Study of quasimolecular states in $^{24}\text{Mg}$ via $d-\alpha$ angular correlation analysis in the $^{12}\text{C}(^{14}\text{N}, d)^{24}\text{Mg} * (\alpha)^{20}\text{Ne}$ reaction

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The methods of calculating the angular correlation functions in the reactions induced by light and semiheavy ions of energy up to 10 MeV/nucleon have been discussed. The differential cross sections and $\alpha$-$d$ angular correlation functions in the reaction $^{12}\text{C}(^{14}\text{N}, d)^{24}\text{Mg} * (\alpha)^{20}\text{Ne}$ involving high-lying states in $^{24}\text{Mg}$ at $^{14}\text{N}$ beam energies $E_{\text{lab}}$ = 29 - 45 MeV in the model of direct $^{12}\text{C}$ transfer the framework of the distort-wave method with finite interaction range and the statistical compound-nucleus model have been analyzed. The reduced width amplitudes for the high-excited states in $^{24}\text{Mg}$ with $^{12}\text{C}\otimes^{12}\text{C}$ quasimolecular structure are extracted and the influence of relative motion of $^{12}\text{C}+^{12}\text{C}^*$ are studied.

Keywords: Angular correlation functions; quasimolecular structure

Métodos teóricos de estudios de correlación angular de partícula-partícula en reacciones nucleares iniciadas por los iones ligeros y semipeusados con energía hasta 10 MeV por nucleón son discutidos. Las secciones transversales diferenciales y las funciones de correlación angular de alfa-deuterón en la reacción $^{12}\text{C}(^{14}\text{N}, d)^{24}\text{Mg} * (\alpha)^{20}\text{Ne}$ involucran los estados de la excitación alta en el modelo de traspaso directo de $^{12}\text{C}$ basado en el método de ondas distorcionadas con interacción de radios finito y en el modelo estadístico de núcleo compuesto son analizadas. Las amplitudes de anchuras reducidas para los estados de alta excitación en $^{24}\text{Mg}$ con estructura cuasi moleculares $^{12}\text{C}\otimes^{12}\text{C}$ están extraídos y la influencia de movimiento relativo de $^{12}\text{C}+^{12}\text{C}^*$ es estudiada.

Descriptores: Funciones de correlacion angular; estructuras cuasimoleculares

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1. Introduction

Experimental and theoretical studies of nuclear molecular states or nuclear quasimolecules in the heavy ion reactions have been conducted since the beginning 1960's, when they were experimentally discovered [1] in the first studies of $^{12}\text{C}+^{12}\text{C}$ interactions. These unexpectedly narrow and well-separated resonances in the interaction of heavy ions were found well correlated in different channels and had a width much greater than the compound-nucleus (CN) one for given excitation energy of the CN system around 15 - 50 MeV. During the last decade other light systems which feature even $N$, even $Z$, $N = Z$ nuclei have been intensively studied [2, 3] primarily in elastic and inelastic scattering, and in reactions leading to $\alpha$-particle transfer channels (the modern state-of-art one can find in reviews [4]). The light-heavy-ion grazing collisions can form states of very high angular momentum that are associated with quasimolecular configurations of the two rotating nuclei [5, 6].

The study of particle-particle angular correlations historically began with investigations of the quasielastic knockout reactions $A(p, 2p)B^*$ on light nuclei [7] and the clustering phenomena in light nuclei by the $(\alpha, ax)$ reaction [8, 9]. Important quantitative spectroscopic information and some new theoretical results were obtained recently in Ref. 10, where the quasielastic knockout of $\alpha$-clusters by intermediate energy protons and ultrarelativistic electrons was studied.

As early as in 1984 [11] experimental data of the measurement of the $d-\alpha$ angular correlation function (ACF) in the $^{12}\text{C}(^{14}\text{N}, d)^{24}\text{Mg}(\alpha)^{20}\text{Ne}$ reaction were obtained which have attracted considerable interest to the study of quasimolecular states of nuclei via particle-particle angular correlations. It was found that for the excited state 13.45 MeV ($J^* = 6^+$) in $^{24}\text{Mg}$ there are oscillations in ACF at forward deuteron emission angles, whose shape is well described by the square of the sixth-order Legendre polynomial and could not be explained in the CN model. The authors interpreted this fact as the proof of direct transfer of 12 nucleons and suggested the presence of quasimolecular $^{12}\text{C}\otimes^{12}\text{C}$ configurations in this state of $^{24}\text{Mg}$. This result was confirmed ones again [12] in 1994.

