Interaction induced oscillations (ININO) in correlated electron transport

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The correlated motion of two electrons in a wire with an externally applied longitudinal electric field is discussed. We show within the tight binding approximation, that the electron-electron interaction induces time dependent oscillations of the mobility whose period depends on the strength and range of the coupling only. The oscillations are interpreted as Rabi-like, taking place between states in a band of paired states, and states in a band of extended states. The role of disorder is also discussed.

Keywords: Electrons transport, low dimensional systems.

Se discute el movimiento correlacionado de dos electrones en un alambre con un campo eléctrico externo longitudinal. Mostramos, con la aproximación tigh binding, que la interaccion electrón-electrón induce oscilaciones temporales de la movilidad cuyo periodo depende sólo de la intensidad y alcance del acoplamiento. Interpretamos las oscilaciones como del tipo Rabi, ocurriendo entre estados en una banda de estados pareados, y estados en una banda de estados extendidos. Tambien, consideramos desorden y discutimos su influencia.

Descriptors: Electrons transport, low dimensional systems.

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There has been much interest recently in the effect of particle interactions on localization due to disorder. Work by Shepelyanski in the two electron problem suggests that in 1d delocalization ensues, a result that has been disputed [1–6]. The quantity usually studied is the localization length of the system response to an external electric field, in order to judge the ability to conduct through the behavior of the mobility. Our main finding is that this quantity oscillates in time with a period determined by the strength and range of the interaction. We suggest that the eventual observation of such oscillations would permit an experimental characterization of the nature of two-body correlations taking place in a disordered background.

We consider two interacting electrons forced to move in a one dimensional chain of lattice parameter $a$, with an electric field $F$ applied along the chain. In the tight binding model, the amplitude $C_{l,m}$ for having one particle at site $l$ and the other at site $m$ at time $t$, is given by

$$-\lambda(C_{l+1,m} + C_{l-1,m} + C_{l,m+1} + C_{l,m-1}) + E_{l,m}C_{l,m} = i\hbar \frac{dC_{l,m}}{dt},$$

where

$$E_{l,m} = \epsilon_l + \epsilon_m + V(l-m) - eFa(l+m),$$

with $\epsilon_l$ the energy at site $l$ and $V(l-m)$ the two body interaction potential. $\lambda$ is the usual hopping energy. In this model either of the two charges can hop to its nearest neighbor sites. The geometry is that of a linear chain. Disorder may be included by making the site energies $\epsilon_l$ random.

The two electron problem described above is equivalent to that of a single particle moving in a square lattice with sites on the plane labeled by the pair $l,m$. The interaction acts like a surface potential, symmetric about the diagonal $l=m$, and the applied external field is parallel to this line. One may solve for the spectrum and eigenstates of the system with no electric field, no disorder and a contact Hubbard interaction $V(l-m) = U\delta_{l,m}$. To obtain the solution, we substitute in Eq. (1)

$$C_{l,m} = e^{i(l+m)ka} f(l-m),$$

where the new amplitudes $f$ depend only on the distance $u = l-m$. It is found that $f$ obeys a one-dimensional equation, with hopping amplitude $-2\lambda \cos(ka)$ and a chain of defects of local potential $U$ along the line $l = m$, acting like an interface separating two identical media. The states bounded to this interface, that we shall call paired states, are then given by

$$f(l-m) = A e^{-|l-m|} \sqrt{U^2 + 16\lambda^2 \cos^2(ka)},$$

with $A$ a normalization constant, and

$$\alpha = -\text{arcsinh}(U/4\lambda \cos(ka)).$$

The range $\pi/2 < ka < 3\pi/2$ defines an energy band covering the interval $U \leq E \leq \sqrt{U^2 + 16\lambda^2}$ for $U > 0$, and a
symmetric negative band exists for
\[ U < 0 \quad \text{and} \quad -\pi/2 < ka < \pi/2. \]

Note that the interaction gives more weight to configurations for which the electrons lie near each other \((l \approx m)\), regardless of the sign of the interaction. Notice also that \(\alpha\) diverges at the lower edge of the \(U > 0\) band (upper edge of the \(U < 0\) band), leading to occupation exclusively along the interface \(l = m\). The wavefunction is symmetric under exchange of particles and is therefore appropriate for a singlet state. Together with this band of paired states there is also a band of pure plane-wave solutions covering the interval \((-4\lambda, 4\lambda)\) which correspond to traveling waves that scatter off the defect line \(l = m\) \([7]\). The two bands giving paired and extended states overlap, except for \(U > 4\lambda\).

