Double beta decay and neutrino masses

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The flux of solar and atmospheric neutrinos has been measured with increasing precision, and offers now direct evidence of the presence of neutrino oscillations between different neutrino flavors. The neutrinoless double beta decay, if detected, would provide the complementary information needed to determine neutrino masses, and would also offer definitive evidence that the neutrino is a Majorana particle. A review is presented on the ongoing and future double-beta-decay experiments, the challenges still present in the calculation of double-beta-decay nuclear matrix elements, and of recent improvements based on the pseudo-SU(3) shell model.

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1. Introduction

Atmospheric and solar neutrino experiments have proven that neutrinos are massive, and the different flavors are largely mixed [1, 2]. The existence of neutrino oscillations allow the determination of the difference between the square of the neutrino masses coming from different families [3]. However, the neutrino mass absolute scale must be obtained from direct kinematical measurements, or from the double beta decay.

The neutrinoless double beta decay ($\beta\beta_{0\nu}$), if detected, would not only provide the complementary information needed to determine neutrino masses, but also offer definitive evidence that the neutrino is a Majorana particle, i.e. it is its own antiparticle [4, 5].

Theoretical nuclear matrix elements are needed to convert experimental half-life limits, which are available for many $\beta\beta$-unstable isotopes [6, 7], into constraints for the effective Majorana mass of the neutrino and the contribution of righthanded currents to the weak interactions. Thus, these matrix elements are essential to understand the underlying physics.

The two neutrino mode of the double beta decay ($\beta\beta_{2\nu}$) is allowed as a second order process in the standard model. It has been detected in ten nuclei [8, 7] and has served as a test of a variety of nuclear models. The calculation of the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ matrix elements requires different theoretical methods. Therefore a successful prediction of the former cannot be considered a rigorous test of the latter. However, in most cases it is the best available proof we can impose to a nuclear model used to predict the $\beta\beta_{0\nu}$ matrix elements. When it is possible, the same models are used to calculate the energy spectrum, electromagnetic and beta decay transitions in the initial, intermediate and final nuclei.

Many experimental groups have reported measurements of $\beta\beta$ processes [8, 9]. In direct-counting experiments the analysis of the sum-energy spectrum of the emitted electrons allowed the identification of the different $\beta\beta$-decay modes [10]. The $\beta\beta_{2\nu}$-half lives have been measured in ten nuclei, ranging between $10^{18} - 10^{22}$ yrs. Limits on the $\beta\beta_{0\nu}$-half lives go up to $10^{26}$ yrs which, based on the theoretical estimates to the nuclear matrix elements, put limits to the Majorana mass of the neutrino close to 0.3 eV.

The structure of the present article is as follows. In Sec. 2 the neutrino masses and mixing are discussed, in Sec. 3 the double beta decay formalism is introduced, and the $\beta\beta$ experiments are reviewed in Sec. 4. Section 5 contains the theoretical calculations of the nuclear matrix elements in deformed nuclei. The pseudo SU(3) formalism is described in Sec. 6, while in Sec. 7 old and new results for the $\beta\beta$ half-lives are presented. Conclusions are drawn in Sec. 8.

2. Neutrino masses

The description of the present results of the atmospheric and solar neutrino experiments requires the neutrinos emitted in weak decay processes like beta decay (i.e. $\nu_e, \nu_\mu, \nu_\tau$) to be a
combination of several mass eigenstates \( \nu_i \) with mass eigenvalues \( m_i \) [11, 12],

\[
\nu_e = \sum_{i=1}^{3} U_{ei} \nu_i.
\]

Neutrino mixing matrix elements are denoted by \( U_{ei} \). It is customarily assumed that \( m_1 < m_2 < m_3 \). The effective electron neutrino mass \( m_\beta \) measured from the tritium \( \beta \)-decay experiments can be written as

\[
m_\beta = \left( |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2 \right)^{1/2}.
\]

Present experimental limits are \( m_\beta < 2.2 \text{ eV} \). It can be reduced to 0.3 eV along the present decade.

