Chaotic interactions and new perspectives in nuclear structure

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We discuss the recent development in application of quantum chaos to mesoscopic systems as nuclei: manifestations of chaos, spectral statistics, wave function equilibration and thermalization, exponential convergence method and geometrical effects including the predominance of the zero ground state spin in systems with random interactions.

Keywords: Quantum chaos; nuclear structure

Discutimos el reciente desarrollo en la aplicación del caos cuántico a sistemas mesoscópicos tales como los núcleos: manifestaciones de caos, estadística espectral, equilibrio y termalización de la función de onda, método de convergencia exponencial y efectos geométricos incluyendo la predominancia del estado base de spin cero en sistemas con interacciones aleatorias.

Descritores: Caos cuántico; estructura nuclear

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1. Introduction

Nuclear structure was successfully studied for the last seventy years, starting with the discovery of the neutron and the primary ideas of strong forces and isospin invariance. Enormous success in qualitative understanding, quantitative explanation and practical applications throughout the periodic table is well known [1]. Developed concepts and theoretical methods turned out to be easily transferable and very useful for other quantum many-body systems in atomic, molecular and condensed matter physics. Not long ago one could hear voices claiming that the field is almost over and only minor details are left for those who still continue their stubborn efforts in nuclear structure. The broad interest to nuclei far from stability and the related ocean of new phenomena, speculations, theoretical suggestions, applications, in particular in astrophysics, and new impressive large-scale experimental designs, refute this opinion. At the same time, even traditional areas of nuclear theory revealed their vitality and inexhaustibility. One of the most rapidly growing branches of physics during last two decades was chaotic dynamics, both in classical and quantum regimes. Atomic and nuclear structure can serve as the testing grounds for the ideas of quantum chaos [2, 3]; on the other hand, those ideas shed new light on physics of the nucleus.

In a classical chaotic system, the phase space trajectories with close initial conditions diverge exponentially with time amplifying all uncertainties of the initial state and leading to unpredictability and necessity of a statistical approach. In the quantum case, the uncertainty relation precludes the infinite divisibility of the phase space. Instead, quantum chaos manifests itself in specific "stochastic" properties of energy levels, wave functions and transition probabilities. Below we discuss exclusively the stationary states of a strongly interacting finite many-body system, such as the nucleus, complex atom, molecule or small solid particle, the world nowadays called mesoscopic. Being built of many particles, such objects can be naturally described in statistical terms. On the other hand, because of the relative smallness of the system, under special conditions it becomes possible to study individual quantum levels, for example isolated neutron resonances in heavy nuclei. The remarkable combination of the global, statistical, consideration, with the local features of individual states makes the quantum chaos approach unique.

In the best studied chaotic problems, such as hydrogen atom in a magnetic field or quantum billiards, the main reason for the development of chaos is the destruction of symmetries. The eigenstates are complicated superpositions of basic functions with different symmetry; now they can be labeled by total energy only. The spectral statistics are close to those in random matrix theory (RMT) [4, 5]. For a time-reversal invariant system the appropriate version is the Gaussian Orthogonal Ensemble (GOE). In nuclei the symmetry of the mean field plays a secondary role in chaosization. The driving force for chaotic mixing of simple configurations is the interaction between the particles. This leads to a number of new effects and a notion of many-body quantum chaos. It is impossible to give a full review of the results in this field which is rapidly expanding with the doubling period of about 3 years in publication rate. Below we give a brief description of typical properties of many-body quantum chaos and consider two examples of unexpected development which open new perspectives in our treatment of such systems (one can argue that this class includes practically all realistic cases).

