Physics around the proton drip–line

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Proton radioactivity observed beyond the proton–drip line in the region of $50 < Z < 82$ is reviewed and discussed.

Keywords: Drip line; proton radioactivity.

Se revisa y discute la radioactividad de proton observada mas alla de la linea de emisión protónica en la region $50 < Z < 82$.

Descriptores: Línea de emisión; radioactividad de protones.

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1. Introduction

The latest research activity in nuclear physics aims to produce exotic nuclei with proton or neutron excess, and reach the limits of stability of matter beyond which a nucleon is no more bound. Whereas most of the neutron drip–line will still be unreachable for some time, since it is still impossible to produce in the lab the heavy elements that by fragmentation could reach the limit of neutron excess, the same is not true for protons. The proton drip–line has been mapped extensively in the region of low and intermediate nuclear charges, and most recently [1, 2] for charges within $50 < Z < 82$.

One of the reasons for this success, is the easier accessibility of the proton drip–line that due to Coulomb force is not so distant from the stability valley.

New physical phenomena emerged from these studies. In the region of light nuclei, candidates for one and two proton halos have been suggested, but there seems to be no experimental support for them. Many reactions at the proton drip–line between light and intermediate nuclei, are of great importance for astrophysical considerations and to understand nucleosynthesis and energy generation. In fact, the path of rapid proton $rp$ processes go through nuclear reactions and excitation of resonances at the border of proton stability.

Exotic decay modes have been observed when approaching the limit of proton stability for higher atomic numbers, like $\beta$–delayed emission of protons, and proton radioactivity found in nuclei lying beyond the proton drip–line. This new form of radioactivity are in the mass region of heavy nuclei, where the Coulomb barrier is very high and the proton is trapped by the barrier in a resonance state which will decay, in a process almost analogous to $\alpha$ decay. The tunnelling through the barrier is quite long, therefore the decay widths of these resonances are quite narrow of the order of $10^{-16}$–$10^{-22}$ MeV. The escape energy of the emitted proton is also very small, around 1 MeV, therefore these resonances lie very low in the continuum, and correspond essentially to single particle excitations, in contrast with what happens in stable nuclei.

Different shapes for proton radioactive nuclei were observed, ranging from spherical emitters [3], to nuclei with quite large deformations. Most part of these decays occur from the ground state, but decay from isomeric excited states of the parent nucleus, have also been observed. For some emitters, the daughter nucleus can have a first excited $2^+$ rotational state very low in energy, to which decay is also possible, referred as fine structure.

The experimental information gathered on the observed exotic decays, and in the particular case of proton radioactivity is invaluable to learn about nuclear structure properties of exotic nuclei, and to predict their resonance properties.

In this work we will briefly review our theory for odd–even and odd–odd deformed proton emitters, based on the exact calculation of Nilsson resonances and discuss the importance of the choice of the single particle potential in the evaluation of decay widths. The importance of the Coriolis coupling and the pairing residual interaction are also addressed.

2. Proton radioactivity

2.1. Spherical emitters

The first approach to study proton emitting nuclei is to consider them as spherical as spherical and discuss tunnelling through the barrier [3]. A simple WKB estimation of the transmission through the Coulomb and centrifugal barriers could already suggest the order of magnitude of the decay rates and the angular momentum of the decaying state. The
most important contribution to the WKB integrand is defined by the Coulomb and centrifugal barriers, and it is almost independent of the nuclear interaction.

It is possible to define experimental spectroscopic factors [3] as the rate between the calculated half-lives, with a simple or an improved version of WKB approach, and the measured ones. Since spherical nuclei display a vibrational spectrum with some unharmonicity, a correlation is possible between the outgoing proton and the lowest $2^+$ excited state of the daughter nucleus, as observed in $^{145}$Tm [4]. The spectroscopic factor is very sensitive to this vibrational coupling, which has to be included in the calculation of half-lives [5].