The analysis of the deuteron angular distributions as well as $d-\alpha$-angular correlation functions [13] performed in the framework of the compound nucleus Hauser-Feshbach formalism confirmed the importance of CN contribution, but, at the same time, it demonstrated the smooth behavior of $d-\alpha$-angular correlation functions for 13.45 MeV, 6$^+$ state in $^{24}\text{Mg}$ for all forward deuteron emission angles. It was found [14, 15] that calculations of the differential cross sections performed in the exact finite-range distorted-wave Bohr approximation (EFR DWBA) for the direct carbon-transfer mechanism in some cases reproduce the experimental data more successfully.
Nevertheless, a number of principal questions still remained: Why was a polynomial structure of the ACFs observed only for one selected state in $^{24}\text{Mg}$ and not observed for other states with high spins near this state? Is it only exception rather than the rule? What is the origin of energy dependence of the ACFs for this state? What is a reaction mechanism, which makes the main contribution to the reaction cross sections and ACFs?

The present study focuses on the theoretical analysis of the quasimolecular states in $^{24}\text{Mg}$ and their influence on the reaction characteristics. We calculate the differential cross sections as well as $d-\alpha$-ACFs in $^{12}\text{C}(^{14}\text{N}, d)'^{24}\text{Mg}$ to obtain the full description of experimental data known today and to answer the questions mentioned above. We extract quantitative values of the reduced width amplitudes for the $^{12}\text{C} \otimes ^{12}\text{C}$ configurations and we investigate how does the relative motion of two $^{12}\text{C}$ clusters effect on the reaction characteristics.

2. Methods of calculating the particle-particle angular correlation functions based on the spin density matrix and its spin tensors

We shall analyze the differential cross sections and the angular correlation functions in reactions induced by light and semiheavy ions using the general theory of angular correlations in terms of the spin density matrix (one can find its detailed description in the classic papers of Biedenharm and Rose [16], and Goldfarb [17]). For a binary nuclear reaction $A(a, b)B^*$ in a case of unpolarized incident particle and target the spin density matrix $\rho_{I_f}(M, M')$ of the nucleus $B^*$ in the state with spin $I_f$ and projection $M_f$ may be written as

$$\rho_{I_f}(M_f, M'_f; \Omega_b) = \frac{1}{(2I_a + 1)(2I_A + 1)} \sum_{M_a, M_A, M_b, M'_b} T_{I_f, M_a, M_A, M_b, M'_b} (\Omega_b) T_{I_f, M_a, M_A, M'_b} (\Omega_b),$$

where $T_{I_f}(\Omega_b)$ is the transition amplitude of reaction.

For practical applications, it is convenient to introduce the spin-tensors $\rho_{I_f}(I_f)$ of the density matrix that can be defined in the standard manner [17] with $0 < k < 2I_f$ and $\kappa = -k, \ldots, k$.

We normalize the density matrix (1) to differential cross section and monopole spin-tensor

$$\text{Tr} \rho_{I_f}(M_f, M'_f; \Omega_b) = \frac{d\sigma}{d\Omega} = \rho_{00}.$$ (2)

Let us consider a two stage nuclear reaction, the first stage of which, $a(I_a) + A(I_A) \rightarrow B^*(I_f) + b(I_b)$, is the binary reaction where the nucleus $B^*$ is produced in the excited state. The second stage, $B^*(I_f) \rightarrow C(I_0) + c(I_2)$, is the decay of the nucleus $B^*$ to the nucleus $C^*$ generally in the excited state with spin $I_0$ and the second emission, for example, particle $c$ (the total nucleus spins are shown in brackets). According to the general formalism [16, 17], the angular correlation function $W(\Omega_b, \Omega_c)$ is the probability for simultaneous detection of particle $b$ in the direction $\Omega_b$ and emitted particle $c$ in the direction $\Omega_c$. It is defined in terms of the spin-tensors $\rho_{I_f}(I_f; \Omega_b)$ of the density matrix and the tensors $\varepsilon_{I_f}(I_f; \Omega_c)$ of the efficiency matrix as

$$W(I_f; \Omega_b, \Omega_c) = \sum_{k\alpha} \rho_{I_f}(I_f; \Omega_b) \varepsilon_{I_f}(I_f; \Omega_c).$$ (3)

In a special case of zero spin $I_r$ of emitted particle (for example, $\alpha$-particle decay of excited nuclear state) and zero spin $I_0$ of the residual nucleus $C$ the efficiency tensor $\varepsilon_{I_f}(I_f)$ is of the most simple form

$$\varepsilon_{I_f}(I_f; \Omega_c) = (-1)^{I_f}(2I_f + 1)[4\pi(2k + 1)]^{-1/2} \langle I_f O I_f 0 \mid k 0 \rangle Y^{*}_{k\alpha}(\Omega_c).$$ (4)