The neutrinoless double beta decay \( (\beta\beta_{0\nu}) \), if detected, would offer definitive evidence that the neutrino is a Majorana particle [4, 5]. This process violates the lepton number conservation, being forbidden in the standard model of elementary particles.

The Majorana mass of the neutrino \( \langle m_\nu \rangle \) appears in the \( \beta\beta_{0\nu} \)-decay in the form

\[
\langle m_\nu \rangle = |U_{e1}|^2 m_1 + e^{i\phi_1} |U_{e2}|^2 m_2 + e^{i\phi_2} |U_{e3}|^2 m_3.
\]

The relative Majorana CP-violating phases are denoted \( \phi_i \). Notice that, even if the mass eigenstates are as large as their upper limits \( \approx 2 \text{ eV} \), the CP-phases could conspire to generate a strong cancellation, causing \( \langle m_\nu \rangle \) to be very small, eventually zero.

From the solar and atmospheric neutrino experiments the squared mass differences \( \Delta m_{ij}^2 = m_i^2 - m_j^2 \) have been determined. In the most probable scenarios, \( \Delta m_{12}^2, |U_{e1}|^2 = \cos^2 \theta_\odot \) and \( |U_{e2}|^2 = \sin^2 \theta_\odot \) are obtained from solar neutrino experiments, \( \Delta m_{23}^2 \) from atmospheric neutrino experiments and \( |U_{e3}|^2 \) is restricted from experiments looking for electron disappearance like CHOOZ. The phases \( \phi_i \) and the mass of the lightest neutrino must be obtained from the \( \beta\beta_{0\nu} \)-decay experiments.

The most probable values of the neutrino parameters, obtained from global analysis of known experimental results [11], are listed in Table I.

Combined with the tritium \( \beta \)-decay data, they establish the following limits to the neutrino masses [11]:

\[
m_i \leq 2.2 \text{ eV}, \quad i = 1, 2, 3, \quad |m_i - m_j| < 0.08 \text{ eV}.
\]

### 3. Double beta decay

The detection of the neutrinoless double beta decay \( (\beta\beta_{0\nu}) \) would imply indisputable evidence of physics beyond the standard model and would be useful in order to select Grand Unification Theories [4]. Theoretical nuclear matrix elements are needed to convert experimental half-life limits, which are available for many \( \beta \)-unstable isotopes [8], into constraints for particle physics parameters such as the effective Majorana mass of the neutrino and the contribution of right-handed currents to the weak interaction. The two neutrino mode of the double beta decay \( (\beta\beta) \) is allowed as a second order process in the standard model. It has been detected in ten nuclei [8, 7] and has served as a test of a variety of nuclear models. It is the best available proof we can impose to a nuclear model used to predict the \( \beta\beta_{0\nu} \) matrix elements.

Many experimental groups have reported measurements of \( \beta \beta \) processes [8, 7]. Nearly for all the cases the ground state (g.s.) to ground state \( (0^+ \rightarrow 0^+) \) decay was investigated. In direct-counting experiments the analysis of the sum-energy spectrum of the emitted electrons allowed the identification of the different \( \beta\beta \)-decay modes.

The inverse half life of the two neutrino mode of the \( \beta\beta \)-decay can be expressed in the form [13]

\[
\left[ \frac{\tau_{2\nu}^{1/2} (0^+ \rightarrow 0^+)}{R} \right]^{-1} = G_{2\nu} | M_{2\nu} |^2.
\]

where \( G_{2\nu} \) is a kinematical factor, which depends strongly on total energy released in the decay.

