Elastic-scattering and reaction cross section analysis of the $^{12}$C+$^{12}$C system at low energies

T.L. Belyaeva$^{a}$, E.F. Aguilera$^{a,b}$, and R. Perez-Torres$^{a,b}$

$^a$Universidad Autónoma del Estado de México, C.P.50000 Toluca, México
$^b$Depto. del Acelerador, Instituto Nacional de Investigaciones Nucleares, Apartado postal 18-1027, C.P. 11801 México, D.F., México

Recibido el 20 de enero de 2004; aceptado el 23 de mayo de 2004

An optical model analysis of the intermediate resonant structure for the $^{90}$-elastic scattering and fusion excitation function at the low energy region $E_{c.m.} = 2.5 \sim 19$ MeV has been performed. Optical potentials with variable geometry were applied on the basis of a regular fit of the angular distributions at these energies. Calculations show that a strongly energy dependent interference picture, observed in the elastic scattering of $^{12}$C+$^{12}$C ions, can be successfully reproduced in the framework of the optical model. The resulting optical potential with an energy dependent adjusted power parameter and an also energy dependent imaginary strength $W$ is valid to give a good description of the individual elastic-scattering angular distributions and to reproduce detailed features of the elastic-scattering and reaction excitation functions.

Keywords: Elastic scattering; fusion; $^{12}$C+$^{12}$C; optical model; resonances.

Se analizó con el modelo óptico la estructura resonante de las funciones de excitación de dispersión elástica en 90° y de fusión, observada a bajas energías $E_{c.m.} = 2.5 \sim 19$ MeV. Se aplicaron potenciales ópticos con geometría variable para ajustar distribuciones angulares de dispersión elástica a estas energías. Los cálculos muestran que la dependencia energética de las secciones eficaces, observada en la dispersión elástica del sistema $^{12}$C+$^{12}$C, se puede reproducir exitosamente usando el modelo óptico. El potencial óptico se obtuvo con una dependencia energética de dos parámetros: el parámetro de geometría de potencial y la intensidad imaginaria, con las cuales se pueden reproducir características detalladas de las distribuciones angulares y posteriormente las funciones de excitación de reacción y de dispersión elástica.

Descriptores: Dispersion elástica; fusión; $^{12}$C+$^{12}$C, modelo óptico; resonancias.

PACS: 24.10.Ht; 25.70.Bc; 25.70.Ef

1. Introduction

Experimental and theoretical studies of a resonant structure observed in the elastic scattering and reaction excitation functions in the $^{12}$C+$^{12}$C system at a broad energy region have been conducted since the beginning 1960’s, when nuclear quasimolecular states were experimentally discovered by Bromley, Kuehner, and Almqvist [1, 2]. Now the experimental data for the $^{12}$C+$^{12}$C system at low energies, down to energies of astrophysical interest? Are they deep or shallow, what geometrical forms are more acceptable? How can we describe correctly the angular distributions and excitation functions of the elastic scattering and fusion simultaneously over a sufficiently
wide energy range? What is the role played by the imaginary potential responsible for nuclear absorption in nonelastic channels at low energies?

These questions are taken up in this paper. We carried out an optical model analysis of the experimental elastic scattering and fusion data, which contain the elastic angular distributions at energies $E_{c.m.}=6.44–18.3$ MeV [14–16], the $90^\circ$–elastic-scattering excitation function in the energy region $E_{c.m.}=3–18$ MeV [17,18] and the fusion excitation function within the energy region $E_{c.m.}=2.5–9$ MeV [8–13]. Optical potentials with variable geometry were applied to perform a regular fit of the angular distributions. We carry out the OM elastic scattering calculations using the computer code OPTICA [18].

