Double beta decay of light nuclei in the shell-model framework

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The two-neutrino double beta decays of nuclei \( ^{36}\text{Ar} \), \( ^{46}\text{Ca} \), \( ^{48}\text{Ca} \), and \( ^{50}\text{Cr} \) have been studied in the framework of the nuclear shell model. Well-established, fitted two-body interaction matrix elements have been used in the study. The structure of the few lowest \( 1^+ \) intermediate states in the two-neutrino double beta decays of calcium isotopes has been found to play a significant role.

**Keywords:** Double beta decay; nuclear shell model; light nuclei.

Los decaíemientos beta con dos neutrinos de los núcleos \( ^{36}\text{Ar} \), \( ^{46}\text{Ca} \), \( ^{48}\text{Ca} \), y \( ^{50}\text{Cr} \) han sido estudiados en el marco del modelo de capas nuclear. Hemos utilizado elementos de matriz de la interacción a dos cuerpos que han sido ajustados previamente y están bien establecidos. Se ha encontrado que la estructura de algunos estados intermedios más bajos \( 1^+ \) en los decaímentes doble beta a dos neutrinos de los isótopos del calcio participan significativamente.

**Descriptores:** Décimiento doble beta; modelo de capas nuclear; núcleos ligeros.

**PACS:** 21.60.Cs; 23.40.Bw; 23.40.Hc; 27.30.+t; 27.40.+z

1. Introduction

The results from the recent neutrino-oscillation measurements in SUPERKAMIOKANDE and SNO have shown that neutrinos have a non-zero mass. At present, the most practical way to access the absolute mass scale and Dirac/Majorana character of the neutrino, is to perform measurements of the neutrinoless double beta \( (\nu\beta\beta) \) decay of atomic nuclei. The \( 0\nu\beta\beta \) decay and the two-neutrino double beta \( (2\nu\beta\beta) \) decays are among the slowest reactions in nature. Both these modes have been studied carefully in the theoretical and experimental frameworks. However, currently only the \( 2\nu\beta\beta \) mode has been confirmed by the experiments.

One of the biggest challenges related to nuclear aspects of the double beta decay lies in the fact that most of the double-beta-decaying nuclei are medium-heavy or heavy nuclei. Therefore, the early attempts for nuclear-structure calculations of double-beta-decay, which were done in the shell-model basis, were done either in a heavily truncated configuration space or in the weak-coupling limit. Nevertheless, despite of the increased computer CPU power and memory capacity, nuclear-structure calculations for most of the double-beta-decaying nuclei in a realistic configuration space are out of reach today. For this reason other, more suitable methods have been widely used, the quasiparticle random-phase approximation and its variants being the most popular ones [1]. However, there are some double-beta-decaying nuclei where the shell-model calculations can be applied without suppressing the model space severely.

In the theoretical description of double beta decay, the transition proceeds via the virtual states of the intermediate double-odd nucleus. For the \( 2\nu\beta\beta \) mode only the \( 1^+ \) states are active, whereas in the case of the \( 0\nu\beta\beta \) mode all the multipole states of the intermediate nucleus are active. A reliable description of the structure of these intermediate states is essential in the theoretical calculations due to the fact that the half-lives of the double beta decay have a strong dependence on their structure. The structure of these intermediate states can be analyzed for example by performing electron capture or \( \beta \)-decay experiments. However, the drawback of these methods appears in the fact that they can be used only to probe the lowest \( J^+ \) state, typically the ground state of the odd-odd intermediate nucleus.

Recently it has been suggested that ordinary muon capture (OMC) reaction could be used to investigate the structure of the intermediate nucleus and its excited states [2, 3]. Due to the large mass of captured muon (roughly 100 MeV) the OMC reaction can lead to (highly) excited states of the final nucleus. However, this also leads to a more complicated theoretical treatment of this process as compared to the beta decay process. In contrast to the beta decay, the large mass of the muon also allows high transition rates for forbidden transitions. This can be used to analyze the structure of other than \( 1^+ \) states in the intermediate nuclei, like the \( 2^- \) states which are sometimes very relevant in the \( 0\nu\beta\beta \) decay.

In the present work we have studied the two-neutrino double-beta-decays of the \( ^{36}\text{Ar} \), \( ^{46}\text{Ca} \), \( ^{48}\text{Ca} \) and \( ^{50}\text{Cr} \) nuclei.