In the case the 0\( \nu \) decay exists, the virtual neutrino must be emitted in one vertex, and absorbed in the other. Since in the standard theory the emitted particle is a right-handed antineutrino and the absorbed one a left-handed neutrino the process requires that: (a) the exchanged neutrino is a Majorana particle and (b) both neutrinos have a common helicity component. The helicity matching can be satisfied in two ways: (i) the neutrinos have a nonvanishing mass and therefore a "wrong" helicity component proportional to \( m_\nu / \Delta \). The decay rate will be proportional to \( (m_\nu)^2 \). Or (ii) the helicity restriction could be satisfied if there is a right handed current interaction. In this case a nonvanishing mass allowing mixing of neutrino types is also required [13, 14].

For massive Majorana neutrinos one can perform the integration over the four-momentum of the exchanged particle and obtain a "neutrino potential" which for a light neutrino \( (m_\nu < 10 \text{ MeV}) \) has the form

\[
H(r, E) = \frac{2R}{\pi r} \int_0^{\infty} dq \frac{\sin(qr)}{q^2 + E},
\]

where \( E \) is the average excitation energy of the intermediate odd-odd nucleus and the nuclear radius \( R \) has been added to

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Table I. Most probable values of neutrino parameters.

<table>
<thead>
<tr>
<th></th>
<th>LOW-QVO</th>
<th></th>
<th>LOW-QVO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta m_{12}^2 ) [eV²]</td>
<td>( U_{e1}^2 )</td>
<td>( \Delta m_{12}^2 ) [eV²]</td>
<td>( U_{e1}^2 )</td>
</tr>
<tr>
<td>3.3 \times 10^{-5}</td>
<td>0.26</td>
<td>9.6 \times 10^{-8}</td>
<td>0.37</td>
</tr>
</tbody>
</table>
make the neutrino potential dimensionless. The final formula, restricted to the term proportional to the neutrino mass, is [13, 15]

\[
(r_{0\nu}^{1/2})^{-1} = \left( \frac{\langle m_\nu \rangle}{m_e} \right)^2 G_{0\nu} M_{0\nu}^2.
\]  

(7)

where \(G_{0\nu}\) is the phase space integral associated with the emission of the two electrons. The nuclear matrix elements \(M_{0\nu}\) are [13]

\[
M_{0\nu} \equiv \left| M^{GT}_{0\nu} - \frac{g_\nu^2}{g_A^2} M^{F}_{0\nu} \right|,
\]  

(8)

with

\[
M^{\alpha}_{0\nu} = (0^+_f \| O^{\alpha} \| 0^+_i),
\]  

(9)

where the kets \(|0^+_f\rangle\) and \(|0^+_i\rangle\) denote the corresponding initial and final nuclear states, the quantities \(g_\nu\) and \(g_A\) are the dimensionless coupling constants of the vector and axial vector nuclear currents, and

\[
O^{GT} = \sum_{m,n} O^{GT}_{mn} = \sum_{m,n} \delta m t_m \cdot \delta n t_n H \left( |f_m - f_n|, E \right),
\]  

(10)

\[
O^{F} = \sum_{m,n} O^{F}_{mn} = \sum_{m,n} t_m t_n H \left( |f_m - f_n|, E \right).
\]  

(11)

The \(\delta\) being the Pauli matrices related with the spin operator and \(t^-\) the isospin lowering operator, which satisfies \(t^-|n\rangle = |p\rangle\). The superindex \(GT\) denotes the Gamow-Teller spin-isospin transfer channel, while \(F\) indicates the Fermi isospin one.

4. Double beta decay experiments

After the first direct detection of the \(\beta\beta_2\nu\)-decay of \(^{82}\text{Se}\) in 1987 using a time projection chamber, this rare decay has been observed in ten nuclei using a variety of experimental techniques [8]. Geochemical detection was the first method employed, allowing the determination of the \(\beta\beta_2\nu\)-half life of \(^{82}\text{Se}, \text{^{96}Zr, ^{128,130}Te}\). The \(\beta\beta_2\nu\)-half life of \(^{238}\text{U}\) was measured using radiochemistry. Time projection chambers were used with \(^{48}\text{Ca}, \text{^{82}Se, ^{100}Mo and ^{150}Nd}\), counting spectrometers with \(^{82}\text{Se}, \text{^{96}Zr, ^{100}Mo and ^{116}Cd}\), and ultrasilicon semiconductor germanium detectors with \(^{76}\text{Ge}\).