2. Many-body quantum chaos in nuclei

2.1. Level equilibration

We usually start our consideration of a nucleus by taking into account Fermi-statistics and putting the nucleus in the shell-
model (mean field, MF) context. This gives us an idea of occupied and valence single-particle orbitals and, by inspecting a scheme of low-lying levels, of spherical or deformed shape. The basis states are independent particle Slater determinants; assuming axial symmetry we can construct them in the \( m \)-scheme. In the spherical case we can consider the groups of states—partitions—corresponding to certain configurations of the particles in the set of \( j \)-levels. Already at this stage one sees that the many-body level density \( \rho(E) \) rapidly increases with excitation energy \( E \) because of the combinatorial growth of the number of possible distributions of this energy over single-particle excitations. When we switch on the residual interaction, it becomes effectively strong for two close configurations. If the mixing matrix elements do not vanish, the original configurations go through a series of avoided crossings as a function of the interaction strength. Complexity of the stationary states with respect to the basis of noninteracting particles increases very fast. As a result of a crossing, the mixed levels repel each other and rapidly establish a quasi-rigid sequence. The entire process is similar to the equilibration in a gas due to the collisions (in our case crossings) of the particles (here-individual levels).

Numerical experiments with realistic single-particle levels and residual interactions, as well as spectroscopic data, show that the equilibration takes place already at a relatively weak interaction, Fig. 1. If we introduce a scaling factor \( \lambda \) for the interaction strength, with \( \lambda = 0 \) for the noninteracting case and \( \lambda = 1 \) for the realistic strength, the spectrum is equilibrated at \( \lambda = 0.2 \pm 0.3 \). The level statistics at this point are close to the predictions of the GOE: the nearest level spacing distribution agrees with the Wigner surmise \( P(s) \propto s \exp(-\text{const.} \cdot s^2) \), where the spacing \( s \) is measured in the units of the local average spacing \( D \), and the fluctuations of the level number in a spectral interval of a certain length \( L \) increase only logarithmically with \( L \). In a system with regular wave functions which belong to several different symmetry classes and do not mix, one would have the Poisson distribution \( P(s) = \exp(-s) \), which was first introduced in the problem of the level statistics by Gurevich in 1939 [6], whereas the fluctuations would increase linearly with \( L \). Even more detailed characteristics of the level statistics, such as the curvature distribution (the analog of the scattering angle in the equilibration problem), agree with the GOE [7].

2.2. Wave function equilibration

A seemingly paradoxical situation when the levels in an ordered system behave disorderly while in a system with chaotic mixing they form a rather regular sequence already for a relatively weak interaction does not signal the end of the evolution. As the interaction strength increases further, the eigenfunctions gradually acquire a greater degree of complexity. The equilibration of the wave functions means that the Percival alternative [8]—are close in energy states similar in structure or they can be completely different and just accidentally get in the same energy range?—is resolved in favor of the first possibility. After chaotic mixing, the generic states in a narrow energy window at high level density turn out to be very similar.

How can one quantify the degree of complexity of a state? Among different options one can choose information (Shannon) entropy as a suitable tool. Let \( \{|k\}\) be the basis of independent particle configurations. Take this set as a reference one. Express the eigenfunctions \( |\alpha\rangle \) of the full Hamiltonian with residual interactions in the reference basis,

\[
|\alpha\rangle = \sum_k C_k^{\alpha} |k\rangle, \quad \sum_k w_k^{\alpha} \equiv \sum_k |C_k^{\alpha}|^2 = 1. \tag{1}
\]

Information entropy of the state \( |\alpha\rangle \) in a given basis \( |k\rangle \) is

\[
I_\alpha = -\sum_k w_k^{\alpha} \ln w_k^{\alpha}. \tag{2}
\]