In discussing transport we consider the time dependent average position
\[ < z > = \sum_{l,m} P(l,m)(l + m)a \]
in the linear chain, where
\[ P(l,m) = C_{l,m}^* C_{l,m} \]
is the probability of finding one electron at site \(l\) and the other at site \(m\). We then take the time derivative of this expression to find the velocity. After some algebraic manipulation we arrive, with the use of Eq. (1) to the following expression for the average velocity:
\[ < v > = -\frac{2\lambda a}{\hbar} \int_m \sum_{l,m} C_{l,m}(C_{l+1,m}^* + C_{l,m+1}^*). \]  

This relation is exact for the infinite chain. In the high field limit an analytic solution for the time evolution of this quantity may be obtained if one assumes the system is initially in a plane wave (Bloch) state, as if the external field is turned on just at \(t = 0\). For \(\lambda \ll eF a\) one then has
\[ C_{l,m}(t) = C_{l,m}(0)e^{-\frac{iE}{\hbar}t} + O(\lambda). \]  

We first discuss the case with no disorder. Replacing in Eq. (4) we obtain, always to \(O(\lambda)\),
\[ < v > = \frac{2\lambda a}{\hbar} \sin(ka + \frac{eF at}{\hbar}) \frac{1}{N^2} \sum_{l,m} \cos(\frac{\delta V_{l,m}t}{\hbar}). \]  

Here \(N\) is the number of sites and
\[ \delta V_{l,m} = V(l + 1 - m) - V(l - m). \]

The factor in front of the sum gives the usual Bloch oscillations. \([8–10]\) The sum has the upper bound \(N^2\), so that the interaction in general reduces the drift velocity. Also, for the contact interaction model only \(\delta V_{l,m} = U\) occurs in the argument of the cosine when finite, showing there is an oscillation of period \(T_f = \hbar/U\). Although the amplitude of this interaction induced oscillation (ININO) depends on the wavenumber \(k\) and the ratio \(U/\lambda\), its period is independent of the hopping amplitude, and is constant throughout the band of paired states. For a long-range interaction several frequencies may be present, although at short times terms with \(l \approx m\) will be dominant.

Next we consider disorder. From Eqs. (4) and (5) it is easy to see that the contribution of a disordered distribution of site energies appears through phase factors of the form
\[ \exp[i(\epsilon_{l+1} - \epsilon_l)t/\hbar], \]
so that the average over disorder yields an overall factor
\[ \ll \cos[(\epsilon_{l+1} - \epsilon_l)t/\hbar] \]
in the drift velocity (4), which at all times is less than one. Thus, disorder decreases the drift velocity of the pair without affecting the period of the interaction induced oscillation discussed above.

The results described above rely on the approximation (5) that holds when \(\lambda \ll eF a\). We have performed numerical calculations to test different ranges of parameters. \([11]\) We find the oscillations to be present with the expected period even at small values of the applied field, when Eq. (5) does not hold.

In summary, we have shown that the electron-electron interaction induces oscillations in the drift velocity of a pair of electrons moving along a chain and subject to an external electric field, with a period determined solely by the interaction range and strength. The problem is identical to that of a single particle moving in a two dimensional square lattice, with a line of defects that divides the space in two symmetric halves. One can take advantage of the equivalence of the two cases to understand the physical origin of the oscillations. With no interactions the single particle in 2D will respond to an external field purely through Bloch oscillations in a band of extended states. The line of defects introduced by the interaction gives rise to a separate band of partially localized surface states, such that electrons may exhibit Rabi-like oscillations between the two bands. This interpretation is supported by our finding through numerical studies that the ININO dissappear when the initial state is entirely in the lower band (extended states for \(U > 0\), paired states for \(U < 0\)).

The equivalence of the 1D and 2D models emphasized in the introduction may also be uselful in checking the effect we are reporting, since the experimental probe could be either a 1D system with two electrons, or one electron moving in a 2D lattice which contains a line of impurities.

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