The expressions (1) and (4) allow one to write the ACF (3) as [18, 19]

$$W(I_f; \Omega_b, \Omega_c) = \frac{(2I_f + 1)^{1/2}}{(2I_a + 1)(2I_A + 1)} \sum_{M_f M'_f} (-1)^{I_f} Y^*_{I_f M_f} (\Omega_c) Y_{I_f M_f} (\Omega_c) \sum_{M_a M_A M_b} \left| T_{I_f, M_a, M_A, M_b, M'_b} (\Omega_b) \right|^2.$$ (5)

Let us consider the methods of the spin-tensor of density matrix and the ACF calculation in the frameworks of two complementary nuclear reaction models: EFR DWBA and the Hauser-Feshbach (HF) formalism of statistical CN model.
In the EFR DWBA the transition amplitude $T_{if}$ of the reaction $A(a, b)B$ may be written as [20, 21]

$$T_{if}(\Omega_b) = \frac{1}{\sqrt{E_i E_f}} \frac{K_b}{K_a} \times \sum_{I_1 M_{12} I_2 M_{12} \Omega_1} (-1)^{I_1 + I_2} \beta_{I M_1 A_{I_1, I_2} I_1 I_2 I_X E_X} (\Omega_b).$$

(6)

The essence of the reaction mechanism is contained in the kinematical amplitudes $\beta_{I M_1 A_{I_1, I_2} I_1 I_2 I_X E_X} (\Omega_b)$, which are the overlap integrals of the incoming and outgoing distorted waves, the wave functions of the relative motion, and the interaction potential $V$. For the one-step direct terms ("break-up" of the projectile) $V$ represents the interaction potential for the direct stripping and heavy knock-on mechanisms. The structure factors

$$\Theta_{A_1 A_2 I_1 I_2 I_X} = (-1)^{I_1} \frac{u(i_1 I_1 I_2 A_2 : I_X)}{\sqrt{2 I_X + 1}} \Theta_{A_1 I_1 I_X}^{B X + A} \Theta_{A_2 I_2 I_X}^{a X + b},$$

(7)

may be expressed [21] in terms of the reduced width amplitudes (RWAs) $\Theta_{A_1 I_1 I_X}^{B X + A}$ and $\Theta_{A_2 I_2 I_X}^{a X + b}$, which describe the probability of formation of the cluster configurations $X + A$ and $X + b$ containing the intermediate nucleus $X$ in the nuclei $B$ and $a$, respectively.

Here we take into account the total angular-momentum coupling schemes

$$I = \Lambda_1 + \Lambda_2 = I_1 + I_2,$$

$$I_a = I_X + \Lambda_2 + I_b = I_X + J_2 = I_2 + I_b,$$

$$I_f = I_X + \Lambda_1 + I_A = I_X + J_1 = I_1 + I_A$$

(8)

and use the coupling scheme for orbital angular momenta:

$$L_a = L_X + \Lambda_2 + L_b = L_X + L_2,$$

$$L_f = L_X + \Lambda_1 + L_A = L_X + L_1 = L_X$$

(9)

where $I$ is the transferred angular momentum, $\Lambda_1$ and $\Lambda_2$ are the orbital angular momenta of relative motion of the clusters $A + X$ and $b + X$ in the nuclei $B$ and $a$, respectively, and $I_1$ and $I_2$ are the total transferred angular momenta.

We use the conventional definition of the cluster spectroscopic amplitude of reduced width for decay probability of the nuclei $a$ and $B$ via the channels $X + b$ and $X + A$, respectively, in terms of the fractional parentage coefficients (FPC) in the translationally invariant shell model technique (see, for example, Refs. 22–26). Without changing the formal structure of the EFR DWBA formalism let us introduce RWAs in the vertexes $B \rightarrow X + A$ and $a \rightarrow X + b$, which describe a possibility of formation of a massive cluster $X$ possessing its own excited states as

$$\Theta_{A_2 I_2 I_X}^{a X + b} = \sum_{j_2} u(i_2 A_2 I_2 I_X : j_2) \zeta_{A_2 I_2 I_X}^{a X + b},$$

(10)

where

$$\zeta_{A_2 I_2 I_X}^{a X + b} = \left( \frac{N_a}{N_X} \right)^{1/2} \sum_{[f_a]} \sum_{L_S \Lambda_S \Lambda_B} a^{L_f T_a}_{[f_a] L_a S_a} a^{L_X T_X}_{[f_x] L_x S_x} a^{L_B T_B}_{[f_b] L_b S_b} \times (-1)^{L_X + L_a + L_b} u(l_2 A_2 L_2 S_2 B) K(\Lambda_2 L_2 : L_2) \int_{L_X M_{T_X} T_X} \int_{L_B M_{T_B} T_B} \int_{L_a M_{T_a} T_a} \left( L_X \ S_X \ I_X \right) \times \left( L_2 \ S_2 \ J_2 \right) \times \left( L_a \ S_a \ I_a \right)$$

