One- and two-nucleon transfer reaction study of $^{196}$Au: supersymmetry in nuclear excitation spectra*

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To provide information for comparison with predictions for a dynamical supersymmetry, the odd-odd nucleus $^{196}$Au was studied via transfer reactions. With a polarized deuteron beam we measured $(d,t)$ and $(d,\alpha)$, with unpolarized beams $(p,d)$, $(^3He,d)$, and $(\alpha,d)$ transfer reactions. From the high resolution $^{197}$Au$(p,d)^{196}$Au spectrum a rather complete set of excitation energies was obtained. Quantum numbers and spectroscopic factors were obtained from angular distributions of single-neutron transfer in $^{197}$Au$(d,t)^{196}$Au, single-proton transfer in $^{195}$Pt$(^3He,d)^{196}$Au, and two-nucleon transfer $^{196}$Hg$(d,\alpha)^{196}$Au. Because of the $J^\pi = 0^+$ target in the case of the $(d,\alpha)$ reaction identified transfer quantum numbers provide unique information about the $J^\pi$ values of the excited states in $^{196}$Au. This is different for the $(d,t)$ and $(^3He,d)$ reactions where, because of the $J^\pi = 3/2^+$ and $1/2^-$ targets, the identification of transferred $lj$ values may provide only restrictions on the $J^\pi$ values in $^{196}$Au. In this way we obtain for 20 out of the 26 observed states with negative parity below 500 keV safe $J^\pi$ assignments. The number of states and the safe or restricted assignments are in agreement with the predictions from the dynamical $U_n(6/12) \otimes U_n(6/4)$ supersymmetric scheme. If we include also our $(\alpha,d)$ data we have spectroscopic factors for four different transfer channels. When model predictions of spectroscopic factors become available, these data will provide a further critical test, to what an extent this symmetry is realized in nature.

Keywords: Transfer reactions; supersymmetry; spectroscopic factors.

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Descriptores: Reacciones de transferencia; supersimetría; factores espectroscópicos.

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1. Introduction

The excitation energy spectra of heavy nuclei, in general, are very complex. Group theoretical symmetry considerations, as introduced by Arima and Iachello (1975) [1], provide a kind of order and thus a means to discuss physics.

The aim is to find "simplicity in complexity", that means to identify a few relevant degrees of freedom. They start from the observation that valence neutrons (and protons) are especially strongly bound if they form pairs with total angular momentum $J = 0$ or $J = 2$.

1.1. Even-even nuclei

In their model for even-even nuclei, the Interacting Boson Approximation (IBA), these pairs, the s and d bosons, are considered as the only relevant degrees of freedom. These bosons have, because of the respective $2J + 1$ magnetic substates, $\nu = 1 + 5 = 6$ allowed eigenstates, hence their interaction is treated within the group formalism of $U(6)$ symmetry.

The Hamiltonian, in general, is the sum of a number of terms of lower symmetry than $U(6)$. Allowed symmetries are $U(6), O(6), U(5), O(5), SU(3), O(3), O(2)$. Each term is the product of a constant giving the energy scale and a Casimir operator representing the respective group properties. Interesting are those cases where some of these terms do not appear and the remaining ones form a chain of subsequently broken symmetry (The $U(6)$ symmetry is broken in a regular way). Then we have analytical solutions both for the eigenenergies and the eigenstates, determined by a scheme of quantum numbers. The energy scale factors remain as the only values to be determined.

Comparing, for example, even-even nuclei with an increasing number of neutron and proton holes with respect to
the closed-shell core $^{208}$Pb, we observe nuclei with $U(5)$, then $O(6)$ and finally $SU(3)$ structure and nuclei with structures in between. Nuclei with a definite symmetry structure are said to exhibit a "dynamical symmetry". The $U(5)$, and $SU(3)$ dynamical symmetries are related to the vibrational and rotational models of Bohr and Mottelson. The intermediate $O(6)$ dynamical symmetry resulted as a prediction. The discovery that $^{196}$Pt, with $Z = 78$ and $N = 118$ or $N_p = 2$ and $N_n = 4$ bosons with respect to $^{208}$Pb as core, is well described by the $O(6)$ symmetry was one of the big achievements of the IBA.

The case of $O(6)$ dynamical symmetry results from a group decomposition and yields a spectrum of excitation energies

$$ E = E_0 + C_1 \sigma (\sigma + 4) + C_2 \tau (\tau + 3) + C_3 L(L + 1) $$

with relations in between the quantum numbers $\sigma, \tau, L$, which are restricted by $N_p + N_n$.

The formalism is analogue to the case of a rigid rotor in an external magnetic field: The kinetic energy of rotation is proportional to the square of the angular momentum $l^2$ and thus invariant against the choice of all the three spacial coordinates: We have $O(3)$ symmetry. An external magnetic field will break this symmetry, but invariance against the choice of the two coordinates perpendicular to the field remains. We have $O(2)$ symmetry and accordingly the energy spectrum

$$ E = E_0 + C_1 L(L + 1)\hbar^2 + C_2 m \hbar $$

and the relation $m = L, L - 1, L - 2, ..., -L$ in between the quantum numbers.