Geochemical and radiochemical methods count the number of daughter nuclei in an old sample, being unable to distinguish between the modes with and without neutrino emission. Direct measurements detect the simultaneous emission of the two electrons, and the shape of the energy spectrum provides a clear signature for the different \(\beta\beta\)-decay modes. These experiments must be performed in a very low background environment, usually provided in underground laboratories, and make use of ultra-pure materials with radioactive contaminants present in less than one part in \(10^{12}\). The \(\beta\beta_2\nu\) decay of \(^{100}\text{Mo}\) to the first excited state in \(^{100}\text{Ru}\) has also been detected.

5. Theoretical calculations for heavy deformed nuclei

The calculation of two neutrino double beta decay matrix elements has proven to be extremely sensitive to the details of the wave functions of the initial and final nuclei [5]. While QRPA calculations are easy to perform, the uncertainties in the residual particle-particle proton-neutron interaction strongly limit their predictive power [5]. Shell model studies in the full \(np\) shell provide reliable matrix elements for \(^{48}\text{Cr}\) and other light nuclei [20]. Matrix elements for \(^{76}\text{Ge, ^{82}Se and ^{96}Xe}\) were obtained using very large shell model spaces, which are however strongly truncated [21]. For heavier nuclei standard shell model calculations are impractical.

The pseudo SU(3) shell model [22, 23] is a microscopic model which allows the description of heavy deformed nuclei in the laboratory frame through the use of a fermionic many-particle basis with good angular momentum. The microscopic Hamiltonian employed includes single-particle energies as well as pairing and quadrupole-quadrupole interactions. Most of the Hamiltonian parameters are fixed by
known systematics. Three “rotor terms” allow a fine tuning of the energy spectra and are fitted for each nuclei. The pseudo SU(3) model has been used to describe many low-lying rotational bands, as well as B(E2) and B(M1) intensities in rare earth and actinide nuclei, both with even-even and odd-mass numbers [24-29]. It has been exhibited as a powerful and predictive tool in the description of heavy deformed nuclei.

The ββ half lives of some of these parent nuclei to the ground and excited states of the daughter ones were evaluated for the two and zero neutrino emitting modes [30-34] using the pseudo SU(3) scheme. The predictions were in good agreement with the available experimental data for 150Nd and 238U. Extending the previous research [30-34] the SU(3) model has been used to describe many low-lying states for two and zero neutrino emitting modes [30-34]. The pseudo SU(3) scheme, is strongly dependent on the occupation numbers for protons and neutrons in the normal and abnormal parity states: \( n^N_A, n^N_N, n^A_A, n^A_N \) [30]. These numbers are determined by filling the Nilsson levels from below, as discussed in Ref. 30. The effect of allowing changes in these numbers is discussed in Refs. 36 and 37 and briefly mentioned in the next section.

In the first series of papers [30-34] we evaluated the ββ matrix elements by taking into account only the leading SU(3) coupled proton-neutron irreducible representation, which in recent calculation was shown to represent around 60% of the wave function in even-even Dy and Er isotopes [29].

### 6. The pseudo SU(3) formalism

In the pseudo SU(3) shell-model coupling scheme [22], normal parity orbitals \( \{\eta\ell, l, j\} \) are identified with orbitals of a harmonic oscillator of one quanta less \( \bar{\eta} = \eta - 1 \). The set of orbitals with \( j = j = \bar{l} + \bar{s} \), pseudo spin \( \bar{s} = 1/2 \), and pseudo orbital angular momentum \( \bar{l} \), define the so-called pseudo space. The orbitals with \( j = \bar{l} \pm 1/2 \) are nearly degenerate. For configurations of identical particles occupying a single \( j \) orbital of abnormal parity, a convenient characterization of states is made by means of the seniority coupling scheme.