2. Analysis and results

In the framework of the optical model the elastic-scattering differential cross sections for identical spin-zero particles may be written as [3]

$$
\frac{d\sigma}{d\Omega_{el}} = |f_C(\theta) + f_C(\pi - \theta)|^2 + \frac{i}{k} \sum_{l=even} (2l+1) \exp(2i\sigma_l) (1 - S_l) P_l(\cos \theta) |^2,
$$

where

$$
f_C(\theta) = \frac{\gamma}{2k} \exp \left(-i\gamma \ln \sin^2 \theta / 2 + 2i\sigma_0 \right),
$$

and $\gamma = e^2 Z^2 / \hbar v = 0.1574 Z^2 \sqrt{\mu / E_{c.m.}}$ is the Coulomb parameter, $\mu$ is the reduced mass, $\sigma_l$ and $S_l$ are the Coulomb phase shifts and S-matrix elements for the partial wave $l$.

Eq. (1) contains a coherent summation over symmetrized Coulomb and nuclear amplitudes (direct and reflected terms). The symmetrized Coulomb amplitude is referred to as a Mott amplitude. Partial-wave S-matrix elements for complex optical potentials

$$
S_l = |S_l| \exp(2i\delta_l) = \eta_l \exp(2i\delta_l)
$$

are expressed in terms of the nuclear reflection coefficients $\eta_l = |S_l|$ and nuclear phase shifts $\delta_l$.

The total reaction cross section for identical particles, in accordance with the OM

$$
\sigma_R = \frac{2\pi}{k^2} \sum_{l=even} (2l+1)(1-\eta_l^2) = \frac{2\pi}{k^2} \sum_{l=even} (2l+1)T_l,
$$

is defined in terms of nuclear transmission coefficients $T_l = 1 - \eta_l^2$ corresponding to the l-wave elastic-scattering $S_l$-matrix elements.

To describe the elastic scattering angular distributions and then to obtain the excitation function, we apply an optical model formalism with the complex nuclear OP of the following form:

$$
V(r) = V f_V(r) + i W f_W(r),
$$

$$
f_n(r) = \{1 + \exp [(r - R_n) / a_n]\}^{-\nu_n}, \ n = V, W
$$

An additional geometrical parameter, the power $-\nu_n$, was introduced to describe with more flexibility a slope of the potential curve in the superficial range. This modification of the conventional Woods-Saxon geometry by the choice of some real adjustable power of the potential form-factor, was made for the first time by Michel and Vanderpoorten [19] in OM analyses of anomalous large-angle scattering (ALAS) of $\alpha$-particles on $^{40}$Ca. We used this type of modification of OM potentials to describe the $\alpha$-particles ALAS on $^{12}$C and $^{27}$Al [20]. Another argument for this choice is that a proper shape and strength of real OPs for the $^{12}$C+$^{12}$C system, which are given by the folding model using a realistic interaction, differ from the conventional Woods-Saxon form [21]. The real potential that has the square of a Woods-Saxon shape was recently used with success by Boztosun and Rae [22] for an inelastic excitation function analysis for the $^{12}$C+$^{12}$C system over a wide incident energy range from 32.0 MeV to 126.7 MeV.

The basic set of OP parameters was chosen in accordance with our previous calculations of the interaction potential, the wave functions and reduced widths for quasistationary states in $^{24}$Mg formed by $^{12}$C+$^{12}$C clusters in the $^{12}$C($^{14}$N,d)$^{24}$Mg reaction [23]. The real part of this deep potential with parameters: $V = 300$ MeV, $R_V = 4.235$ fm, $a_V = 0.6$ fm, is appropriate to reproduce the excited energies of a number of quasistationary cluster states in $^{24}$Mg. The real strength of this OP is in perfect accordance with the OP found by Kondo, Brandan and Satchler [24]. Also it is in accordance with the argumentation of Brandan and Satchler [6] that the potentials found appropriate for light heavy-ion systems must be deep and diffuse. The imaginary part of our OP has a standard Woods-Saxon shape ($\eta_{\nu} = 1$) with fixed geometric parameters $R_{\nu} = 6.25$ fm, $a_{\nu} = 0.8$ fm. The Coulomb interaction is represented by the potential of a uniformly charged sphere with a radius of 6.34 fm.

