Fingerprint of a QCD string in baryon spectra

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Recibido el 10 de enero de 2001; aceptado el 28 de febrero de 2001

Multi-spin valued states of the Rarita-Schwinger type $(k/2, k/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$ with $k = 1, 3, 5$ are found to be realized in the excitation spectra of the light-quark baryons. We conjecture that the above multi-resonance clusters may take their origin from a QCD string described by means of a linear action.

Keywords: Barionic spectrum; QCD string

Los estados valorados con multiples spins del tipo Rarita-Schwinger $(k/2, k/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$ con $k = 1, 3, 5$ se encuentran en los espectros de las excitaciones de los baryones con quarks ligeros. Conjeturamos que los múltiples de Lorentz pueden tomar su origen de una cuerda de QCD descrita por medio de una acción lineal.

Descriptores: Espectro bariónico; cuerda de QCD

PACS: 98.80.Cq

1. Introduction

The specification of particles by the mass- and spin-quantum numbers is one of the most important paradigms in contemporary field theory. It takes its origin from the frame-independent invariants of the Poincaré group as was first realized by Wigner in his work of late thirties [1]. On that ground, quantum states of free particles in a Poincaré covariant framework transform for different inertial observers according to

$$|\psi'\rangle = \exp [i (\varepsilon^\mu P_\mu - \theta^{\mu\nu} I_{\mu\nu})] |\psi\rangle. \quad (1)$$

Here $P_\mu$ and the totally antisymmetric tensor $I_{\mu\nu}$ with $\mu(\nu) = 0, 1, 2, 3$ are the generators of the Poincaré group which satisfy the commutation relations (Poincaré algebra)

$$[I_{\mu\nu}, I_{\rho\sigma}] = i (g_{\nu\rho} I_{\mu\sigma} - g_{\nu\sigma} I_{\mu\rho} + g_{\mu\rho} I_{\nu\sigma} - g_{\mu\sigma} I_{\nu\rho}), \quad (2)$$

$$[P_\mu, I_{\rho\sigma}] = i (g_{\mu\rho} P_\sigma - g_{\mu\sigma} P_\rho), \quad [P_\mu, P_\nu] = 0, \quad (3)$$

where $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the metric tensor, and $\varepsilon^\mu$ and $\theta^{\mu\nu}$ are continuous parameters. Furthermore, $P_\mu$ generate translations, $I_{kj} = \varepsilon_{kj} I_j$ with $k, (j, l) = 1, 2, 3$ are the generators of rotations in the $kj$-plane, while $I_{0l} = K_l$ generate the boost along each of the $l$-axis. Wigner postulated that particle states transform as classical unitary, and therefore infinite-dimensional representations of the Poincaré group. Later on, Weinberg argued [2] that quantized non-unitary 2(2j + 1)-dimensional representations can also be given particle interpretation, once unitarity of the corresponding quantized field operators is ensured. Weinberg’s argument allowed one to consider massive states of pure spin. Indeed, as the Poincaré group has $P_\mu P^\mu$ and $W_\mu W^\mu$ as Casimir operators (with $W_\mu$ standing for the so called Pauli-Lubanski vector), particles can be characterized by mass $(m)$ and spin $(j)$ according to

$$W_\mu W^\mu |\psi\rangle = -j(j + 1)m^2 |\psi\rangle, \quad (4)$$

$$W_\mu = -\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} T^{\nu\rho} P^\sigma. \quad (5)$$

The specification of particles by the mass- and spin-quantum numbers is one of the most important paradigms in contemporary field theory. The purity of this paradigm was doubted latest by neutrino physics experiments, which provide evidence [3] that a neutrino of a given flavor, $|\nu_\ell\rangle$, instead of being of definite mass, $|m_\ell\rangle$, rather exists in a linear superposition of three states of different masses

$$|\nu_\ell\rangle = \sum_{i=1}^{3} U_{\ell i} |m_i\rangle, \quad \ell = e, \mu, \tau. \quad (6)$$