From the theoretical point of view, the spectroscopic factors can be evaluated within the independent quasiparticle BCS approach, where they simply represent the probability that the spherical orbital is empty in the daughter nucleus. A comparison between experimental and theoretical spectroscopic factors for odd $Z$ and even or odd $N$ proton emitters has shown [5] a good correlation between them, specially for nuclei with charge $Z > 68$. However, there are exceptions indicating a different tunnelling probability or fragmentation of the single particle strength, which was attributed to deformation as predicted from theoretical considerations [6]. Decay from deformed emitters cannot be described by a WKB calculation, but requires an appropriate theory.

2.2. Decay widths for deformed Odd-Z even-N emitters

Nuclei on the drip–line have a Fermi level very close or even immersed in the continuum, therefore, decay of odd–Z even–N nuclei has been interpreted as decay from a single particle Nilsson resonance close to the Fermi surface, of the unbound core–proton system. These resonances are obtained solving exactly the Schrödinger equation [7] with outgoing wave boundary conditions for each partial wave. The parent nucleus can be considered within the one particle plus rotor in the strong coupling limit [8]. This corresponds to a rotor with infinite moment of inertia, and a degenerate excitation spectrum. The partial decay width can be determined from the overlap between the initial and final states, which in the case of decay to the ground state

$$\Gamma_{j_p}^m(r) = \frac{\hbar^2 k}{\mu(j_p+1/2)} \left| \frac{RI_{j_p}^m(r)}{G_{j_p}(kr) + iF_{j_p}(kr)} \right|^2 u_m^2,$$

(1)

since only the component of the s.p. Nilsson wave function with the same angular momentum as the ground state of the parent nucleus contributes. The functions $F$ and $G$ are the regular and irregular Coulomb functions, respectively, and $RI_{j_p}^m$ the component of the single particle wave function with momentum $j_p$, equal to the spin of the decaying nucleus, $j_p = J_i = m$. The probability that the single particle level $m$ is empty in the daughter nucleus is given by $u_m^2$, evaluated in the BCS approach.

The decay width of Eq. (1) depends on deformation, and is very sensitive to the wave function of the decaying state. Therefore, if it is able to reproduce the experimental value, will give clear indication of deformation and properties of the decaying state. In the case of decay to excited states, few combinations are permitted for $l_{j_p}$ according to angular momentum coupling rules, and consequently different components of the parent wave function are then tested.

These calculations rely on single particle potentials to describe the deformed mean field, and have a phenomenological origin. There are different choices that equally give reasonable fits of the data on single particle properties of deformed nuclei, as seen in Ref. [9]. The comparison of s. p. energies evaluated from them [10] show a different ordering of spherical levels, a fact important for decay, since the Fermi level is the level occupied by the decaying nucleon its position is crucial.

Inserting in Eq. (1) the decay functions corresponding to Nilsson states lying close to the Fermi level, for the various parameterizations of the s. p. potentials, the decay widths, and corresponding half–lives, can be determined and compared with the experimental data. The deformed spin-orbit part of the potential, requires some transformations in order to obtain very precise resonance solutions of the Schrödinger equation [7], needed for a safe comparison with the experimental data.

The total half–lives for decay to ground state, obtained with the various potentials are shown in Fig. 1 for $^{131}$Eu, a well deformed proton emitter, with a predicted quadrupole deformation $\beta_2 = 0.33$ [6], and a sizeable branching ratio for decay to the $2^+$ excited state of $^{130}$Sm known as fine structure. The half–lives are perfectly described inside the limits of experimental uncertainties, with a deformation close to the predictions of Ref. [6] using any of the different parameterizations of the s. p. potentials, with the exception of the Becchetti–Greenlees potential [11]. It has a quite small radius parameter and interactions with larger radii were usually adopted uniformly for spherical and deformed systems in a consistent determination of the experimental spectroscopic factors [5]. The ordering of levels for this potential is very odd, therefore the Nilsson wave functions and decay widths become very unreasonable. It was fitted to scattering data and it is difficult to expect that its application to proton radioactivity, should work well.