Figures 1-3 show some examples of the OM analysis of the angular distributions for the elastic $^{12}$C+$^{12}$C scattering at $E_{c.m.} = 6.44 – 18.3$ MeV. Based on the calculations of the individual angular distributions (solid lines in Figs. 1-3), we will construct later the excitation functions and make a conclusion about the energy dependence of the varying parameters. Also in Figs. 1-3 we show OM calculations (dashed lines) that were done by applying one of the optical parameter sets presented in [25]. This potential set was developed from an analysis of higher energy data, above $E/A = 6$ MeV, and its application for the high energy range gave excellent results. Extrapolation of this OP set to the lower energy range presents some troubles, since the elastic angular distributions are not reproduced adequately. This is one of the problems.
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Figure 1. Comparison between OM calculations of the elastic scattering angular distributions for the $^{12}$C+$^{12}$C system and the experimental data [14] at $E_{c.m.} = 6.44 \sim 6.99$ MeV. The solid and dashed lines show the results obtained using the OP sets from the present paper and from [25]. The dashed-dotted curve shows calculations performed with the pure Coulomb interaction ($V = W = 0$), the dotted curve shows calculations performed with the Coulomb plus the nuclear real potential ($W = 0$), the dashed-dotted-dotted curve shows calculations performed with the Coulomb interaction plus the nuclear absorptive potential ($V = 0$).

that stimulated our search for optical potentials more acceptable at the low energies.

The comparison of the results of OM calculations with the detailed experimental data obtained at Yale University (Korotky et al. [14]) at $E_{c.m.} = 6.44 \sim 6.99$ MeV, in the vicinity of the Coulomb barrier top is shown in Fig. 1. We take into account the barrier height of 6.72 MeV and radius of 7.78 for the $^{12}$C+$^{12}$C system, which were approximated in [26]. The main structure of the angular distributions presented in Fig. 1 is formed due to a constructive interference of the direct and reflected Coulomb amplitudes (Eq. (1)), but the detailed values of the cross sections in the minima and maxima depend strongly on the contribution of the nuclear interaction. To illustrate this statement we present in Fig. 1 for $E_{c.m.} = 6.64$ MeV some calculations performed with the pure Coulomb interaction ($V = W = 0$) (the dashed-dotted curve), with the Coulomb interaction plus the nuclear absorptive potential ($V = 0$) (the dashed-dotted-dotted curve), and with the Coulomb interaction plus the nuclear real potential ($W = 0$) (the dotted curve). Comparison between the results obtained with our OP set and OP from [25] presented in Fig. 1 shows that despite the reasonable description that both OP sets produce, the main difference between these two OM calculations is shown in the minima and in the maxima of the angular distributions, which are more sensible to the nuclear interaction.

Figures 2,3 show a comparison between calculated and experimental angular distributions at energy regions $E_{c.m.} = 10.5 \sim 14$ MeV and the experimental points are from Ref. [15].

Figures 2,3 show a comparison between calculated and experimental angular distributions at energy regions $E_{c.m.} = 10.5 \sim 14.1$ MeV and $14.8 \sim 18.3$ MeV, respectively, using the same sets of OP parameters. The experimental data obtained by Emling et al.[15] and Ledoux et al. [16], respectively, were used. Calculations performed without nuclear interaction (pure Coulomb potential) and with the Coulomb

plus the nuclear real potential \((W = 0)\) are shown in Fig. 2 by dashed-dotted and dotted curves. These examples demonstrate that for the \(^{12}\text{C} + ^{12}\text{C}\) system at energies approximately less than 11-12 MeV the angular distributions are formed due to the Coulomb interaction and afterwards due to an interference of Coulomb and nuclear amplitudes. At higher energies the structure of the angular distributions near 90° is formed predominantly by the nuclear interaction, whereas the Coulomb interaction dominates at forward angles (and also at backward angles for identical ion systems). Our calculations showed that the energy value of 11-12 MeV (about twice the Coulomb barrier value) indicates approximately a threshold dividing the low and the higher energy regions.