The massive four-dimensional representation space $(1/2, 1/2)$ of the Lorentz group is the simplest example for a non-pure spin state. This follows from the fact, that the only non-
vanishing matrix elements of, say, $K_x$ in this multiplet are
\[ \langle j' = 0^+, m = 0 | K_x | j = 1^-, m = 0 \rangle = \delta = \langle j' = 1^-, m = 0 | K_x | j = 0^+, m = 0 \rangle. \] (6)

It is obvious that the boost does not preserve the quantum number of spin and demands the $(1/2, 1/2)$ representation space to be in general covariantly inseparable into pure spin-0 and spin-1 states. A more detailed analysis of the massive four-dimensional Lorentz multiplet was performed in Ref. 5. There, the $(1/2, 1/2)$ space was constructed ab initio as $(1/2, 0) \otimes (0, 1/2)$ direct product. The separation between spin-0 and spin-1 states was found to be possible only in the kinematically restricted settings of the rest frame, on the one hand, and the helicity frame, on the other hand. In the most general case of a boost along a direction different but the axis of rotation, the $(1/2, 1/2)$ space was found to covariantly bifurcate solely into a singlet and a triplet of opposite parities, without that these state carry any definite spin. In [5] it was further found that the $P^\mu A_\mu = 0$ condition, usually viewed as an auxiliary condition for eliminating one out of the four degrees of freedom of the $(1/2, 1/2)$ space, is in fact trivially satisfied for the triplet part where it simply reduces to the mass-shell condition $P^2 - m^2 = 0$, while it can not be fulfilled at all for its singlet subspace. Thus, the $(1/2, 1/2)$ space of finite-mass is in general two-spin-valued.

The four vector is at the very heart of the so-called Rarita-Schwinger fields
\[ \Psi_{\mu_1 \mu_2 \ldots \mu_k} := \left( \frac{k}{2} \frac{\kappa}{2} \right) \otimes \left[ \left( \frac{1}{2} \frac{0}{2} \right) \oplus \left( 0 \frac{1}{2} \right) \right]. \] (7)

The latter are described in terms of totally symmetric traceless rank-$k$ Lorentz tensors with Dirac spinor components and satisfy the Dirac equation for each Lorentz index associated with a $(1/2, 1/2)$ space
\[ (i\partial_\lambda \gamma^\lambda - M) \Psi_{\mu_1 \mu_2 \ldots \mu_k} = 0. \] (8)

The simplest field of the type in Eq. (7), with $k = 1$ is built on top of the four-vector. It was considered more than two decades earlier by Rarita and Schwinger [6] and applied to the description of a spin-3/2 particle.

The RS fields are compilations of fermions of different spins and parities. This statement is best illustrated at the level of the group $O(4)$. The Wick rotated compact form of the Lorentz group. For the sake of concreteness, we consider the coupling of, say, a positive parity Dirac fermion to an $O(4)$ boson composed of $O(3)$ states of either natural ($\eta = +$), or, unnatural ($\eta = -$) parities. These mass degenerate $O(3)$ states carry all integer internal angular momenta, $l$, with $l = 0, \ldots, \sigma - 1$ and transform with respect to the space inversion operation $P$ as
\[ P \sigma_{\eta, l, m} = \eta e^{i\eta l} \sigma_{\eta, l, -m}, \]
\[ l^P = 0^\eta, 1^{-\eta}, \ldots, (\sigma - 1)^{-\eta}, \quad m = -l, \ldots, l. \] (9)

In coupling now the Dirac spinor to $(k/2, k/2)$ from above, the following spin $(J)$ and parity $(P)$ quantum numbers are created
\[ J^P = \frac{1}{2}, \frac{1}{2}, \ldots, \frac{k}{2}, \frac{k}{2}. \] (10)

In the following, we will use for the spin-sequence in Eq. (10) the short-hand notation $\sigma_{\eta, l, m}$, with $l$ standing for the isospin of the states considered, and $\sigma = k + 1$.