This quantity can change from 0 (with no interaction \( w_k^{\alpha} = \delta_k^{\alpha} \) to \( \ln N \) for a wave function uniformly spread over all \( N \) basis states, \( w_k^{\alpha} = 1/N \). It is easy to understand that, for the case of time-reversal invariance when all amplitudes \( C_k^{\alpha} \) can be taken real, the orthogonality requirements do not allow a noticeable number of states to have maximum entropy. The average value for the GOE is \( I_\alpha = \ln (0.48 N) \).
Large-scale shell model diagonalization in truncated space invariably shows that the eigenstates of the full effective Hamiltonian are indeed extremely complex combinations of the original configurations. However, information entropy (2) is, in a class of states with fixed exact quantum numbers $J^\pi T$, a smooth bell-shape function of excitation energy. It starts with a relatively low value, reaches a maximum close to (usually lower than) the GOE value in the middle and then symmetrically goes down to the highest end of the spectrum. All neighboring states look the same as expected for strong mixing. We can bring information entropy for the majority of states to the GOE level by artificially increasing the ratio of the interaction strength to the single-particle level spacing.

An important message comes from the dependence of $I_0$ on the basis $|\alpha\rangle$. In fact we measure the relative complexity of two orthonormalized complete sets. The shell model, or more generally MF, basis is the natural reference set since it separates in an optimal way the regular features from chaotic local fluctuations and correlations. In this basis one sees a regular behavior of growing complexity. In a randomly taken basis, for practically all $|\alpha\rangle$, information entropy is uniformly at the GOE value and does not reveal any spectral evolution. The generic behavior can be violated by a sharp change of the MF at some excitation energy, for example, by an appearance of another (e.g. superdeformed) potential well. Here the original basis may be inadequate. Another measure of degree of complexity can be formulated in an invariant way: for this purpose one can use correlational (von Neumann) entropy [9, 10]

$$S_\alpha = -\text{Tr}\{\rho^\alpha \ln(\rho^\alpha)\}, \quad \rho^\alpha_{kl} = C^\alpha_k C^\alpha_l,$$

where the averaging is assumed over a range of values of a random parameter in the Hamiltonian. $S_\alpha$ is representation-independent, sensitive to the relative phases of the amplitudes, and strongly changes in the phase transition regions; it reflects in fact a number of exciton classes mixed, or an effective order of perturbation theory, rather than a number of principal components.

In a “normal” case of a smooth evolution of information entropy, each eigenstate $|\alpha\rangle$ is characterized by a localization length $l_\alpha = \exp(I_0) \approx 0.48 N_\alpha$, where $N_\alpha$ is an effective number of principal components in this spectral region. This quantity can be used for estimating probabilities of processes which involve those complicated states. Thus, from the fact that $|C^\alpha_k|^2 \approx 1/\sqrt{N_\alpha}$ it follows that the matrix elements of a simple, let say one-body, operator between chaotic states of a similar degree of complexity $N$ are suppressed $\propto 1/\sqrt{N}$ compared to typical matrix elements of the same operator between simple configurations $|k\rangle$ ($N$-scaling [11]). This has an important consequence that in the region of high level density and strongly mixed states the effect of a small perturbation is enhanced; roughly speaking, the mixing matrix element is small as $1/\sqrt{N_\alpha}$ while the levels are squeezed by a factor $\propto 1/N_\alpha$ so that the resulting enhancement factor in the admixture is $\sim \sqrt{N_\alpha}$. An observed (first theoretically predicted) strong enhancement of parity nonconservation [12] in the mixture of $s$- and $p$-wave neutron resonances has this nature (plus the kinematic factors due to the large width ratio $\Gamma_0/\Gamma_p$). Especially bright this mechanism works in the large fragment asymmetry for the fission induced by slow polarized neutrons where the kinematic enhancement is absent. In a sense, this is similar to classical chaos: chaotic mixing amplifies small admixtures to the wave functions.