An interpolation and extrapolation of the above OP parameters for the whole energy region from 2.0 MeV up to 19 MeV allows us to reproduce the 90°− elastic scattering excitation function. A comparison between our OM calculations and the experimental data [1,17] for \(E_{\text{c.m.}} = 3-18.3\) MeV is shown in Fig. 4(a). We assume that only two adjusted parameters of the proposed phenomenological OP exhibit an energy dependence. We note also a strong sensitivity of the calculated cross sections to the optical-potential parameters that describe a potential curve slope in a surface nuclear region, \(i.e.,\) the real diffuseness \(a_V\) and the power parameter \(\nu_V\). Our calculations of the angular distributions with \(\chi^2\)-fits showed that the potential families with varying \(W\) and \(\nu_V\) or \(W\) and \(a_V\) pairs, as adjusted parameters, present the best results. In this paper we discuss results obtained for the first pair of parameters only, taking into account the energy dependence of two parameters: the imaginary strength \(W\) responsible for nuclear absorption processes, and the power parameter \(\nu_V\) related to a dynamic

**Figure 3.** Same as in Fig. 1 except that \(E_{\text{c.m.}} = 14.8\sim 18.3\) MeV and the experimental points are from Ref. [16].

**Figure 4.** (a) Comparison of OM calculations of the 90° elastic scattering excitation function with the experimental data [1,17]. The energy points corresponding to an incorporation of the next even grazing partial wave are indicated by arrows. (b) The deduced energy dependence of the imaginary strength \(W\) (circles) and an analytical approximation to the average behavior of \(W\) (the dotted line). The dashed curve corresponds to the energy dependence of \(W\) obtained in [25]. (c) The deduced energy dependence of the power parameter \(\nu_V\) (squares) and its analytical approximation (the solid line).
deformation of the nuclear shape during a collision. The deduced energy dependence of the imaginary strength \( W \) is shown in Fig. 4(b). An average \( W \) behavior for this energy region can be interpolated by an analytical expression, for example, of the Boltzman-type: 

\[
-\frac{dW}{dE} (\text{MeV}) = A_2 + \frac{A_1 - A_2}{1 + \exp\left(\frac{E - E_0}{dE}\right)},
\]

where \( A_1 = 0.0, \quad A_2 = 3.71, \quad E_0 = 8.72 \text{ MeV}, \quad dE = 1.01 \text{ MeV} \) (the dotted line in Fig. 4(b)). This approximation is available for the low energies up to 2 MeV and describes a rapid localized increase of \( |W(E)| \) between \( E_{\text{cm}} = 6 \) and 11 MeV. For the greater energies \( W(E) \) tends to level off, which indicates a saturation of absorption after a rapid increase of the direct processes. The energy dependence of the power parameter \( \nu_W \) at \( E_{\text{cm}} = 2 - 19 \text{ MeV} \) is shown in Fig. 4(c). This set of OP parameters is characterized by a smooth variation of \( \nu_W \) between 1.0 and 1.28 with the maximum value near \( E_{\text{cm}} = 10 \text{ MeV} \), near a threshold dividing the low and the higher energy regions.