### 1.2. Odd-even nuclei and supersymmetry

For the description of odd-A nuclei a fermion needs to be coupled to the $N$ boson system. Because of the interaction the fermion will create excitations of the core. This can be done within a semi-microscopical approach which relies on seniority in the nuclear shell model [3]. An alternative to this interacting boson-fermion approach is the construction of Hamiltonians exhibiting dynamical Bose-Fermi symmetries that are analytically solvable. In both approaches the boson-fermion space is spanned by the irreducible representation (irrep) $[N]$ of $U^B(6) \otimes U^F(M)$, where $M$ is the dimension of the single-particle space.

A significant step towards unification was made in the early eighties when Iachello and coworkers embedded the Bose-Fermi symmetry into a graded Lie algebra $U(6/M)$ [4,5]. This is the algebra invented for particle physics beyond the standard model, relating to each Boson a Fermion and vice versa. In our case the supersymmetric algebra is applied to the Hamiltonian, thus we discuss a "dynamical" symmetry.

The supersymmetric irrep $[N]$, then, spans a space that describes both an even-even nucleus with $N$ bosons and an odd-A nucleus with $N - 1$ bosons and an odd fermion. In some cases, the dynamical supersymmetry leads to an analytically solvable algebraic Hamiltonian with fixed parameters for both nuclei. If this is the case, one concludes that these nuclei exhibit a supersymmetry.

One successful case is $U(6/12)$ in which the fermion can occupy the orbits with $j = 1/2, 3/2$ and $5/2$. Considering those as arising from the coupling of a pseudo spin part with $s^f = 1/2$ with a pseudo orbital part with $l^f = 0$ and 2, the following reduction is obtained: $U^F(12) \supset U^F(6) \otimes U^F(2)$ which allows the coupling of the pseudo orbital part with the bosonic generators at the $U(6)$ level [6]. Another one is $U(6/4)$ which uses the isomorphism between the $U^F(4)$ group describing the space for a $3/2$ fermion, and the bosonic $O(6)$ group [5].

### 1.3. Odd-odd Nuclei and the extended supersymmetry

A step further is the extended supersymmetry [2, 9] which deals with boson-fermion and neutron-proton degrees of freedom, allowing the description of a quartet of nuclei, using the same algebraic form of the Hamiltonian. The quartet consists of an even-even nucleus with $(N'_{p} + N'_{n})$ bosons, an odd-proton and an odd-neutron nucleus with $(N'_{p} + N'_{n}) - 1$ bosons and an odd-odd nucleus with $(N'_{p} + N'_{n}) - 2$ bosons and a proton and neutron. Thus supersymmetry, if it works, relates the often very complex structure of the odd-odd nucleus to the much simpler even-even and odd-A systems.

If the Hamiltonian is built out of Casimir operators of groups forming a group chain, its eigenvalues are analytical as a function of the quantum numbers classifying the irreps. In case of $U_{v}(6/12) \otimes U_{v}(6/4)$ this leads to the expression [2]:

$$ E = A[N_1(N_1 + 5) + N_2(N_2 + 3)] $$

$$ + B[\Sigma_1(\Sigma_1 + 4) + \Sigma_2(\Sigma_2 + 2)] $$

$$ + B'[\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2) + \sigma_3^2] $$

$$ + C[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] $$

$$ + D L(L + 1) + E J(J + 1) \quad (1) $$

with $A, B, B', C, D$ and $E$ being free parameters to set the energy scales and $[N_1, N_2], <\Sigma_1, \Sigma_2>, <\sigma_1, \sigma_2, \sigma_3>, (\tau_1, \tau_2), L, J$ the quantum numbers correlated to the irreducible representations of $U(6), O(6), O(5), O(3)$ and $\text{Spin}(3)$, respectively. The reduction rules then lead to the level schemes in the different nuclei. These can be found in [5–7]. In addition to the analytic expressions for the excitation energies the supersymmetric scheme also provides analytic results for the wave functions. These do not depend on the parameters given above and can be tested via the calculation of electromagnetic transition rates and transfer reaction amplitudes. Especially the transfer experiments shall provide a very stringent test of the existence of supersymmetry via the distribution of the transferred nucleons in the predicted wave functions.
This model, which exhausts all aspects of symmetry, has to be considered as a minimalistic concept. A number of interactions and degrees of freedom which are expected in a fully microscopic description, are neglected. Thus the model provides a kind of benchmark one may compare with, and it is very interesting to study to what extent it is realized in nature.