The uniformity of information entropy makes the degree of complexity a well defined function of excitation energy, i.e. thermodynamic variable. Statistical quantities, such as single-particle occupation numbers, follow the evolution of complexity. This reveals a close connection between chaos and thermalization [13, 14]. The equilibrated wave functions of adjacent states have similar macroscopic properties, a necessary condition for thermodynamic equilibrium: the global observables, for a given class of states, depend only on energy rather than on exact population of neighboring states by the initial excitation. The interaction plays the role of a heat bath; as a rule, mean occupancies of the orbitals are close to the Fermi-Dirac distribution although the system is far from a perfect gas [3, 15]. This means that Fermi-liquid theory is valid in average sense even for high excitation energy and not only near the Fermi-surface. Various temperature scales are possible in a small closed strongly interacting system: a conventional one based on the level density and thermal entropy, single-particle using the Fermi-Dirac distribution, information entropy scale and so on. In the region of strong mixing all of them are practically equivalent, Fig. 2.

![Figure 2. Thermodynamic temperature found from the Gaussian approximation for the level density of $J^\pi T = 0^+0$ states in the shell model for $^{28}$Si, solid hyperbolic lines (the centroid of the spectra corresponds to infinite temperature); single-particle temperatures of individual states from the fit of the occupation numbers by the Fermi-Dirac distribution, dots.](image)
3. Exponential convergence

Here we give an example of an application of the ideas of quantum chaos to "everyday life", namely to the computational problem of diagonalization of huge shell model matrices. Consider a Hamiltonian matrix for $A$ valence particles with a MF and pairwise effective interaction. The dimensions of such matrices dramatically grow with the particle number and the energy interval included in the truncated space. Only a limited number of low-lying states are really of interest for a detailed calculation and comparison with data. The rest can usually be taken into account statistically. One needs a practical approach to finding just those states explicitly and avoiding the impossible full diagonalization. The trivial idea of truncation turns out to be quite effective if supplemented by information on properties of chaotic wave functions. Since in the realistic situation the spacings between the single-particle levels are of the same order of magnitude as the strongest matrix elements of the residual interaction, we expect that the lowest noninteracting configurations are noticeably represented in the wave functions of low-lying states. How far in excitation energy should we go in order to account for the main admixtures of higher configurations due to the residual interactions?

A natural measure of this "range of influence" comes from the strength function

$$F_k(E) = \sum_\alpha u_k^\alpha \delta(E - E_\alpha) = \langle k | \delta(E - H) | k \rangle,$$

an energy spread of a "simple" basis state $|k\rangle$. The lowest moments of the strength function give average information about the degree of nonstationarity of the state $|k\rangle$. They can be found from the matrix of $H$ without a diagonalization. The centroid of $F_k$ is the diagonal matrix element

$$\bar{E}_k = \int dE E F_k(E) = H_{kk},$$

while the width is determined by the sum of the off-diagonal elements squared over the $k$th row of the matrix $H$,

$$\sigma_k = \int dE (E - \bar{E}_k)^2 F_k(E) = \sum_{l \neq k} |H_{kl}|^2.$$ (5)

We will not discuss in detail an important question of the generic shape of the strength function. If the centroid is located already in the chaotic region, the shape evolves from the Breit-Wigner (BW) one in the so-called standard model [1] predicting the golden rule spreading width

$$\Gamma_s = 2\pi \frac{\langle V^2 \rangle}{D},$$

where $V$ are typical coupling matrix elements and $D$ the spacing between the mixed states (it is valid for the relatively weak coupling when the width is smaller than the range of strong mixing matrix elements), to the Gaussian for the strong coupling case. The realistic strength of interaction is closer to the Gaussian case except for the very collective states as giant resonances which have the BW or intermediate shape [17]. Multiple giant resonances in the case of weak anharmonicity, as for the isovector dipole case, can be considered with the help of convolution of individual strength functions [18, 19]. The intermediate situation between the BW and Gaussian leads to the width scaling with the overtone number $n$ as $n^q$ with $q = 1$ for BW and $q = 1/2$ for the Gaussian. Therefore the higher overtones turn out to be narrower than it could be expected; according to the central limit theorem the deviations from the BW shape are amplified by the convolution.