(11)

In (11) $a^{L_f T_a}_{[f_a] L_a S_a}$ are the intermediate coupling coefficients, $K(\Lambda_2 L_2 : L_2)$ are the general Talmi coefficients that extract the intrinsic wave function of nucleus $b$ in the given state with orbital momentum $\Lambda_2$ from the wave function of $b$ nucleons with angular momentum $L_2$ [22, 23]. We wrote the expressions (10) and (11) for vertex $a \rightarrow X + b$ (we refer to subscript 2 for vertex $B \rightarrow X + A$ and subscript 1 for vertex $B \rightarrow X + A$) taking in the mind that for vertex $B \rightarrow X + A$ the expressions are the same with the substitution $2 \leftrightarrow 1, a \leftrightarrow B, b \leftrightarrow A$.

Finally, substituting (6) into (5) we write the expression for the angular correlation function in the EFR DWBA for the case when the spins of emitted particle $c$ and the residual nucleus $C$ are zeros

$$W(I_f; \Omega_b, \Omega_c) = \frac{(2 I_f + 1)^{1/2}}{(2 I_A + 1)^{1/2}} \left( \sum_{l_2 m_2 M_A} \sum_{M_f} M_{i_2 M_2 M_A} M_f (\Omega_b) \times \gamma_{I_f M_f} (\Omega_c) \right)^2,$$

(12)

where
\[ M_{I_2M_2M_A}^{I_fM_f}(\Omega_b) = \sum_{l_1M_1} (2I_1 + 1)^{1/2} \left\langle I_A M_A I_1 M_1 | I_f M_f \right\rangle \sum_{lm_1} i^l \left\langle I_2 - M_2 I_1 M_1 | lm_1 \right\rangle \times \sum_{\Lambda_1 \Lambda_2 I_2 E_X} (-1)^{I_1 + \Lambda_1 + \Lambda_2} \Theta_{\Lambda_1 \Lambda_2 \Lambda_2 \Lambda_2 I_2 I_2 E_X} \beta_{lm_1 \Lambda_1 \Lambda_2} \theta_{\Omega_b}(\Lambda_1, \Lambda_2, I_2, E_X) (\Omega_b). \] (13)

Let us now consider the method of the spin-tensor of the density matrix and ACF calculation in the modified statistical CN model [27], which allows to investigate interaction between nuclei with arbitrary spins including the spin-orbit interaction. Writing the transition amplitude \( T_{I_f} \) of the reaction \( A(a, b)B \) in the usual manner
\[ T_{I_f} = \langle \Psi_f | \Psi_C \rangle \langle \Psi_C | \Psi_i \rangle, \] (14)
we use the reduced width technique and express the wave functions \( \Psi_i, \Psi_f, \) and \( \Psi_C \) of initial, final, and compound quasistationary state \( \lambda \) with spin \( I_C \), level energy \( E_\lambda \) and total width \( \Gamma_\lambda \) in the form
\[ \Psi_i = \Psi_A \Psi_a \chi^{(+)}(K_a r_a), \]
\[ \Psi_f = \Psi_B \Psi_b \chi^{(-)}(K_b r_b), \]
\[ \Psi_C(r, E_\lambda) = \frac{1}{(E_\lambda - E) + \frac{1}{2} \Gamma_\lambda} \Psi_C(r), \]
\[ \Psi_C(r) = \sum_{I_1M_1I_2M_2I_CMC} \left( I_1 M_1, I_2 M_2 | I_1 M_1 \right) \langle I_1 M_1, I_2 M_2 | I_2 M_2 \rangle \beta_{I_1 I_2}^{C(\lambda)A+a} \Psi_A \Psi_a \theta_{I_1 I_2}^{C(\lambda)A+a}(r_a) \]
\[ = \sum_{I_2M_2I_CMC} \left( I_2 M_2, I_2 M_2 | I_2 M_2 \right) \langle I_2 M_2, I_2 M_2 | I_2 M_2 \rangle \beta_{I_2 I_2}^{C(\lambda)B+b} \Psi_B \Psi_b \theta_{I_2 I_2}^{C(\lambda)B+b}(r_b), \]
where \( \chi^{(+)}(K_a r_a) \) and \( \chi^{(-)}(K_b r_b) \) are the distorted waves in the entrance and exit reaction channels, \( \Psi_A, \Psi_B, \Psi_A, \) and \( \Psi_a \) are the internal wave functions of correspondent nuclei, \( \Psi_a(r_a) \) and \( \Psi_b(r_b) \) are the wave functions of relative motion, \( \beta_{I_1 I_1}^{C(\lambda)A+a} \) and \( \beta_{I_2 I_2}^{C(\lambda)B+b} \) are the reduced width amplitudes of the decay of the CN quasistationary state \( \lambda \) via the channels \( C \rightarrow A + a \) and \( C \rightarrow B + b \).