Now, it was initiated by Weinberg around mid sixties [2] to consider the lower-spin components of the RS fields as redundant, unphysical states that have to be projected out by a set of suitably chosen Lorentz covariant auxiliary conditions. In doing so, one eventually would describe a baryon of single spin-$J = k + 1/2$ and fixed parity as the highest-spin state of the $\Psi_{\mu_1 \mu_2 \ldots \mu_k}$ field and benefit from the linear differential equation [Eq. (6)]. Immediately, one notes that such a description is not necessarily compatible with the general nature of the boost generator. Indeed, since Lorentz and spinor indices of the RS fields always factorize, the boost of this representation space is most naturally obtained in boosting its bosonic part independently from the fermionic one. The properties of the boost to not preserve the quantum number of spin, in combination with Eqs. (9) and (10), clearly illustrate that the presence of the lower-spin components in $(k/2, k/2)$ and thereby in the associated RS cluster, guarantee in a simple manner its covariance with respect to Lorentz transformations. It is little wonder, that truncated RS fields suffer various pathologies (see Ref. 11 for details). So one has to face the unavoidable conclusion of the principal difficulty to describe pure higher-spin states of fixed parity in terms of linear differential equations. Note, that the problem arose out of the tacit assumption that nature produces baryons as pure higher-spin states. A glance at the baryon spectra teaches us that actually Nature strongly favors the excitations of multi-spin-valued resonance clusters over that of pure higher-spin states.

We now show that the new interpretation of the RS fields is supported by the data on baryonic spectra.

3. Rarita-Schwinger fields as multi-resonance clusters

One of the basic quality tests for any model of composite baryons is the level of accuracy reached in describing the nucleon and $\Delta$ excitation spectra. In that respect, the knowledge on the degeneracy group of baryon spectra appears as a key tool in constructing the underlying strong-interaction dynamics. To uncover it, one has first to analyze isospin by isospin how the masses of the resonances from the full listing in Ref. 12, spread with spin and parity. Such an analysis has been performed in our previous work [9] where it was found that Breit-Wigner masses reveal on the $M/J$ plane a well pronounced spin and parity clustering. There, it was further shown that the quantum numbers of the resonances belonging to a particular cluster fit into Lorentz group representations of...

The full bricks stand for three-to-four-star resonances, while the triangles represent states that are "missing" for the completeness of the three RS clusters. Note that "missing" $F_{17}$ and $H_{3,11}$ nucleon excitations in Fig. 1a appear as four-star resonances in the $\Delta$ spectrum of Fig. 1b. The "missing" $\Delta$ excitations $P_{33}, P_{33},$ and $D_{33}$ from $6_{3,-}$ are one-to-two star resonances in the nucleon counterpart $6_{1,-}$. The $\Delta(1600)$ resonance (shadowed oval) drops out of our RS cluster systematics and we view it as an independent hybrid state.