The width of the strength function is rather stable in the process of spectral evolution [20] since the numerator and the denominator are both decrease inversely proportionally to the degree of complexity $N$. Therefore one expects the uniformity of the widths (6), $\sigma_k \approx \bar{E}$ for different initial states of the same symmetry class. This is indeed revealed by the shell model analysis [3]. (In the spherical shell model another reason helps the uniformization, see the next section). The stability of the width and shapes of generic chaotic states allows one to construct an analytic self-consistent theory [17]. On a practical side, it gives a hope that a truncation on the level of $(3-4)\bar{E}$, with an exact diagonalization of the truncated matrix, can catch all the important admixtures to the low-lying states. This is indeed the case as shown by the shell model calculation [21]. Such a truncation provides eigenvalues close to the exact values and the wave functions with the $\sim 90\%$ overlap with the exact solutions. For many practical purposes such precision may be sufficient.

Going beyond the diagonalization of truncated matrices, it is possible to study the gradual convergence of the results in the process of progressive inclusion of remote parts of the original matrices. Because of the variational character of the procedure, the convergence is monotonous. Moreover, it turns out to be almost precisely exponential [22]. An accurate numerical analysis [16] shows that, although the strength function near the centroid evolves from the BW to the Gaussian, the wings have an exponential behavior. This reminds the exponential localization of electron wave functions in disordered solids. The exponential tails come from the short-time nonexponential decay of the simple state which makes the exact BW shape impossible; they correspond to the self-similar solution of the hierarchy of mixings when the strength function at energy $E$ far away from the centroid is proportional to the full remaining strength,

$$F(E) = \text{const} \cdot \int_0^{\infty} dE' F(E') \propto e^{-\text{const} \cdot E}.$$ (8)

This means that the convergence of the eigenvalues of a generic chaotic system should be exponential in the process of increasing the matrix dimension. This was tested [22] for full and banded random matrices as well as for realistic shell model examples which allow the exact diagonalization. It was also confirmed [23] that onset of exponential convergence occurs after the truncation at the distance of $(3-4)\bar{E}$. Thus,
we get at our disposal a powerful tool for shell model matrices of an intractable size. In practice [23] we use the spherical shell model basis with $jj$-coupling and decomposition into partitions. We order the partitions by their centroid energies $E(p)$ and the width $\sigma(p)$ determines the primary truncation. Few initial diagonalization steps lead to the point where the remaining approximation of energy can be achieved by the exponential extrapolation, Fig. 3. Preliminary results show that the wave functions also converge which allows one to determine the primary truncation. The points where the exponential convergence is not known in mathematical literature. It was shown roughly equidistant unperturbed spectrum. Complete understanding was achieved [22] for tridiagonal matrices with smoothly changing matrix elements, diagonal $\epsilon_n$ and off-diagonal $V_{n,n+1} = V_{n+1,n} \equiv V_n$. Here the exponential convergence takes place if $\lambda^2_n = V_n^2/(\epsilon_n\epsilon_{n-1}) \to \lambda^2 < 1/4$ for $n \to \infty$.

4. Chaos and geometry of a finite system

Usually chaotic systems are considered as deprived of all symmetries so that the states can be labeled only by the energy eigenvalues. Finite systems without external fields are rotationally invariant and preserve angular momentum. (In nuclei parity and isospin are also conserved in a good approximation.) The first impression might be that this is hardly important because the different symmetry classes do not mix so that the only manifestation of reducibility will be the Poisson distribution of the nearest level spacings if all states are put together. However, such a conclusion is correct only for systems of the GOE type where the Hamiltonian interconnects all basis states within a certain class, and all matrix elements are independent uncorrelated quantities. Then the mixings within different classes are absolutely unrelated.