Here we define the following angular-momentum coupling scheme
\[ I_1 = I_a + I_A, \quad I_2 = I_b + I_f, \]
\[ I_C = I_1 + \bar{I}_a = I_2 + \bar{I}_b, \] (16)
where \( \bar{I}_a \) and \( \bar{I}_b \) are the orbital angular momenta of the particle relative motion in the entrance and exit channels, and \( I_1 \) and \( I_2 \) are the total spins of the entrance and exit channels.

Inserting the partial-wave expansion of the distorted waves, we can obtain the transition amplitude and the spin-tensors \( \rho_{kn}(I_f) \) of the density matrix in the case of forming only one CN resonance. In the region of the quasicontinuous spectrum, where resonances overlap strongly, it is necessary to average the contribution of each resonance over the energy and to sum all resonances in the averaging interval. As a result, we have the transmission coefficients \( T_{I_1}^{I_1} \), which are related to the average partial width amplitudes
\[ \gamma_{I_1 I_1}^{C(\lambda)A+a} = \beta_{I_1 I_1}^{C(\lambda)A+a} \int d^2\chi_{I_1}(k_a r_a) \Psi_a(r_a) Y_{I_1 M_1}(\theta_{r_a}, \varphi_{r_a}) r_a dr_a \] (17)
and the average spacing \( D_{I_C} \) between levels as
\[ T_{I_1}^{I_1} = 2\pi \frac{\gamma_{I_1 I_1}^{C(\lambda)A+a}^2}{D_{I_C}}. \] (18)

Finally, the spin-tensors of the density matrix in the statistical limit of the CN model take the form

$$\rho_{kn}(I_f) = \frac{(2I_f + 1)^{1/2}}{2K^2_R(2I_a + 1)(2I_a + 1)} \sum_{l_1 l_2 l_1' l_2' m_1 m_1'} \left\{ \left( -1 \right)^{2I_C + I_f + I_b} (2I_C + 1)^2 [(2I_1 + 1)(2I_2 + 1)]^{1/2} (-1)^{m_1 + I_1 + I_2 + I_2'} \right. \\
\times \frac{T_{l_1}^{I_C} T_{l_1'}^{I_C} P_{l_1 m_1} (\theta_1) P_{l_1' m_1'} (\theta_1) [(2l_1 + 1)(2l_2 + 1)(2I_a + 1)(2I_b + 1)]^{1/2} \langle I_m | I_{l_1}^* - m_1 | k \rangle \langle I_{l_1} | I_{l_1'} - m_1 | k \rangle \langle I_1 | I_{l_2}^* - m_1 | k \rangle \langle I_2 | I_{l_2'} - m_1 | k \rangle \\
\left. \times \langle I_1 | O_{l_1} | m_1 | l_1' m_1' \rangle \times w(I_f I_2 I_f' I_2' : I_b k) w(I_1 l_1' l_2' : I_1 k) w(I_1 I_2 l_2 b : I_1 l_2 b) w(I_1 l_2 l_2 b : I_1 l_2 b) \right\} $$

The total decay width \( G(I_C) \) includes all the energetically allowed decay channels of the CN and the transmission coefficients \( \Gamma_{l l}^{I_C} \) can be determined through the elastic scattering matrices of the entrance and exit channels, and may be calculated, for example, in the optical model of elastic scattering.

Finally, we are able to calculate ACFs by using the spin-tensors of the density matrix (19) and the appropriate expression for the efficiency tensors \( \varepsilon_{k n} (I_f) \).

3. \(^{12}\text{C}(^{14}\text{N},d)^{24}\text{Mg}\ast(\alpha)^{20}\text{Ne}\) reaction calculations and quasimolecular \(^{12}\text{C} \otimes ^{12}\text{C}\ast\) states in \(^{24}\text{Mg}\)

3.1. Differential cross sections

The first results of studies of nuclear reactions induced by nitrogen ions at incident energies up to 100 MeV on \(^{1p-}, ^{2s-}\) \(^{1d}\)-shell nuclei performed in the early 1970s [28, 29] suggested that these reactions could be treated as ones involving the transfer of large group of nucleons. Nevertheless, after further analysis of the spectra, the excitation functions, and, in particular, the observations of resonances-like structure in the excitation functions from elastic scattering and from reactions, it was concluded that the mechanism of CN formation plays the dominant role in these reactions [30–32]. However, after analyzing the differential cross sections for these reactions, in Ref. 32 and later in Ref. 14, it was noticed that the angular distributions have a clear oscillated structure. This indicates that the statistical CN model describes the smooth, but substantial background, on which oscillations or irregularities associated with other reaction mechanisms are superimposed, the contribution of which can be comparable to that of CN formation.

