economical framework, in which properties of nuclear matter and finite nuclei, as well as the dynamics of heavy-ion collisions, can be calculated (for a recent review see Ref. 1). In comparison with conventional non-relativistic approaches, relativistic models explicitly include mesonic degrees of freedom and describe the nucleons as Dirac particles. Non-relativistic models and the relativistic mean-field theory predict very similar results for many properties of beta stable nuclei. However, cases have been found where the non-relativistic description of nuclear structure fails. An example is the anomalous kink in the isotope shifts of Pb nuclei [10]. This phenomenon could not be explained neither by the Skyrme model, nor by the Gogny approach. Nevertheless, it is reproduced very naturally in relativistic mean-field calculations. A more careful analysis [11] has shown that the origin of this discrepancy is the isospin dependence of the spin-orbit term.

In the following we present results for the chain of Sn and Ni isotopes. We find that in the framework of relativistic mean field theory, the magnitude of the spin-orbit potential is
or interaction become weaker. This result in a reduction of the neutron number. The effective one-body spin-orbit potential is considerably reduced in light drip line nuclei. With an increase of the neutron number, the effective one-body spin-orbit interaction becomes weaker. This result in a reduction of the energy splittings between spin-orbit partners. The reduction of the spin-orbit potential is especially pronounced in the surface region, and does not depend on a particular parameter set used for the effective Lagrangian. These results are at variance with those calculated with the non-relativistic Skyrme model. It has been shown that the differences have their origin in the isospin dependence of the spin-orbit terms in the two models. If the spin-orbit term of the Skyrme model is modified in such a way that it does not depend so strongly on the isospin, the reduction of the spin-orbit potential is comparable to that observed in relativistic mean-field calculations.

In Fig. 1 we display the one- and two-neutron separation energies

\[ S_1(Z,N) = B_1(Z,N) - B_1(Z,N-1), \]

\[ S_{2n}(Z,N) = B_{2n}(Z,N) - B_{2n}(Z,N-2), \]

for Ni (24 \( \leq N \leq 50 \)) isotopes, respectively. The values that correspond to the self-consistent RHB ground-states are compared with experimental data and extrapolated values from Ref. 12. The theoretical values reproduce in detail the experimental separation energies. The model describes not only the empirical values within one major neutron shell, but it also reproduces the transitions between major shells (for details see Ref. 13).

In Fig. 2 we show the self-consistent ground-state neutron densities for the Sn and Ni nuclei. The density profiles display shell effects in the bulk and a gradual increase of neutron radii. In the insert of Fig. 2 we include the corresponding values for the surface thickness and diffuseness parameter. The surface thickness \( t \) is defined to be the change in radius

\[ \rho(r) = \rho_0 \left[ 1 + \exp \left( \frac{r - R_0}{\alpha} \right) \right]^{-1}, \]

where \( R_0 \) is the half-density radius. By adding more units of isospin the value of the neutron surface thickness increases and the surface becomes more diffuse. The increase in \( t \) and \( \alpha \) is more uniform in Sn, and both parameters increase approximately forty percent from \( ^{100}\text{Sn} \) to \( ^{132}\text{Sn} \). A somewhat smaller increase in the surface thickness is observed for Ni isotopes. The diffuseness parameter for Ni is essentially a step function: \( \alpha \approx 0.45 \) fm for \( N < 40 \) and \( \alpha \approx 0.50 \) fm for neutrons in the \( \text{1g}_{9/2} \) orbital. We will show that the observed changes in surface properties result from the reduction of the spin-orbit term in the effective single-nucleon potential.

In Ref. 14 we have shown that in the framework of the relativistic mean-field model the magnitude of the spin-orbit term in the effective single nucleon potential is greatly reduced for light neutron rich nuclei. With the increase of the number of neutrons the effective spin-orbit interaction becomes weaker and this result in a reduction of the energy splittings for spin-orbit partners. The reduction in the surface region was found to be as large as \( \approx 40\% \) for Ne isotopes at the drip-line. The spin-orbit potential originates from the addition of two large fields: the field of the vector mesons (short range repulsion), and the scalar field of the sigma meson (intermediate attraction). In the first order approximation, and assuming spherical symmetry, the spin orbit term can be writ-
Figure 3. Radial dependence of the spin-orbit term of the potential in self-consistent solutions for the ground-states of Sn (50 ≤ N ≤ 82) nuclei.