the RS type. To be specific, one finds the three RS clusters with $k = 1, 3,$ and 5 in both the nucleon (N) and $\Delta$ spectra. In terms of the notations introduced above, all reported light-quark baryons with masses below 2500 MeV (up to the $\Delta(1600)$ resonance that is most probably an independent hybrid state), have been shown in Ref. 13 to be completely accommodated by the RS clusters $2_{2I,+}, 4_{2I,-},$ and $6_{2I,-},$ having states of highest spin-$3/2^-, 7/2^+$, and $11/2^+$, respectively (see Fig. 1). In each one of the nucleon, $\Delta$, and $\Lambda$ hyperon spectra, the natural parity cluster $2_{2I,+}$ is always of lowest mass. We consider it to reside in a Fock space, $F_+,$ built on top of a scalar vacuum. From Eqs. (10) and (11) follows that the $2_{2I,+}$ clusters, where $I = 1/2, 3/2,$ and 0, always unite the first spin-$1/2^+, 1/2^-$, and $3/2^-$ resonances. For the non-strange baryons, $2_{2I,+}$ is followed by the unnatural parity clusters $4_{2I,-}$ and $6_{2I,-},$ which we view to reside in a different Fock space, $F_-,$ that is built on top of a pseudoscalar vacuum. Tobe specific, one finds all the seven $\Delta$-baryon resonances $3_{31}, P_{33}, P_{33}, D_{33}, D_{35}, F_{35},$ and $F_{37}$ from $4_{3,-}$ to be squeezed within the narrow mass region from 1900 MeV to 1950 MeV, while the $I = 1/2$ resonances paralleling them, of which only the $F_{17}$ state is still "missing" from the data, are located around 1700$^{20}_{50}$ MeV (see Fig. 1). Therefore, the $F_{17}$ resonance is the only non-strange state with a mass below 2000 MeV which is "missing" in the present RS classification scheme. Further paralleling baryons from the third nucleon and $\Delta$ clusters with $\sigma = 6,$ one finds in addition the four states $H_{3,11}, P_{33}, P_{33},$ and $D_{33}$ with masses above 2000 MeV to be "missing" for the completeness of the new classification scheme. The $H_{3,11}$ state is needed to parallel the well established $H_{3,11}$ baryon, while the $\Delta$-states $P_{33}, P_{33},$ and $D_{33}$ are required as partners to the less established $P_{13}(2100), P_{13}(1900),$ and $D_{13}(2080)$ nucleon resonances. For $\Lambda$ hyperons, sparse data prevent a conclusive analyses. Even so, see Fig. 2, the RS pattern is already apparent in the reported spectrum. The (approximate) degeneracy group of baryon spectra as already suggested in Ref. 13 and further justified here, is, therefore, found to be $SU(2) \otimes O(1, 3)_L,$ i.e., isospin$\otimes$space-time symmetry.

Within our scheme, the inter-cluster spacing of 200 to 300 MeV is larger by a factor of 3 to 6 as compared to the mass spread within the clusters. For example, the $2_{1,+}, 2_{3,+}, 4_{1,-},$ and $4_{3,-}$ clusters carry the maximal mass splitting of 50 to 70 MeV. The above considerations establish that

1. Observed excited light-flavor baryons preferably exist as multi-resonance clusters that are described in terms of the three RS multiplets $2_{2I,+}, 4_{2I,-},$ and $6_{2I,-}.$

2. The above RS clusters exhaust all the resonances observed in the $\pi N$ scattering channel [up to the $\Delta(1600)$ state]. They constitute an almost accomplished excitation mode in its own rights, as only 5 resonances are "missing" for the completeness of this structure.

3. As long as the internal parity of the clusters changes from natural for the first one, to unnatural, for the subsequent two, the question arises whether this change signals a phase transition for baryons. In Ref. 14 we further showed how presence or absence of the independent cluster excitation sequence $3_{2I,-}, 3_{2I,+},$ and $5_{2I,+},$ could probe the scale of the chiral phase transition for baryons. In this way, the "missing" state search program of the CLAS collaboration at JLAB [11] obtains an additional motivation, that is conceptually different from the $SU(6)_{SF} \otimes O(3)_L$ classification scheme.

4. Empirical linear mass relation for the multi-resonance clusters

The reported mass averages of the resonances from the RS multiplets with $k = 1, 3,$ and 5 are well described by means of the following simple empirical recursive relation

$$M_{\sigma'} - M_{\sigma} = m_1 \left[ \frac{1}{\sigma^2} - \frac{1}{(\sigma')^2} \right] + \frac{1}{2} m_2 \left[ \frac{(\sigma')^2 - 1}{2} - \frac{\sigma^2 - 1}{2} \right],$$

(11)
where, again, $\sigma = k + 1$. The two mass parameters take the values $m_1 = 600$ MeV, and $m_2 = 70$ MeV, respectively. The first term on the RHS in Eq. (12) is the typical difference between the energies of two single particle states of principal quantum numbers $\sigma$, and $\sigma'$, respectively, occupied by a particle with mass $m$ moving in a Coulomb-like potential of strength $\alpha_c$ with $m_1 = \alpha_c^2 m/2$. The term
\[
\frac{\sigma^2 - 1}{2} = 2 \frac{k}{2} \left( \frac{k}{2} + 1 \right),
\] (12)
in Eq. (12) is the generalization of the three-dimensional $j(j + 1)$ rule (with $j = k/2$) to four Euclidean dimensions and describes a generalized $O(4)$ rotational band (see Fig. 2).