2. Theory

The \( 2\nu\beta\beta \) decay proceeds via the \( 1^+ \) states of the intermediate double-odd nucleus and the corresponding expression for the inverse half-life can be factorized as

\[
\left[ \langle \frac{1}{2}, J^\pi \rangle \phi_{(2\nu)}^{f} \phi_{(0\nu)}^{i} \right]^{-1} = G_{DGT}^{(2\nu)} | M_{DGT}^{(2\nu)} |^2, \tag{1}
\]

where \( G_{DGT}^{(2\nu)} \) is an integral over the phase space of the leptonic variables [1, 4]. The nuclear matrix element \( M_{DGT}^{(2\nu)} \), corresponding to the \( 2\nu\beta\beta \) decay, can be written as

\[
\langle \frac{1}{2}, J^\pi \rangle \phi_{(2\nu)}^{f} \phi_{(0\nu)}^{i} \]
\[ M^{(2\nu)}_{\text{DGT}} = \sum_n \left\langle \frac{1}{2} Q_{\beta\beta} + E_n - M_i \middle| M_i \middle| 0^+ \right\rangle, \]

where the transition operators are the usual Gamow-Teller operators, \( Q_{\beta\beta} \) is the \( 2\nu\beta\beta \) \( Q \) value, \( E_n \) is the energy of the \( n \)th intermediate state, \( M_i \) is the mass energy of the initial nucleus, and \( m_e \) is the rest-mass of the electron.

The calculations of all the nuclear states of the double-beta-decay processes were done by using the shell-model code OXBASH [5]. The calculated one-body transition densities were then inputs to the double-beta-decay nuclear matrix-elements. In this work we have used USD [6] and SDPOTA [7] interaction for the \( sd \)-shell nuclei \(^{36}\text{Ar}, \) \(^{36}\text{Cl}\) and \(^{36}\text{S}\). The nuclear structure for \( fp \)-shell nuclei were calculated by the FPBP [8] and FPMCC [9] interactions.

For the \( A = 50 \) nuclei, the \( JT \)-dimensions of the nuclear structure calculations were reduced heavily by requiring the minimum particle occupancy in the \( f_{7/2} \) orbital to be 8. However, it is not clear how accurate the results are after this kind of heavy truncation. Therefore, these results should be taken as order-of-magnitude estimates for the double-beta-decay half-life.

In the case of \( A = 36 \) all the intermediate \( 1^+ \) states have been taken into account. For the \( A = 46 \) isobar, the 75 lowest \( 1^+ \) intermediate states have been calculated and in the case of the \( A = 48 \) isobar, the 150 lowest \( 1^+ \) intermediate states have been taken into account. For the \( A = 50 \) isobar, the 300 lowest \( 1^+ \) intermediate states have been taken into account. The higher energy states do not yield much contribution to the double Gamow-Teller matrix element due to the energy denominator in the Eq. (2).

3. Results and discussion

To begin with, all our calculated \( 2\nu\beta\beta \)-decay results are listed in Table I. In all of our calculations we have adopted the renormalized value \( g_A = -1.0 \) for the axial-vector coupling constant. For the \( 2\nu\beta\beta \) decay of \(^{36}\text{Ar}\) the cumulative matrix elements are plotted in Fig. 1. In the earlier calculations of Nakada, Sebe and Muto [10] the effective Gamow-Teller operator has been used with the USD interaction. The resulting matrix element, \( M^{(2\nu)}_{\text{DGT}} \), is almost same as in the case of the SDPOTA interaction although the shapes of the curves are a bit different.

The cumulative sum of matrix element \( M^{(2\nu)}_{\text{DGT}} \) for \( 2\nu\beta\beta \) decay of \(^{46}\text{Ca}\) has been plotted in Fig. 2, for both the FPBP and FPMCC interactions. As can be seen, these interactions produce almost the same value for the matrix element, even though the cumulative curves have a different shape. This same phenomenon can also be seen in the other calculated \( fp \)-shell nuclei. From the cumulative curves of the matrix element \( M^{(2\nu)}_{\text{DGT}} \), we note that the few lowest intermediate states yield the biggest contribution to the total value of the matrix.

**Table I.** Calculated double-beta-decay matrix elements \( M^{(2\nu)}_{\text{DGT}} \), the Q-values in units of electron rest mass, phase-space factors in units of inverse years, and the corresponding half-lives in units of a year for the studied double-beta-decay transitions. The renormalized value \( g_A = -1.0 \) has been used in all cases.

