Holographic cosmology in 2+1 dimensions

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We examine a closed Friedmann-Robertson-Walker (FRW) model in 2+1 dimensions in order to investigate the cosmic holographic principle suggested by Fischler and Susskind. The matter content of the model is composed of two perfect fluid, with a \(\gamma\)-law equation of state. We found that a closed universe satisfy the holographic principle for dust and exotic matter with a negative pressure. Our results was derived rigorously, differing from the previous one found by Wang and Abdalla.

\textbf{Keywords:} Cosmology, lower dimensions.

In this work we study the cosmic holographic principle suggested by Fischler and Susskind, in a model of closed Friedmann-Robertson-Walker (FRW) in 2+1 dimensions. The matter content of the model consists of two perfect fluids, with an \(\gamma\)-law equation of state. We found that a closed universe with dust and exotic matter with a negative pressure satisfy the holographic principle. Our results were derived rigorously, differing from the previous one found by Wang and Abdalla.

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and $0 < \chi < \infty$, for $\kappa = 0, -1$. In the formulation of FS for the HP, the entropy contained within a volume coordinate size $V_c$ should not exceed the area of the horizon in Planck units. In terms of the (constant) comoving entropy density $\sigma$ this condition can be written for a closed universe as

$$\frac{S(t)}{A} = \sigma \frac{V_c(\chi)}{A(\chi)} = \sigma \left( \frac{1 - \cos \chi}{a(\chi) \sin \chi} \right) < 1.$$  

(6)

We emphasize again that $\chi$ is not zero at the initial time ($\sigma \sim a_0^2$). The condition $S/A < 1$ implies that the HP is satisfied if

$$a(\chi) > \frac{1 - \cos \chi}{\sin \chi}.$$  

(7)

In Ref. 5 it is assumed that closed universes containing exotic matter collapses. In the $2 + 1$ dimensional case, since the acceleration depends only on the pressure, an inflationary solution for the scale factor is obtained if the total pressure is negative. For a dust filled universe with exotic matter density the scale factor is always accelerating and there is no collapse (see [6] for a discussion from the point of view of the specific energy conditions). The solution $a(\chi)$ for a closed universe filled with dust and a exotic fluid with $\gamma = 1/2$ is given by

$$a(\chi) = 4\alpha \frac{\xi}{\beta_{1/2}} \exp \alpha \chi \left( \frac{1}{\xi - \exp \alpha \chi} \right)^2.$$  

(8)

where

$$\beta_{1/2} \equiv 2\pi \rho_{1/2} a_0, \quad \xi \equiv 2\pi \rho_1 a_0^2 - 1 \quad \text{and} \quad \alpha \equiv \left( \sqrt{\xi + \beta_{1/2} a_0} + \sqrt{\xi} \right) / \left( \sqrt{\xi + \beta_{1/2} a_0} - \sqrt{\xi} \right).$$

$\rho_{1/2}$ and $\rho_1$ denotes the initial density of the exotic matter and dust respectively.

From the condition given in Eq.(7) it is easy to see that the HP is satisfied. Our conclusion is that closed universes may satisfy HP if the matter content is dust and exotic matter with a negative pressure, contrary to the results found in Ref. 5. Other studies in $3 + 1$ dimensions, found that HP is satisfied for a closed universe filled with two fluids, where one of them has an equation of state, $p = w\rho$, with $w < -1/3$ [7].

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