The situation is different for realistic systems with mainly two-body interactions. The Hamiltonian matrix contains many correlated entries. Indeed, for any two-body interaction amplitude $(1, 2 \to 3, 4)$ there are many spectator configurations of the remaining $A - 2$ particles on the orbitals different from $1, 2, 3, 4$. In all such cases the many-body matrix elements are identical. Now, the two interacting particles preserve their total angular momentum $L$ which can be coupled to the background angular momentum into the total spin $J$ of the system. Then it is clear that the matrix elements of the many-body Hamiltonian are strongly correlated for different $J$-classes. This is a new aspect of many-body quantum chaos which was never earlier discussed—correlations between the classes of states which belong to different representations of inviolable symmetry but are governed by the same Hamiltonian.

One hypothetical manifestation of such correlations would be the existence of suggested by Motelson compound, or ergodic, rotational bands. At excitation energy around 1 MeV above the yrast line in deformed even-even nuclei, the bands built on certain particle configurations in a deformed MF, are strongly mixed by the residual interaction [25]. Then the “laminar” deexcitation flow of stretched quadrupole transitions $J \to (J - 2) \to (J - 4) \ldots$ along a specific band gives place to a “turbulent” flow via many mixed paths. However, if the mixing in the $J$-class is similar to that in the $(J - 2)$-class, it can still happen that any strongly mixed $J$-state has its counterpart, an analogously mixed state in the next class, so that the radiation transitions prefer a single path (or few paths). Such compound bands are found in shell model calculations but not yet in the experiment. A similar phenomenon would be an observation of a pairing rotation band $A \to A \pm 2 \to \ldots$ with the pair transfer reactions.

Another face of the same problem is geometric chaos [26]. Even within a single partition of $A$ valence particles in a spherical MF, there are dozens or hundreds of states with the same total spin $J$ and correspondingly different vector addition schemes for $\sum_{a} j_a = J$ leading to orthogonal states through various intermediate steps $J_{12}, J_{123}, \ldots.$ The majority of such schemes look as a random walk in the many-body spin space. The idea of random spin coupling was employed by Bethe for the derivation of the Fermi-gas level density [27, 28] which is based on the Gaussian distribution of the resulting total $J_s$ values. This is also related to the complicated statistical properties of the $3nj$-symbols, recoupling schemes and fractional parentage coefficients studied by Wigner [29] and Ponzano and Regge [30]. Because of the rapidly increasing complexity of the coupling schemes (no dynamic interaction!), the correct linear combinations even within a single partition consist of many $m$-scheme Slater determinants with nearly random phases. This is an important factor in the above mentioned uniformization of the widths.
of the $J$-projected basis states $|k\rangle$. The geometric chaoticity can be used in many-body theory as a nonperturbative tool for discriminating high order effects.

Recently the geometric correlations between separate classes of states were emphasized by the discovery of the predominance of the ground state spin zero in an ensemble of finite fermionic [31, 32] or bosonic [33, 34] systems with random but rotationally invariant two-body interactions. Typically one assumes a set of degenerate $j$-levels and the residual interaction

$$H = \sum_{LA:1234} V_L \langle 1234 | a_1^\dagger a_2^\dagger a_3 a_4 \rangle_{LA} | a_1 a_2 a_3 a_4 \rangle_{LA},$$

where the unified labels $1, \ldots$ include all quantum numbers except for the magnetic ones ($J_L$), and $V_L$ are random numbers taken from an ensemble with equal probabilities for attraction and repulsion. The effect appears already in the simplest case of $A \geq 4$ identical particles on a single $j$-level. After averaging over a large number of realizations $\{V_L\}$, the fraction of cases with the ground state spin $J_0 = 0$ is by an order of magnitude higher than anticipated from the multiplicity of states $J = 0$ in the system. In many cases the probability of the ground state with the maximum possible spin $J_{\text{max}}$ is also enhanced. These results are robust and insensitive to the details of the ensemble, Fig. 4.