The many-particle states of \( n_\alpha \) nucleons in a given shell \( \eta_\alpha \), \( \alpha = \nu \) or \( \pi \), can be defined by the totally antisymmetric irreducible representations \( \{1^N_\alpha \} \) and \( \{1^A_\alpha \} \) of unitary groups. The dimensions of the normal \( (N) \) parity space is \( \Omega^N_\alpha = (\bar{\eta}_\alpha + 1)!(\bar{\eta}_\alpha + 2)! \) and that of the unique \( (A) \) space is \( \Omega^A_\alpha = 2\bar{\eta}_\alpha + 2 \) with the constraint \( n_\alpha = n^A_\alpha + n^N_\alpha \). Protons and neutron states are coupled to angular momentum \( J^N \) and \( J^A \) in both the normal and unique parity sectors, respectively. The wave function of the many-particle state with angular momentum \( J \) and projection \( M \) is expressed as a direct product of the normal and unique parity ones, as:

\[
|JM\rangle = \sum_{J^N J^A} \left| J^N \right\rangle \otimes \left| J^A \right\rangle^J\rangle_M^J, \tag{12}
\]

Since we are interested in describing low-lying energy states, only pseudo spin zero configurations are taken into account in the normal parity space and only seniority zero configurations in the abnormal parity space. This simplification implies that \( J^A = J^A_\nu = 0 \). This is a strong assumption, but one that is physically motivated and very useful for simplifying the calculations. Recent research show that the quasi SU(3) formalism can be used to improve the description of the intruder sector [38]. Its application to the \( \beta\beta \) decay are under development.

### 7. Predicted \( \beta^-\beta^- \) half-lives

In all of the calculations only one active shell was allowed for protons, and likewise, only one for neutrons. This is a very strong truncation. For the \( \beta\beta \) decay this implies that only one uncorrelated Gamow-Teller transition is allowed: that which removes a neutron from a normal parity state with maximum angular momentum and creates a proton in the intruder shell \( h_{\nu/2}^\nu \rightarrow h_{1/2}^{1/2} \) in rare earth nuclei, \( t^{1/2}_{1/2} \rightarrow t^{13/2}_{13/2} \) in actinides). This unique Gamow-Teller transition controls the \( \beta\beta \) decay. Under these assumptions, if the occupation of the Nilsson levels is such that the number of protons in the abnormal parity states does not change for the initial and final state configurations, the decay is forbidden.

The published results for the six allowed two neutrino \( \beta^-\beta^- \) emitters are given in Table II.

<table>
<thead>
<tr>
<th>Transition</th>
<th>( \beta_{1/2} ) [yr]</th>
<th>( \beta_{0} ) [yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {}^{146}\text{Nd} \rightarrow {}^{146}\text{Sm} )</td>
<td>2.1 \times 10^{21}</td>
<td>1.18 \times 10^{28}</td>
</tr>
<tr>
<td>( {}^{148}\text{Nd} \rightarrow {}^{148}\text{Sm} )</td>
<td>6.0 \times 10^{20}</td>
<td>6.75 \times 10^{24}</td>
</tr>
<tr>
<td>( {}^{150}\text{Nd} \rightarrow {}^{150}\text{Sm} )</td>
<td>6.0 \times 10^{18}</td>
<td>1.05 \times 10^{24}</td>
</tr>
<tr>
<td>( {}^{186}\text{W} \rightarrow {}^{186}\text{Os} )</td>
<td>6.1 \times 10^{24}</td>
<td>5.13 \times 10^{25}</td>
</tr>
<tr>
<td>( {}^{192}\text{Os} \rightarrow {}^{192}\text{Pt} )</td>
<td>9.0 \times 10^{25}</td>
<td>3.28 \times 10^{26}</td>
</tr>
<tr>
<td>( {}^{238}\text{U} \rightarrow {}^{238}\text{Pu} )</td>
<td>1.4 \times 10^{21}</td>
<td>1.03 \times 10^{24}</td>
</tr>
</tbody>
</table>