As Fig. 4 indicates there exists a strong correlation between resonances of the elastic excitation function and oscillations of the absorption potential (the imaginary strength \( W \)) as well as oscillations of the power parameter \( \nu_W \). It should be noted that the maxima of the elastic excitation functions correspond to the energy regions more transparent to interacting nuclei (the minima of \( |W(E)| \)), and vice versa, the minima of the elastic excitation functions indicate an increased absorption, corresponding, for example, to formation of some short-living quasistationary states (the maxima of \( |W(E)| \)). The resonant structure of the excitation function of 1- to 4-MeV width may be understood by examining the partial wave of maximum angular momentum participating in the interaction as a function of the bombarding energy. This phenomenon is referred to as orbiting resonances [27], related to an incorporation of a new grazing partial wave \( l_{gr} \) with a given energy increase. One can detect this fact by calculating the corresponding partial transmission coefficient \( T(l_{gr}) \) for sufficient energy increase. For the \( ^{12}\text{C}+^{12}\text{C} \) scattering the energy points corresponding to an incorporation of the next even grazing partial wave are indicated by arrows in Fig. 4(a). Oscillations in the energy dependence of \( \nu_W \) are, generally, out of phase with oscillations of \( |W(E)| \). Thus, the energy dependencies of the imaginary strength \( W(E) \) and the real part of the OP (through the \( \nu_W(E) \)) are in accordance with the phenomenon known as “threshold anomaly”, describing the variation with energy of the imaginary and real parts of the interaction potential near any energetic “threshold”. Fig. 5 compares the energy dependence of the obtained real and imaginary parts of the OP for the \( ^{12}\text{C}+^{12}\text{C} \) system at \( R = 8.1 \text{ fm} \), near the strong absorption radius [6]. A rapid growth of \( |W(E)| \) at the energies just above the Coulomb barrier reflects the fact of a rapid increase of the open nonelastic channels at this energy interval. Simultaneously, as it follows from general dispersion relations, a rapid variation with energy of the absorptive potential implies a corresponding variation of the real potential. The energy dependence of \( V(r) \) is caused by the energy dependence of a slope of the potential curve in the superficial range due to a dynamic deformation of the nuclei during an ion-ion interaction. Indeed, quasistationary, short-living states, which are manifested as resonances in the excitation functions for the \( ^{12}\text{C}+^{12}\text{C} \) system, are formed by excited carbon clusters, arranged in some deformed molecular-like configurations. These processes are simulated effectively by the power parameter \( \nu_W \) energy dependence.

It is interesting to compare the obtained phenomenological OP set with other OPs in terms of the values of the volume integrals \( J_V \) and \( J_W \) per interacting ion pair and mean-square radii, defined as

\[
J_f = -\frac{4\pi}{A_p A_t} \int f_E(r) r^2 dr,
\]

\[
\langle r^2 \rangle_f = \left( \int f_E(r) r^4 dr \right) \left( \int f_E(r) r^2 dr \right)^{-1},
\]

where \( f = V, W \) and \( A_p, A_t \) denote the projectile and target mass numbers, respectively.

Figure 6 presents the energy dependence of the imaginary volume integrals per interacting ion pair for the \( ^{12}\text{C}+^{12}\text{C} \) system at \( E_{\text{cm}} = 2.0 - 19.0 \text{ MeV} \). An analytical approximation of the \( J_W \) energy dependence may be presented as the Boltzman-type expression:

\[
J_W = A_2 + \frac{A_1 - A_2}{1 + \exp\left(\frac{E - E_0}{dE}\right)},
\]

where \( A_1 = 0.0, \quad A_2 = 29.85, \quad E_0 = 8.55 \text{ MeV}, \quad dE = 1.01 \text{ MeV} \) (the solid line in Fig. 6). These \( J_W \) values, ranging between 0.15 and 40 MeV fm\(^3\), are consistent with the result reported in [6,24,25]. The values of the real volume integrals \( J_V \) (\( \geq 660 \text{ MeV fm}^3 \) at \( E_{\text{cm}} = 2.0 - 19.0 \text{ MeV} \)) calculated with our OP set are significantly larger than those proposed in [22,24,25] from the folding and phenomenological analysis at higher energies. Large real volume integrals are not exceptions for the deep OPs developed for the scattering of light projectiles with \( A_p = 1 - 6 \). For example, one can compute corresponding \( J_V \) using the well known optical model parameter compilation of Perey and Perey [28]. The double folding potentials...
of radial parameters (for example, by decreasing $R_V$ and increasing $\nu_V$), which is reasonable in view of a greater penetration of colliding ions into each other at high energies.