<table>
<thead>
<tr>
<th>( \text{Element} )</th>
<th>( G^{(2\nu)}_{\text{DGT}}(\beta^-\beta^-) ) ([y^{-1}])</th>
<th>( Q(\beta^-\beta^-) ) ([m_e])</th>
<th>( M^{(2\nu)}_{\text{DGT}} )</th>
<th>( t_{1/2} ) ([y])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{46}\text{Ca}, \text{FPBP})</td>
<td>(4.8 \times 10^{-23})</td>
<td>1.94</td>
<td>0.108</td>
<td>(1.7 \times 10^{24})</td>
</tr>
<tr>
<td>(^{46}\text{Ca}, \text{FPMCC})</td>
<td>(1.0 \times 10^{-17})</td>
<td>8.36</td>
<td>0.058</td>
<td>(1.8 \times 10^{19})</td>
</tr>
<tr>
<td>(^{48}\text{Ca}, \text{FPBP})</td>
<td>(1.6 \times 10^{-17})</td>
<td>8.36</td>
<td>0.055</td>
<td>(2.1 \times 10^{19})</td>
</tr>
<tr>
<td>(^{48}\text{Ca}, \text{FPMCC})</td>
<td>(6.4 \times 10^{-28})</td>
<td>0.84</td>
<td>0.117</td>
<td>(1.4 \times 10^{29})</td>
</tr>
</tbody>
</table>

**Table II.** Calculated double-beta-decay matrix elements \( M^{(2\nu)}_{\text{DGT}} \), the Q-values in units of electron rest mass, phase-space factors in units of inverse years, and the corresponding half-lives in units of a year for the studied double-beta-decay transitions. The renormalized value \( g_A = -1.0 \) has been used in all cases.

<table>
<thead>
<tr>
<th>( \text{Element} )</th>
<th>( G^{(2\nu)}_{\text{DGT}}(\beta^-\beta^-) ) ([y^{-1}])</th>
<th>( Q(\beta^-\beta^-) ) ([m_e])</th>
<th>( M^{(2\nu)}_{\text{DGT}} )</th>
<th>( t_{1/2} ) ([y])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{36}\text{Ar}, \text{USD})</td>
<td>(5.1 \times 10^{-28})</td>
<td>0.84</td>
<td>0.117</td>
<td>(1.4 \times 10^{29})</td>
</tr>
<tr>
<td>(^{50}\text{Cr}, \text{FPMCC})</td>
<td>(4.9 \times 10^{-25})</td>
<td>2.27</td>
<td>0.097</td>
<td>(2.1 \times 10^{26})</td>
</tr>
</tbody>
</table>
element. Partly this can be explained by the energy denominator of Eq. (2). However, even without the energy denominator, these states are the most dominant ones. This can be seen in Fig. 3 where we have plotted the product of the two Gamow-Teller matrix elements, which appear in the Eq. (2), for the FPBP and FPMCC interactions. The beta strength functions for the case of $A = 46$ can be found in Fig. 4.

Here, the $B(GT^-)$ denotes the Gamow-Teller strength distribution in the transitions $^{46}$Ca $\rightarrow$ $^{46}$Sc and $B(GT^+)$ in the transitions $^{46}$Ti $\rightarrow$ $^{46}$Sc.

The $2\nu\beta\beta$ decay of $^{48}$Ca is the only case where our calculated results can be compared to the experimental data. According to Ref. [11] the experimental $2\nu\beta\beta$ half-life of $^{48}$Ca is $4.2^{+3.3}_{-1.3} \times 10^{19}$ y. By comparing the experimental result and our calculated results at Table I, we note that both interactions tend to underestimate the experimental value by a factor of two. Thus, the value of the matrix element $M_{DGT}^{(2\nu)}$ is overestimated. The cumulative sum of the matrix element $M_{DGT}^{(2\nu)}$ for the $2\nu\beta\beta$ decay of $^{48}$Ca can be seen in Fig. 5, for the cases of the FPBP and FPMCC interactions. Here, one can see that the contribution from the few lowest states is quite significant as compared to the other, higher-lying intermediate states, as in the case of the $2\nu\beta\beta$ decay of $^{46}$Ca.