The spin-orbit potential is given by

\[ V_{s.o.} = \frac{1}{r} \frac{\partial}{\partial r} V_{ls}(r), \]

where \( V_{ls} \) is the spin-orbit potential [15]

\[ V_{ls} = \frac{m}{m_{\text{eff}}} (V - S), \]

\( V \) and \( S \) denote the repulsive vector and the attractive scalar potentials, respectively, \( m_{\text{eff}} \) is the effective mass

\[ m_{\text{eff}} = m - \frac{1}{2} (V - S). \]

Using the vector and scalar potentials from the NL3 self-consistent ground-state solutions, we have computed from (11)–(13) the spin-orbit terms for the Ni isotopes. They are displayed in Fig. 3 as a function of the radial distance from the center of the nucleus. The magnitude of the spin-orbit term \( V_{s.o.} \) decreases as we add more neutrons, i.e. more units of isospin. If we compare \(^{56}\text{Ni}\) with \(^{78}\text{Ni}\), in Fig. 3, the reduction is \( \approx 35\% \) in the surface region. This implies a significant weakening of the spin-orbit interaction. The minimum of \( V_{s.o.} \) is also shifted outwards, and this reflects the larger spatial extension of the scalar and vector densities, which become very diffuse on the surface. The effect is reflected in the calculated spin-orbit splittings of the neutron levels in the canonical basis

\[ \Delta E_{ls} = E_{n,l,j=l-1/2} - E_{n,l,j=l+1/2}. \]

In Fig. 4 we display the energy splittings of spin-orbit neutron partners for Ni and Sn, respectively. The calculated spacings are shown as function of the neutron number. We only include the spin-orbit doublets for which one of the partners is an intruder orbital in a major shell. These doublets display the largest energy splittings. We notice in Fig. 4 that the spacing between spin-orbit partners decreases with neutron number. The effect is stronger in Ni than in Sn.

Figure 4. Energy splittings between spin-orbit partners for neutron levels in Ni and Sn isotopes, as functions of neutron number.

Figure 5. The density distribution of the halo nucleus \(^{11}\text{Li}\) as compared to that of the core \(^{9}\text{Li}\). The shaded area corresponds to the experimental error bars.

4. Halo-phenomena at the neutron drip-line

In Fig. 5 we show the calculated density distribution for the neutrons in the isotopes \(^9\text{Li}\) and \(^{11}\text{Li}\). It is clearly seen that the increase of the matter radius is caused by a large neutron halo in the nucleus \(^{11}\text{Li}\). Its density distribution is in very good agreement with the experimental density of this isotope show with its error bars by the shaded area.

As shown in Fig. 6, the microscopic structure of this halo can be understood by the fact that the Fermi level for the neutrons is very close to the continuum limit in close vicinity to the \(\nu 1p_{1/2}\) below the continuum and to the \(\nu 2s_{1/2}\) level in the continuum. Pairing correlations cause a partial occupation of both the \(\nu 1p_{1/2}\) and the \(\nu 2s_{1/2}\) level, i.e. a scattering of Cooper pairs into the continuum. This is in contrast to ear-
later calculations using Skyrme forces and relativistic mean field without pairing, where the last occupied $\nu 1p_{1/2}$ level had to be shifted artificially very close to the continuum limit by a adjustment of the potential. In contrast to these investigations the halo is not formed by two neutrons occupying the $1p_{1/2}$ level very close to the continuum limit, but is is formed by Cooper-pairs scattered mainly in the two levels $1p_{1/2}$ and $2s_{1/2}$. This is made possible by the fact that the $2s_{1/2}$ comes down close to the Fermi level in this nucleus and by the density dependent pairing interaction coupling the levels below the Fermi surface to the continuum. In contrast to earlier explanations which use the accidental coincidence that one single particle level is so close to continuum threshold that the tail of its wave function forms a halo, this is a much more general mechanism, which could possibly be observed also in other halo nuclei also. One only needs several single particle levels with small orbital angular momenta and correspondingly small centrifugal barrier close, but not directly at, the continuum limit.

In fact going along the neutron drip line there are several such regions, in particular the region where the $2p$ and the $3p$ levels come close to the continuum limit. In the first case a multi-particle halo in the region of heavy Ne-isotopes has been predicted in Ref. 5.

5. Conclusions

Summarizing we can conclude that we have to go beyond the simple relativistic mean field model in order to describe halo nuclei properly. We have to take into account pairing correlations and the coupling to the continuum in the framework of relativistic Hartree Bogoliubov theory. A density dependent force of zero range has been used in the pairing channel, whose strength is adjusted for the isotope $^{7}\text{Li}$ to a similar calculation with Gogny’s force D1S. Good agreement with experimental values is found for the total binding energies and the radii of the isotope chain from $^{6}\text{Li}$ to $^{11}\text{Li}$. In excellent agreement with the experiment we obtain a neutron halo for $^{11}\text{Li}$ without any artificial adjustment of the potential, as it was necessary in earlier calculations.

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