The parameter $1/m_2 = 2.82$ fm corresponds to the moment of inertia $J = 2/5MR^2$ of some “effective” rigid-body resonance with mass $M = 1085$ MeV and a radius $R = 1.13$ fm. Note that while the splitting between the Coulomb-like states decreases with increasing $\sigma$, the difference between the energies of the rotational states increases linearly with $\sigma$ so that the net effect is a slightly increasing spacing between the baryon cluster levels.

5. QCD string with linear action

One of the frontier problems of contemporary theoretical physics is to understand how the properties of low-energy hadron physics relate to QCD—the gauge theory of strong interaction. Because of the non-Abelian character of the color gauge group $SU(3)_c$, the associated gauge bosons, the gluons, possess various self-interactions and bring highly non-linear terms into the QCD Lagrangian. This circumstance introduces serious difficulties in making direct QCD predictions, and obtaining solutions that describe the hadronic degrees of freedom. As a possible ansatz for such a solution, a string in four space-time dimensions was considered [12, 13]. A particular type of this solution is known under the name “gonihedric” string, or simply, QCD string. The gonihedric string has been defined as a model of random surfaces with an action which is proportional to the linear size of the surface [13]. In the continuum limit, its equation describes the propagation of a state that is a multi-fermion system as it spreads over a space contour. When the surface shrinks to a single world line, the action becomes proportional to the length of the path and the method resembles Feynman’s path integral for a point-like Dirac fermion. The propagation of a multi-fermion state that contains a spin-1/2 fermion in the point-particle limit can be described by means of a Dirac-type equation for an infinite dimensional wave function
\[
(i\gamma^\mu \partial^\mu - m)\Psi = 0.
\] (13)

The requirement of Lorentz covariance of Eq. (14) restricts $\Psi$ to be a Lorentz multiplet construct and imposes conditions onto the form of the $\Gamma_\mu$ matrices. To be specific, these matrices have to satisfy Majorana’s commutation relations
\[
[\Gamma_\mu, I_{\lambda\rho}] = g_{\mu\lambda}\Gamma_\rho - g_{\mu\rho}\Gamma_\lambda.
\] (14)

In Ref. 18 various solutions of Eq. (14) at the matrix element level were suggested. In the following we shall focus on one of them, that deserves our special attention as it applies directly to low-energy hadron physics. It is the so-called “generalized solution” of Ref. 18, that predicts a mass spectrum characterized by a $\sqrt{2\sigma - 1}$-fold degeneracy according to
\[
[M_{j_1j_2}^{(\sigma)}] = m^2 \left( \frac{1 - 1}{4\sigma^2} \right) [j_{\max}^{(\sigma)}]^2, \quad \sigma = 2, 4, 6, \ldots
\] (15)
with $j^{(\sigma)}$ ranging from $j_{\min}^{(\sigma)} = 1/2$ to $j_{\max}^{(\sigma)} = \sigma - 1/2$. Though a similar degeneracy is known from the hydrogen atom, where $\sigma$ acts as the principal quantum number of the Coulomb problem, the quadratic mass Eq. (16) should not be confused with the Balmer-series, where the energy enters linearly.

The difference between the squared masses of the spin-clusters following from Eq. (16) can be cast into the form
\[
[M_{j_1j_2}^{(\sigma+1)}] - [M_{j_1j_2}^{(\sigma)}] = 4m^2 [j_{\max}^{(\sigma)} + 1]
\] (16)
+ $\frac{m^2}{4} [j_{\max}^{(\sigma)}]^2 \left( \frac{1}{\sigma^2} - \frac{1}{\sigma^2 + 2} \right)$. Now the question arises, how the theory of the gonihedric QCD string relates to hadron spectroscopy.