We performed calculations of the differential cross sections at \(^{14}\text{N}\) bombarding energies of 29 and 35 MeV for the different excited states in \(^{24}\text{Mg}\), using the statistical CN model and direct massive cluster transfer model in the framework of the formalism presented in Sec. 2.

The computer code CNCOR [33] based on the modified HF formalism of the CN model was used to obtain a corresponding contribution of the statistical CN mechanism to the reaction cross section. The CNCOR code allows one to calculate all the components of the spin-tensors of the density matrix for any state of the final nucleus \(^B\ast\), the differential cross-section of the reaction, particle-particle and particle-\(\gamma\)-quantum angular correlation functions. The detailed discussion of parameter choice in the CN calculation was presented in our previous work [13].

Let us now consider the deuteron angular distributions in the \(^{12}\text{C}(^{14}\text{N},d)^{24}\text{Mg}\ast\) reaction in the direct massive cluster transfer model and let us start with the RWA (10) and (11) calculation for the \(^{14}\text{N} \rightarrow ^{12}\text{C} + d\) vertex. We use the shell model wave functions of nuclei with the intermediate coupling coefficients \(a_{f_1 f_2 l_1 l_2 S_n}\), obtained in Ref. 35 by the diagonalization of the Hamiltonian with nucleon-nucleon interaction. The calculated RWAs \(\Theta_{A_{A_{l x} l x}}^{14\text{N} \rightarrow ^{12}\text{C} + d}\) for the transfer of massive cluster \(^{12}\text{C}\) possessing two intermediate states [g.s. (0\(^\ast\)) and 4.33 MeV (2\(^\ast\))] are represented in Table I. The RWAs \(\Theta_{A_{A_{l x} l x}}^{24\text{Mg} \rightarrow ^{12}\text{C} + ^{12}\text{C} \ast}\) were considered as adjusted parameters, which then were used for the structure factor \(\Theta_{A_{A_{l x} l x}}^{14\text{N} \rightarrow ^{12}\text{C} + d}\) (7) calculation. Reaction cross section calculations for the direct transfer mechanism were carried out in the EFRW DBA using the OLYMP-5 computer code [33], which we modified to perform summation over spin \(I_x\) of the intermediate nucleus and total angular momenta \(I_1\) and \(I_2\). The optical-potential and interaction-potential parameters are shown in Table II.

Figure 1 represents the comparison of the results of CN and EFRW DBA calculations with the experimental differential cross sections for the \(^{14}\text{C}(^{14}\text{N},d)^{24}\text{Mg}\ast\) reaction involving different excited states in \(^{24}\text{Mg}\) at beam energy 29 MeV. The theoretical curves are the incoherent sum of the CN model and direct massive transfer calculations. We took into
account a stripping mechanism of direct transfer of the $^{12}\text{C}^*$ cluster in the g.s. and in the first excited 4.44 MeV (2$^+$) state.

Our analysis showed that a direct transfer of the $^{12}\text{C}$ cluster provides approximately 80–85% of the cross section at forward angles for the g.s. (0$^+$), 70% for the 1.37 MeV (2$^+$) level, 50–70% for the 4.12 MeV (4$^+$) and 8.11 MeV (6$^+$) level, 55–60% for the 13.45 MeV (8$^+$) level at 35 and 29 MeV. The CN mechanism is a leading one at big angles. For the 13.45 MeV (6$^+$) state a direct transfer mechanism dominates at all angles, because the CN cross section is small due to the small $I_{\alpha}^{\text{max}}$ and $I_{\beta}^{\text{max}}$ values.

We could estimate $\Theta_{A_1'I_1I_1'}^{24\text{Mg} \rightarrow 12\text{C}+12\text{C}}$ values by comparison of the experimental and calculated differential cross sections and ASFs. The estimated values of the RWDs $\Theta_{A_1'I_1I_1'}^{24\text{Mg} \rightarrow 12\text{C}+12\text{C}}$ are shown in Table III. Naturally, they depend on a choice of the model parameters and, in particular, on the radius of $^{12}\text{C}+^{12}\text{C}$ interaction variation. For example, a sufficient increase of the $^{12}\text{C}+^{12}\text{C}$ interaction radius (following to Ref. 36) leads to a decrease of the $\Theta_{A_1'I_1I_1'}^{24\text{Mg} \rightarrow 12\text{C}+12\text{C}}$ values by factor 3–5. However, these RWA values are one–two order larger than the analytical ones calculated in Ref. 36. This fact gives rise a question of whether the conventional technique of cluster spectroscopic amplitude calculation is applicable to describe multinucleon quasimolecular spectroscopic amplitudes. It should also be noted, that the angular distributions, their forms and absolute values, depend on the relative contribution of the $^{12}\text{C} \otimes^{12}\text{C}$ configurations with different $\Theta_{A_1'I_1I_1'}^{24\text{Mg} \rightarrow 12\text{C}+12\text{C}}$.