The dominance of the edge ground state spin values in the random interaction ensemble is the brightest example of the correlations between the separate classes of states governed by the same Hamiltonian (the number of parameters is small compared to the number of many-body matrix elements). Although it is clear that the correlations are present, the mechanism of the phenomenon is not obvious. Thus, the idea of the largest statistical width of the states with $J = 0$ does not give an explanation but rather reformulates the fact in different terms (actually, the excess of the width for $J = 0$ is not universal and too small in magnitude). Below we briefly describe the statistical mechanism based on geometric chaoticity [35] using the simplest example of a single-$j$ fermionic level.

Any specific realization of random interaction provides a “normal” rotationally invariant two-body Hamiltonian. The interaction creates a MF, and in the body-fixed frame the particles occupy their orbitals $|jm\rangle$. Let us consider aligned many-body states $|JM\rangle$ with the maximum projection $M = J$. The particle occupancies $n_m$ for $J = M \neq 0$ should violate time-reversal invariance (only for spin $J = 0$ the $n_m$ do not depend on $m$). On average, the most probable occupancies for the "equilibrium" state in the ensemble are given by the condition of maximum thermodynamic entropy under constraints for the total particle number $N = \sum_m n_m$ and total spin projection $M = \sum_m m n_m$. This leads to the Fermi-Dirac distribution with two Lagrange multipliers,

$$n_m = \frac{1}{\exp(\gamma m - \mu) + 1}.$$  

In this rough approximation we assume that the average MF is not deformed and all differences in occupancies $n_m$ are only due to the requirement $M \neq 0$. This approach is similar to the cranking model around the symmetry axis, and the parameter $\gamma$ plays the role of the cranking frequency. With Eq. (10) we can approximately calculate energy of this statistically favorable state,

$$\langle H \rangle = \sum_L V_L \langle C_{mm'}^{L\dagger} C_{m'm} \rangle \langle n_m n_{m'} \rangle.$$  

An especially simple result can be obtained if the occupancies only slightly differ from the uniform ones, and we use the expansion with respect to $\gamma$, $n_m = \bar{n} + \alpha m + \beta m^2 + \cdots$. Here $\bar{n} = A/(2j + 1)$, $\alpha$ is proportional to the cranking frequency, and $\beta$ is the second order correction to the chemical potential. Expressing, with the same accuracy, $\gamma$ and $\mu$ in terms of $A$ and $M = J$, and substituting in Eq. (11) the average product of the occupation numbers by the product of average values (10), we come to the series

$$\langle H \rangle = \sum_L (2L + 1) \times V_L \langle h_0(L) + h_2(L) J^2 + h_4(L) J^4 + \cdots \rangle,$$

where $h_{2k}(L)$ are geometrical factors for spin channels $L$ of the pair. Now it becomes clear that the average description is reduced to the yrast-line (only rotational quantum numbers survive) with the effective moment of inertia defined by $h_2(L)$. Because of the linearity in $V_L$, we predict the ground state spin $J_0 = 0$ in approximately 50% of cases, independently of the multiplicity of states $J = 0$. In a similar way one can consider the ferromagnetic state with $J = J_{\text{max}}$. In the single-$j$ model this state is unique, and its wave function is known regardless of the interaction; it favors the effective negative moment of inertia [but Eq. (12) is not valid here]. In

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CONTRAST TO THE CASE OF \( J_0 = 0 \), WHICH DOES NOT PREFER ANY SPECIAL SET \( \{ V_L \} \), THE CASE \( J_0 = J_{\text{max}} \) OCCURS WHEN THE PARAMETERS \( V_L \) FOR LARGE \( L \) CORRESPOND TO A STRONG ATTRACTION.