The agreement between the theoretical two neutrino half lives [30] with the available data for \( {}^{150}\text{Nd} \) \( (\beta_{1/2} = 9.17 \times 10^{18} \) yr) and \( {}^{238}\text{U} \) \( (\beta_{1/2} = 2 \times 10^{21} \) yr) is good. For the theoretical \( \beta_{0} \) half lives [31] we assumed \( (m_\nu) = 1 \) eV.

As mentioned above, the number of nucleons in normal and unique parity orbitals is determined by the filling of the deformed Nilsson orbitals. In this way the theory predicts the complete suppression of the \( \beta\beta \) decay for the following five nuclei: \( {}^{154}\text{Sm}, {}^{160}\text{Gd}, {}^{170}\text{Yb}, {}^{237}\text{Th} \) and \( {}^{244}\text{Pu} \) [30]. It was
expected that these forbidden decays would have, in the best case, matrix elements that would be no greater than 20% of the allowed ones, resulting in at least one order of magnitude reduction in the predicted half-life [34]. Experimental limits for the $\beta\beta_{0\nu}$ decay of $^{160}$Gd have been reported [39, 40]. Recently it was argued that the strong cancellation of the $2\nu$ model in the $\beta\beta_{0\nu}$ decay of $^{160}$Gd would suppress the background for the detection of the $0\nu$ mode [41].

The pairing interaction allows the mixing of states with different occupancies in the normal and intruder sectors. In this way it is possible to calculate a finite $\beta\beta_{0\nu}$ half-life for the nuclei where this decay was previously reported as forbidden. In Refs. 35 and 36 it was assumed that the final nuclei wave function has two components with different occupancies: one to which the $\beta\beta_{2\nu}$ is forbidden, and the other to which it is allowed. The $\beta\beta_{0\nu}$ has finite contributions for the two components with different occupation numbers in the final state. There are two terms in the $\beta\beta_{0\nu}$ decay: one to the basis state which has allowed $\beta\beta_{2\nu}$ decay, and one to the state with forbidden $\beta\beta_{2\nu}$ decay. The results, taken from Ref. 36, are presented in Table III. A detailed study of the $\beta\beta$ decay of $^{160}$Gd, including the mixing of different occupancies in the parent and daughter nuclei, can be found in Ref. 37.

As a consequence of the explicit inclusion of deformation in the present model, the $\beta\beta_{0\nu}$ half-lives are larger than those reported in Ref. 42. In $^{160}$Gd the $\beta\beta_{0\nu}$ decay half-life is at least three orders of magnitude larger than the $\beta\beta_{2\nu}$ decay half-life. It implies that the background suppression due to a large $\beta\beta_{0\nu}$ half-life would be effective, although not as noticeably as was optimistically envisioned in Ref. 41. In any case, the results presented strongly suggest that the planned experiments using GSO crystals [41] would be able to detect the $\beta\beta_{0\nu}$ decay of $^{160}$Gd, and to establish competitive limits to the $\beta\beta_{0\nu}$ decay.

8. Conclusions

We have reviewed the compelling evidence exhibiting the neutrinos as massive particles. The observation of the neutrinoless double beta decay would complement these data by fixing the mass scale. Running and planned experiments will push the upper limits of the Majorana mass of the neutrino to 0.01 eV.

The pseudo SU(3) predictions for the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ half-lives have been extended by the inclusion of the pairing mixing between states with different occupancies in the normal and intruder sectors. The nuclei $^{160}$Gd and $^{244}$Pu emerge as viable candidates where these decays can be detected.

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Referencese