6. Signature for QCD string in baryon excitations

Comparison of Eqs. (12) and (11)–(17) reveals a striking similarity between the spectrum of the gonihedric QCD string and the reported spectra of the non-strange baryons. This similarity concerns

i) The $(\sqrt{2\sigma - 1})$-fold degeneracy of the states.

ii) The occurrence of even values of $\sigma$.

iii) The $-1/\sigma^2$ pattern in the mass distribution.

Within this context, one is rather tempted to conjecture that the observed hydrogen-like degeneracy in baryon spectra may take its roots in the gonihedric QCD string.

This statement is further supported by the realistic mass splitting of the baryon spin-clusters concluded from the QCD string theory on the grounds of Eq. (17). To fix the unknown parameter $m^2$ there, one may fit the difference between the squared masses of the 2nd and 1st clusters to data. From the empirical observation
\[
(M_{j_1}^{(6)})^2 - (M_{j_1}^{(4)})^2 = 0.61 \text{ GeV}^2,
\]
one finds $m^2 = 0.061 \text{ GeV}^2$. In using this value to calculate $(M_{j_1}^{(6)})^2 - (M_{j_1}^{(4)})^2$ we find
\[
(M_{j_1}^{(6)})^2 - (M_{j_1}^{(4)})^2 = 1.1 \text{ GeV}^2.
\]
This number is close to the empirical one, where one encounters
\[
(M_{j_1}^{(6)})^2 - (M_{j_1}^{(4)})^2 \approx 1.56 \text{ GeV}^2.
\]
Moreover, also the linear mass difference \( M^\text{string}_{j+2} - M^\text{string}_{j} \) can be calculated and compared to data. This is done in Table I which also contains the predictions following from Eq. (12). The undertaken comparison shows that the QCD string model leads to an approximate equidistant spacing between the spin-clusters. At the first glance, on the experimental side, the equidistant mass spacing does not seem as well pronounced as on the theoretical side. Note, however, that most of the resonances participating the \( \sigma = 6 \) spin-cluster are not well established and the experimental estimate of the averaged mass here may not be realistic. A reliable knowledge of the precise locations of the resonances in this mass region is urgently desirable and a challenge to experimental resonance programs. Finally, the absolute values of the cluster masses following from the QCD string theory can be found also in Table I, where one finds them strongly underestimated though with increasing mass, the effect noticeably decreases.

7. Summary and Outlook

We conclude that the lower-spin components of the Rarita-Schwinger fields are forced upon by the properties of the Lorentz boost and should not be projected out. These fields are realized in the spectra of the light-flavor baryons, where the landscape of the excitations is well structured along the RS classification scheme, rather than populated by randomly distributed resonances. The (approximate) degeneracy symmetry turned out to be \( SU(2)_f \otimes O(1,3)_s \). Within this context, the \( 2_{21,++} \), \( 4_{21,-} \), and \( 6_{21,-} \) RS clusters observed so far in the \( \pi N \) scattering channel by the LAMPF at LANL, constitute an almost accomplished excitation mode with only five states “missing”. The dynamical origin for the RS clustering is still lacking a unique explanation and it remains a challenge for future research. The factorization of isospin from the space-time symmetry in \( SU(2)_f \otimes O(1,3)_s \) is strongly supported by QCD, where the isodoublet light quarks and the isosinglet heavy-flavor quarks are the established isospin degrees of freedom. On the other hand, any QCD solution has necessarily to be a Lorentz covariant object. In Ref. 14 the bosonic parts of the Lorentz clusters, such as \((k/2, k/2)\) from the RS field in Eq. (17), were considered as independent fundamental bosonic degree of freedom of baryon structure, and the term “hyperquark” was coined for them. There, the clustering was modeled after a \( O(4) \) invariant quark-hyperquark correlation. From a slightly different QCD perspective, the clusters can also be viewed as string solutions associated with a linear action [14]. On the whole, our view is that a structured baryon spectrum that shares common flavor and relativistic symmetries with QCD is more likely to be linked via an appropriate effective theory to first principles of strong-interaction dynamics than a spectrum with states distributed at random.

Acknowledgments

Work supported by CONACyT Mexico.