Fine structure can be also studied using the Nilsson resonances [12], and the corresponding branching ratio is also shown in Fig. 1. The strong oscillating behavior of the Becchetti–Greenlees results are related to a change of sign of the $d3/2$ component of the wave function [13].

We have applied [13–15] our model to all measured odd–even deformed proton emitters including isomeric decays. The results are shown in Table I. The experimental half–lives are perfectly reproduced by a specific state, with well defined quantum numbers and deformation, thus leading to unambiguous assignments of the angular momentum of the decaying states [13, 14]. Extra experimental information provided by isomeric decay observed in $^{117}$La, $^{141}$Ho and $^{151}$Lu, and fine structure in $^{131}$Eu can also be successfully accounted by the model. The experimental half–lives for decay from the
FIGURE 1. Total half-lives for decays from the $3/2^+$ ground state of $^{131}\text{Eu}$, and branching ratio for the decay of $^{131}\text{Eu}$ to the $2^+$ states of $^{130}\text{Sm}$, calculated with different potentials. The experimental result [2, 17, 23] is within the shaded area, and the arrow indicates the deformation predicted in Ref. [6].

TABLE I. Total angular momentum and deformation that reproduce the experimental half-lives for the measured deformed odd–even proton emitters compared with the predictions of [6]. The theoretical results are from Refs. [13–15]. The label $m$ refers to decays from isomeric states.

<table>
<thead>
<tr>
<th>Proton decay</th>
<th>Möller–Nix</th>
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<tbody>
<tr>
<td>$^{109}\text{I}$</td>
<td>1/2+ 0.14</td>
</tr>
<tr>
<td>$^{113}\text{Cs}$</td>
<td>3/2+ 0.15 0.20</td>
</tr>
<tr>
<td>$^{117}\text{La}$</td>
<td>3/2+ 0.20 0.30</td>
</tr>
<tr>
<td>$^{117m}\text{La}$</td>
<td>9/2+ 0.25 0.35</td>
</tr>
<tr>
<td>$^{131}\text{Eu}$</td>
<td>3/2+ 0.27 0.34</td>
</tr>
<tr>
<td>$^{141}\text{Ho}$</td>
<td>7/2− 0.30 0.40</td>
</tr>
<tr>
<td>$^{141m}\text{Ho}$</td>
<td>1/2+ 0.30 0.40</td>
</tr>
<tr>
<td>$^{151}\text{Lu}$</td>
<td>5/2− −0.18 −0.14</td>
</tr>
<tr>
<td>$^{151m}\text{Lu}$</td>
<td>3/2+ −0.18 −0.14</td>
</tr>
</tbody>
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excited states were reproduced in a consistent way with the same deformation that describes ground state emission.

2.3. Decay from odd-Z odd-N nuclei

Emission from deformed systems with an odd number of protons and neutrons can be discussed in a similar fashion [16]. The decaying nucleus is described by a wave function of two particles–plus–rotor model in the strong coupling limit represented in terms of the single particle functions of the odd nucleons. However, in contrast with decay from ground state of odd-even nuclei where the proton is forced to escape with a specific angular momentum, many channels will be open due to the angular momentum coupling of the proton and daughter nucleus, $\vec{J}_d + \vec{j}_p$, giving the total width for decay as a sum of partial widths allowed by parity and momentum conservation,

$$\Gamma_{J_d} = \sum_{j_p = \max(|J_d - K_T|, K_p)} \Gamma_{I_p j_p}^{J_d K_T}$$

where the width for decay in the channel $I_p j_p$ is given by,

$$\Gamma_{I_p j_p}^{J_d K_T} = \frac{\hbar^2 k}{\mu} \left( \frac{2J_d + 1}{2K_T + 1} \right) |\langle J_d, K_n, j_p, K_p | K_T, K_T \rangle|^2 \times \frac{|R_{I_p j_p}^{K_T}(r)|^2}{|G_i(kr) + iF_i(kr)|^2} u_{K_p}^2$$

with $u_{K_p}^2$ the BCS spectroscopic factor. The quantity in brackets is a Clebsch-Gordan coefficient resulting from the angular momentum coupling of the odd nucleons, and $K_T = |K_p \pm K_n|$ the spin of the bandhead state of the decaying nucleus. Since the neutron intrinsic state does not change during decay $K_d = K_n$. The total decay width depends on the quantum numbers of the unpaired neutron which cannot be considered only a spectator, but influences significantly, with its angular momentum, the decay.