The angular distributions for the low-lying levels [g.s., 1.37 and 4.24 MeV (2$^+$)] have the most pronounced structure. In particular, the best agreement with the experimental data for the states 1.37 MeV and 4.24 MeV (2$^+$) was obtained by proposing that the components with $A_1 = I_F = 2$, $I_X = 0, 2$ introduce the main contribution, whereas the $A_1 = 0, I_X = 2$ component is negligible. Due to the contribution of the major component with $A_1 = 4$ ($I_X = 2$) a position of the first angular distribution maximum becomes more correct.
clear polynomial angular dependency. A form of the ACF angular distribution is determined by the relative populations of the \( M_f \) projections. At zero-angle deuteron emission the angular momentum projections \( M_f = M_z = 0, \pm 1, \pm 2 \) only are possible. Meanwhile, the ACF calculations performed with different \( \Theta_{A_1 I_x} \) values have demonstrated strong dependence on the \( M_f \)-projection populations on the relative contribution of different \( [^{12}C \otimes ^{12}C^*]_{A_1 I_x} \) configurations, which are characterized by the angular momentum coupling scheme (8). Our present calculations show that the agreement with the experimental data at \( ^{14}N \) bombarding energy of 33 MeV can be reached only if a coherent summation of two \( [^{12}C \otimes ^{12}C^*]_{A_1 I_x} \) configurations with \( A_1 = 4, 6 \) and \( I_x = 2 \) (i.e., \( ^{12}C \)-cluster is transferred in the first excited state) is carried. The \( ^{12}C \) (g.s.) transfer with \( I_x = 0 \) should be limited because the only \( A_1 = I_x \) is allowed in this case. If the beam energy increases up to 42 MeV, the experimental ACF struc-

The angular distributions for the high-lying levels \( (E^* \geq 8 \text{ MeV}) \) do not have any distinguish structure, and so to extract the RWAs we analyzed them simultaneously with the correspondent angular correlation functions.

### 3.2. \( d-\alpha \) Angular correlation functions

The direct \( ^{12}C \) transfer calculations for ACF in the framework of the formalism developed above were carried out for the excited states in \( ^{24}Mg \) beginning from the 4.12 MeV (4\(^+\)) state. Making a comparison of the calculated ACFs and the experimental ones at different incident energies we were able to restore the cluster structure of the given state. In Fig. 2 we show the calculated and experimental (11, 12) ACFs for the 13.45 MeV (6\(^+\)) state in \( ^{24}Mg \) at \( ^{14}N \) bombarding energies of 33 (a) and 42 MeV (b) at deuteron angle \( \theta_{lab} = 0^\circ \). One can see that the experimental ACFs at \( E_{lab} = 33 \) manifest

<table>
<thead>
<tr>
<th>( I_x ) (g.s.)</th>
<th>( A_1 )</th>
<th>( I_1 )</th>
<th>( I_x )</th>
<th>( 29 \text{ MeV} )</th>
<th>( 35 \text{ MeV} )</th>
<th>( 42 \text{ MeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(^+), g.s.</td>
<td>0</td>
<td>0</td>
<td>0.84</td>
<td>0.84</td>
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<tr>
<td>2(^+), 1.37</td>
<td>2</td>
<td>2</td>
<td>0.66</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2(^+), 4.24</td>
<td>2</td>
<td>2</td>
<td>0.66</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and</td>
<td>2</td>
<td>2</td>
<td>0.66</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2(^+), 4.12</td>
<td>4</td>
<td>4</td>
<td>0.35</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and</td>
<td>4</td>
<td>4</td>
<td>0.35</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6(^+), 8.11</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and</td>
<td>6</td>
<td>4</td>
<td>0.447</td>
<td>0.447</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6(^+), 8.44</td>
<td>6</td>
<td>6</td>
<td>0.447</td>
<td>0.447</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and</td>
<td>6</td>
<td>8</td>
<td>0.447</td>
<td>0.447</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6(^+), 13.45</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td></td>
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<tr>
<td>and</td>
<td>6</td>
<td>4</td>
<td>0.114</td>
<td>0.28</td>
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<tr>
<td>8(^+), 13.21</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td></td>
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<td>and</td>
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<td>6</td>
<td>0.35</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8(^+), 14.15</td>
<td>8</td>
<td>8</td>
<td>0.35</td>
<td>0.35</td>
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</tr>
<tr>
<td>8(^+), 14.15</td>
<td>8</td>
<td>10</td>
<td>0</td>
<td>0.35</td>
<td></td>
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</tr>
</tbody>
</table>