This can be seen in Fig. 6, where we have plotted the product of the two Gamow-Teller matrix element of Eq. (2) in the cases of FPBP and FPMCC interaction. For this reason it is essential to test the structure of these states for example by performing OMC experiments [12]. These experimental results can be used to determine which interaction works best or how the present interactions should be modified. The beta strength functions, $B(GT^-)$ and $B(GT^+)$, for the $A = 48$ isobar can be found in Fig. 7.

Like noted earlier in the text, in the case of $A = 50$ isobar the $JT$-dimensions of nuclear-structure calculations were reduced. It is not clear how this truncation affects the nuclear matrix elements, and therefore these results should be taken as order-of-magnitude estimates. The cumulative matrix element has been plotted in the Fig. 8 for the FPBP and FPMCC interactions. Here, the contribution of the few lowest intermediate states to the total value of the matrix element

![Figure 2. The matrix element $M_{DGT}^{(2\nu)}$ summed up to the excitation energy $E_x$ of the intermediate nucleus $^{46}$Sc for the transition $^{46}$Ca($0^+_g.s.$) $\rightarrow$ $^{46}$Ti($0^+_g.s.$) by using the FPBP and FPMCC interactions.](image-url)

![Figure 3. Calculated values of the product $(0^+_i | \sigma | 1\lambda_i) (1\lambda_i | \sigma | 0^+_f)$ of the Gamow-Teller matrix elements in Eq. (2) for the transition $^{46}$Ca($0^+_g.s.$) $\rightarrow$ $^{46}$Ti($0^+_g.s.$) by using the FPBP and FPMCC interactions.](image-url)

![Figure 4. $GT^-$ strength function of $^{46}$Ca (above the abscissa) and $GT^+$ strength function of $^{46}$Ti (below the abscissa) in the case of the FPBP interaction.](image-url)
$M_{\text{DGT}}^{(2\nu)}$ is not so big as in the cases of the $2\nu\beta\beta$ decays of $^{46}\text{Ca}$ and $^{48}\text{Ca}$. The experiment of Ref. [13] yielded the result $t_{1/2} > 1.3 \times 10^{18}$ y for the $\beta^+\text{EC}$ decay of $^{50}\text{Cr}$ at 95% CL. This result is still far from the results of our shell-model calculation, shown in Table I.

![Figure 5](image1.png)

**Figure 5.** The matrix element $M_{\text{DGT}}^{(2\nu)}$, summed up to the excitation energy $E_x$ of the intermediate nucleus $^{48}\text{Sc}$ for the transition $^{48}\text{Ca}(0^+_2, s) \rightarrow ^{48}\text{Ti}(0^+_2, s)$ by using the FPBP and FPMCC interactions.

![Figure 6](image2.png)

**Figure 6.** Calculated values of the product $(0^+_1 || \sigma || 1^+_2)(1^+_1 || \sigma || 0^+_1)$ of the Gamow-Teller matrix elements in Eq. (2) for the transition $^{48}\text{Ca}(0^+_2, s) \rightarrow ^{48}\text{Ti}(0^+_2, s)$ by using the FPBP and FPMCC interactions.

![Figure 7](image3.png)

**Figure 7.** GT$^-$ strength function of $^{48}\text{Ca}$ (above the abscissa) and GT$^+$ strength function of $^{48}\text{Ti}$ (below the abscissa) in the case of the FPBP interaction.

![Figure 8](image4.png)

**Figure 8.** The matrix element $M_{\text{DGT}}^{(2\nu)}$ summed up to the excitation energy $E_x$ of the intermediate nucleus $^{50}\text{V}$ for the transition $^{50}\text{Cr}(0^+_2, s) \rightarrow ^{50}\text{Ti}(0^+_2, s)$ by using the FPBP and FPMCC interactions.

In a summary, the two-neutrino double-beta-decay half-lives for $^{36}\text{Ar}$, $^{46}\text{Ca}$, $^{48}\text{Ca}$, and $^{50}\text{Cr}$ has been calculated in the shell-model framework. It has been found that for the calculated $fp$-shell nuclei the FPBP and FPMCC interactions give almost the same result for the matrix elements, although the shapes of the cumulative curves are different. For the double-beta-decaying calcium isotopes the dominance of the few lowest $1^+$ intermediate states in the total sum of matrix element $M_{\text{DGT}}^{(2\nu)}$ has been noted.

12. Ch. Briancon, *et al.*, the R-97-03 experiment and its extension $\mu CR42/\beta$ at PSI.