The present OM analysis shows that the obtained set of OP parameters provides a good agreement between the experimental data [8-13] and theory for the fusion excitation function at low energies $E_{c.m.} = 2.5-9$ MeV (see Fig. 7). At these energies the fusion cross section $\sigma_{Fus}$ is almost equal to the reaction cross section $\sigma_R$, and we are able to use Eq.(4) to calculate the latter and to make a comparison with the experimental fusion data. This is not correct for the higher energies, where many direct, quasi-direct, etc. channels become available, and $\sigma_{Fus} \ll \sigma_R$. So the OM reaction cross sections in the higher energy region, in general, are greater than the experimental total fusion ones.

3. Conclusions

Our calculations show that the strongly energy-dependent interference picture, observed in the elastic scattering of $^{12}$C+$^{12}$C ions, can be successfully reproduced in the framework of the optical model by doing two simple modifications of the standard OM: inclusion of an energy dependent adjusted power parameter $\nu_V$ and an also energy dependent imaginary strength $W$. The developed phenomenological optical potential allowed us to reproduce the individual elastic angular distributions at energies $E_{c.m.} = 6.44-18.3$ MeV, the detailed features of the $90^\circ$-elastic-scattering excitation function in the energy region $E_{c.m.} = 3-19$ MeV and the fusion excitation function within the energy region $E_{c.m.} = 2.5-9$ MeV. This OP set is characterized by a deep real potential strength and a relatively large real interaction radius, which makes it similar to the light-ion optical potentials at low energies. A variable geometrical form of the real potential, defined by an additional geometrical parameter, the power $\nu_W$, allows us to describe with more flexibility a slope of the potential curve in the superficial range. Our calculations demonstrate a strong sensitivity of the calculated cross sections to the slope of the potential curve in the surface nuclear region, which in our case led to the power parameter $\nu_W$ variation. The deduced energy dependence of the imaginary strength $W$ and the corresponding analytical approximation are available for the low energies down to 2 MeV, a value that has not been reached yet experimentally. The energy dependence of $W$ is not linear, and is described by a small absorption for very low energies ($\leq 4$ MeV) and by a rapid localized increase of $|W(E)|$ between $E_{c.m.} = 6$ and 11 MeV. For the greater energies $|W(E)|$ tends to level off, which indicates a saturation of absorption after a rapid increase of the direct processes. The energy dependence of the power parameter $\nu_W$ at $E_{c.m.} = 2-19$ MeV is characterized by a smooth variation of $\nu_W$ between 1.0 and 1.28 with the maximum value near $E_{c.m.} = 10$ MeV.

The OM analysis showed that there exists a strong correlation between resonances of the elastic excitation function.
and oscillations of the imaginary strength $W$ as well as oscillations of the power parameter $\nu_V$. The latter are, generally, out of phase with oscillations of $|W(E)|$. This indicates that the energy dependencies of the imaginary strength $W(E)$ and the real part of the OP (through the $\nu_V(E)$) are in accordance with the dispersion relations. A rapid growth of absorption at the energies just above the Coulomb barrier is accompanied by a corresponding decrease of the real potential. The energy dependence of $V(r)$ is caused by the energy dependence of the slope of the potential curve in the superficial range, which simulates a dynamic deformation of the nuclear surface during an ion-ion interaction.

Acknowledgments

The authors are grateful to Professor D. Shapira and Professor M. S. Hussein for helpful discussions at the XXVII Symposium on Nuclear Physics (Taxco, Mexico, 2004). This work was partially supported by CONACyT project 44251-F and UAEM project 1833/2004.

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