\( ^{24}Mg \) reduced widths \( \Theta_{A_1 I_x} \).
structure becomes smoother and does not show clear oscillations. To explain this behavior of experimental ACF, we proposed that at the high incident energy the contribution of the higher relative motion angular momenta \( \Lambda_1 = 6, 8 \) becomes more important, and it causes a smoothing of the ACFs. Assuming that the physical motivation for the unexpectedly big RWA values is the orbital relative motion of two \(^{12}\text{C}\) nuclei, it is not a surprise that ACFs exhibit the pronounce energy dependence, which is shown in Fig. 3. Moreover, it should be stressed, that it is the energy dependence of the ACFs, which indicates that the orbital motion of two \(^{12}\text{C}\)-clusters is responsible for the formation of this quasimolecular state. Indeed, for the reaction involved the given excited state the transferred momentum \( \Delta K = K_a - K_b \) increases as the beam energy increases. Since when multiplied by the interaction radius, it gives the maximum transferred angular momentum \( l = \Lambda_1 + \Lambda_2 \), we see that at the high incident energy the relevant contribution of the higher allowed transferred angular momentum \( l \) and the higher relative motion angular momenta \( \Lambda_1 \) becomes more important.

The absence of oscillations in ACFs for the levels different from the 13.45 MeV \((6^+)\) state, which was discussed in Ref. 13, requires reasonable explanation and justification. Let us consider the results of direct-transfer model ACF calculation for the other high-lying excited states in \(^{24}\text{Mg}\) between 4 and 14 MeV excitation energy. Figure 4a shows the results of ACF calculations obtained with different \( \Theta^{24\text{Mg} \rightarrow ^{12}\text{C} + ^{12}\text{C}} \) for the 13.21 MeV \((8^+)\) state. One can see that the ACFs calculated with equivalent contribution of the configurations with \( \Lambda_1 = 6, 8, 10 \) and \( I_X = 2 \) do not contain any oscillated structure and are sufficiently smooth. The ACFs presented in Fig. 4b were calculated at different \(^{14}\text{N}\) bombarding energies \( E_{\text{lab}} = 29, 33, 35 \) and 42 MeV using \( \Theta^{24\text{Mg} \rightarrow ^{12}\text{C} + ^{12}\text{C}} \) values from Table III. One can see that clear oscillations in ACFs for the 13.21 MeV \((8^+)\) state may appear at high incident energies, for example, at 42 MeV.

Finally, we can conclude that the study of the energy dependence of ACFs gives the most important information about the relative contribution of the different \(^{12}\text{C} \otimes ^{12}\text{C}^*\) configurations to the quasimolecular states in \(^{24}\text{Mg}\). The full and detail presentation of the obtained results is published in Ref. 37.

### 4. Summary and conclusion

Based on the general theory of the spin density matrix and its spin-tensors we have presented the methods for ACF calculations. The theoretical analysis of the differential cross sections and ASFs in the \(^{12}\text{C}(^{14}\text{N}, d)^{24}\text{Mg}(\alpha)^{20}\text{Ne}\) reaction was performed under the assumption that direct transfer of \(^{12}\text{C}^*\) cluster and CN mechanisms are realized in the reaction. Using the total transferred angular momentum coupling scheme we have calculated the RWAs for the light vertex \(^{14}\text{N} \rightarrow ^{12}\text{C}^* + d\) involving g.s. and 4.44 MeV \((2^+)\) intermediate states in \(^{12}\text{C}^*\). It made possible to obtain quantitative agreement between the theoretical calculations and the experimental data. The contribution of the CN mechanism calculated with the critical CN angular momentum \( I_{\text{efr}} \), which sat-
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ologizes the selected rules in the entrance and exit reaction channels, does not exceed 50% of the total cross section for all the studied excited states in the final $^{24}$Mg* nucleus at beam energies 29 and 35 MeV. Our calculations confirmed that the mechanism of $^{12}$C-cluster direct transfer plays the dominant role in this reaction because of the presence of quasimolecular $^{12}$C$\otimes^{12}$C configurations in few states in $^{24}$Mg.

The consistent calculations of the differential cross sections and ACFs allowed us to estimate RWAs for the $^{24}$Mg* $\rightarrow ^{12}$C+ $^{12}$C* vertex, assuming different contributions of various $[^{12}$C $\otimes^{12}$C*] $\Lambda_l I_x$ configurations. The extracted $\Theta_{^{24}$Mg* $\rightarrow ^{12}$C+ $^{12}$C*} values are found to be sufficiently larger (by a factor of $10^2$) than calculated in the framework of the cluster model [36]. The big RWA values as well as the energy dependence of the ACFs are consistent with the suggestion about the orbital relative motion of two $^{12}$C with angular momenta $\Lambda_1$ in the certain quasimolecular configuration.

Acknowledgments

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