The proton and neutron single particle Nilsson levels in $^{112}\text{Cs}$ are depicted in Fig. 2. The neutron Fermi level is at

**FIGURE 2.** Proton a) and neutron b) Nilsson levels in $^{112}$Cs. The dotted lines denote the Fermi surface.

**FIGURE 3.** Half-life for decay from the ground state of $^{112}$Cs as a function of deformation, for neutrons in $K_n = 3/2^+$ and $5/2^+$ states, with $K_T = K_p + K_n$, (a), and $K_T = |K_p - K_n|$, (b). The experimental value [22] is within the shaded area, while the bars drawn at a generic $\beta$, indicate typical uncertainties coming from the error in the experimental energy and $u^2$.

levels with $K_n = 1/2^+, 3/2^+$ or $5/2^+$ according to deformation, whereas for protons we took $K_p = 3/2^+$ since it reproduces [13] the decay of $^{113}$Cs. The corresponding half-lives evaluated from Eq. (2) and (3), are shown in Fig. 3, including the BCS spectroscopic factors calculated according Ref. [14].

They are displayed separately in the two possible coupling cases $K_T = K_p \pm K_n$. Since in each coupling the neu-
tron single particle level leads to quite different factors and intermediate partial widths, the half–lives depend strongly on these quantities. The experimental value is reproduced considering the odd neutron in states with $K_n = 3/2^+, 5/2^+$, with corresponding deformations $\beta > 0.1$ with $K_T = 3^+$, and $\beta > 0.2$ with $K_T = 4^+$, respectively. When the proton and neutron are antiparallel, the same neutron states give $\beta >$ with corresponding deformations $\beta >$ and $\beta >$ with experiment found in the adiabatic context was lost. The results differ by factors of three or four from the experiment, and even the branching ratio for fine structure decay is not reproduced [18–20]. Decay rates in deformed nuclei, are extremely sensitive to small components of the wave function. The Coriolis interaction mixes different Nilsson wave functions, and can be responsible for strong changes in the decay widths. We proved [21] that a correct treatment of the pairing residual interaction in the BCS approach, can modify this mixing of states which should be considered between quasiparticle states instead of particle ones as used in Refs. 18-20. Such calculations bring back the perfect agreement with data of the strong coupling limit.

4. Conclusions

We have shown how proton radioactivity from deformed nuclei can be well understood within a consistent theoretical approach, as decay from single particle Nilsson resonances. All available experimental data on even–odd and odd–odd deformed proton emitters from the ground and isomeric states, as well as the data on fine structure, were accurately and consistently reproduced, identifying the decay level and deformation of the decaying nucleus. For decay from odd–odd deformed nuclei, there is a strong dependence of the decay width on the quantum numbers of the unpaired neutron state, which cannot be considered only a spectator. The resonance states and their corresponding half–lives were evaluated exactly using single particle potentials that fit large sets of data on nuclear properties. Aside from the Bechetti–Greenlees model, most parameterizations of the single particle mean field, can reproduce the experimental data on exotic nuclei in the region of the proton drip line within experimental uncertainties. This holds for both spherical and deformed nuclei, providing a unified description of decay. We have also shown that it is possible to go beyond the description of the decaying nucleus as a particle–plus–rotor in the strong coupling limit and take into account the effect of Coriolis mixing.

The various techniques developed to understand proton radioactivity, will certainly be useful to interpret future data, and to explore the structure of exotic nuclei in the region of the proton drip line.

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