THE COMPARISON OF ACTUAL ENERGIES OF EIGENSTATES WITH EQUATION (12), FIG. 5, SHOWS A STRONG CORRELATION WHICH BECOMES NEARLY COMPLETE IF INSTEAD OF THE PRODUCT OF THE AVERAGE OCCUPANCIES WE USE THE CORRELATED OCCUPATION NUMBERS FROM THE SAME WAVEFUNCTIONS. ACTUAL \( n_m \), FIG. 6, OSCILLATE AROUND THE STATISTICAL VALUES \( (10) \) WHICH IS THE SIGNATURE OF AN EFFECTIVE, USUALLY SLIGHTLY DEFORMED, MF WHICH EXISTS IN ANY CONCRETE REALIZATION OF \( V_L \). THIS CAN BE TAKEN INTO ACCOUNT BY INTRODUCING EFFECTIVE DEFORMED ENERGIES \( \epsilon_m = \partial \langle H \rangle / \partial n_m \) AND CORRECTING THE EXPONENT IN THE OCCUPATION NUMBERS BY ADDING \( \epsilon_m / T \) WITH EFFECTIVE SINGLE-PARTICLE TEMPERATURE (RECALL SECTION III) WHICH MINIMIZES THE ENERGY. 

Thus, geometrical reasons related to chaoticity of coupling of individual spins made \( J_0 = 0 \) FAVORABLE.

ANOTHER ASPECT OF THE PROBLEM IS RELATED TO TIME-REVERSAL INVARIANCE. THE DIRECT INTRODUCTION OF A RANDOM INTERACTION VIOLATING THIS INVARIANCE \( \langle 36 \rangle \) DOES NOT INFLUENCE THE PREDOMINANCE OF \( J = 0 \) GROUND STATES BECAUSE THE ENSEMBLE AVERAGING REMOVES THE EFFECTS LINEAR IN THIS INTERACTION. HOWEVER THE GROUND STATE OF A NON-ZERO SPIN MEANS IN A SENSE THE SPONTANEOUS SYMMETRY BREAKING; UNDER ANY SPECIFIC CHOICE OF THE PROJECTION \( J_z \), THE SYMMETRY OF THE GROUND STATE IS LOWER THAN THAT OF THE HAMILTONIAN. WE HAVE USED THIS FACT ABOVE IN OUR GEOMETRICAL CONSIDERATION RELATED TO THE ALIGNED STATE WITH \( J_z = J \) WHERE THE CHOICE OF THE SENSE OF ROTATION DESTROYS THE TIME-REVERSAL INVARIANCE.

Now, the standard statement that the ground states of all even-even nuclei have zero spin because of the pairing forces is not quite precise because even without pairing the probability of \( J_0 = 0 \) is significantly enhanced. However, the structure of the wave functions with random interactions is very different from that governed by coherent pairing and multipole forces in nuclei \( \langle 38 \rangle \). The overlap of ground states with those in the realistic shell model is only slightly higher than one would expect for pure chaotic wave functions; this enhancement is associated with the off-diagonal pairing matrix elements whose second order effects are always coherent for the ground state (the phonon collectivity is absent as well). This consideration is also important for the ongoing discussion of interaction and spin effects in quantum dots and Bose-Einstein condensates for nonzero spin bosons.

5. Conclusion

Here we summarize the selected important aspects of our discussion.

(i) Many-body quantum chaos describes a generic behavior of all realistic systems.

(ii) Spectral statistics agree with the predictions of random matrix theory.

(iii) The structure of the wave functions is chaotic; the adjacent states have similar complexity; the degree of complexity can be quantified by information entropy and/or invariant correlational entropy.

(iv) Chaos provides an intrinsic mechanism for thermalization in a closed mesoscopic system.

(v) The Fermi-liquid description based on average occupation numbers is valid even far from the Fermi-surface.

(vi) Small perturbations are enhanced in the chaotic region and strongly amplify such effects as parity nonconservation.

(vii) The spreading width of simple states is stable with respect to the interaction strength and excitation energy;
its shape evolves from BW to Gaussian with observable consequences for giant resonances.

(viii) The wings of the strength functions are exponential which allows one to develop exponential convergence methods for practical calculations.

(ix) Geometric properties of the finite chaotic system lead to the predominance of the zero ground state spin and correlations between the blocks of states with different conserved quantum numbers.

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