With respect to the truncation of the fermion space to \( j = 1/2, 3/2 \) and \( 5/2 \) valence neutrons and \( j = 3/2 \) protons (or vice versa), demanded by the model, Jolie and others realized from the beginning that the ultimate candidate for the test is the low lying negative parity spectrum of the odd-odd nucleus \(^{196}\text{Au}\) [2] as part of the quartet \(^{194,195}\text{Pt}\) (due to the neutron \( p_{3/2} \) or \( f_{5/2} \) orbitals), and \(^{195,196}\text{Au}\), formed by the additional \( d_{3/2} \) proton orbital. The spectrum of \(^{195}\text{Pt}\), compare Fig. 1, is known as the best example of the U(6/12) supersymmetry [8, 12].

2. Transfer reaction studies

When we had been asked by Jan Jolie to study \(^{196}\text{Au}\) in transfer reactions [9] no safely assigned negative-parity states except for the \( 2^- \) ground state in \(^{196}\text{Au}\) were known, despite their in-beam gamma-ray and conversion electron spectroscopy experiments at the cyclotrons of the PSI and the University of Bonn and, more recently, their \( \gamma \gamma \) correlation studies at the Yale accelerator [10]. The difficulty was to relate the transitions to a level scheme.

Because of their excellent energy resolution (4 keV FWHM), the (p,d) transfer reactions at the Q3D magnetic spectograph of the Munich tandem accelerator was used to provide the energy calibration of the \(^{196}\text{Au}\) spectra, using \(^{195}\text{Pt}\) data measured in addition to establish a correlation between measured channels and excitation energies. The achieved uncertainties of the excitation energies are less than 1 keV. These spectra establish a new and almost complete level scheme of \(^{196}\text{Au}\). In total, 47 states were resolved for the first time in the energy range of 0 to 1350 keV including the resolved ground state doublet with an energy spacing of approximately 6 keV, as shown in Fig. 2. These excitation energies allowed later to set the observed \( \gamma \) transitions [10].

Below 500 keV eight new states were detected, thus we identified 26 states of negative parity in this range, in agreement with the supersymmetric model prediction, resulting from the reproduction of the \( A = 194 \) and 195 spectra. The next step to support the model further, is the determination of the \( J \) values.

In this respect we studied three different reactions, \((\vec{d},t)\) and \((\vec{d},\alpha)\) with polarized deuteron beam and \((^3\text{He},d)\) with unpolarized beam.
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**Figure 2.** Part of the $^{198}$Hg($\vec{d},\alpha$)$^{196}$Au and the $^{197}$Au(p,$d$)$^{196}$Au spectra at an angle of 25° measured with 18 MeV vector polarized deuterons on a 37 $\mu$g/cm$^2$ $^{198}$Hg target and with 26 MeV protons on a 67 $\mu$g/cm$^2$ $^{197}$Au target, respectively. Shown is the excitation energy range between 0 and 500 keV.

**Figure 3.** Incremental plot of the observed $s_{1/2}$ and $d_{3/2}$ strengths in ($^{3}$He,d).

Quantum numbers and spectroscopic factors were obtained from angular distributions of single-neutron transfer in $^{197}$Au($\vec{d},t$)$^{196}$Au, single-proton transfer in $^{195}$Pt($^{3}$He,d)$^{196}$Au, and two-nucleon transfer $^{198}$Hg($\vec{d},\alpha$)$^{196}$Au. Because of the $J^\pi = 0^+$ target in the case of the ($\vec{d},\alpha$) reaction identified transfer quantum numbers provide unique information about the $J^\pi$ values of the excited states in $^{196}$Au. This is different for the ($\vec{d},t$) and ($^{3}$He,d) reactions where, because of the $J^\pi = 3/2^+$ and $1/2^-$ targets, the identification of transferred $l\gamma$ values may provide restrictions only on the $J^\pi$ values in $^{196}$Au.

In ($^{3}$He,d) to $^{196}$Au, compare Fig. 3, the $d_{3/2}$ proton strength spreads rather homogeneously over the 0 to 600 keV excitation range, in agreement with the SUSY model. Most of the $s_{1/2}$ strength is located near 1000 keV, the observed much weaker $s_{1/2}$ satellite near 200 keV results from coupling of about 25 percent of the $s_{1/2}$ strength to the ($d_{3/2}^{194}$Pt(2$^+_1$)$\gamma$)$d_{1/2}$ state in $^{195}$Au. The energy of this satellite is low also because of additional residual interactions as the quadrupole-quadrupole interaction and level repulsion. Due to its nature of a comparatively weak admixture in this low energy range this coupling to the $s_{1/2}$ strength - which is outside the SUSY model - cannot cause additional “intruder” states. It will cause modifications only of the wave functions and the excitation energies.

A detailed report on these transfer experiments will be given in a forthcoming paper [12].

Including also the information from $\gamma$ and conversion electron spectroscopy we finally obtain for 20 out of the 26 observed states with negative parity below 500 keV safe $J^\pi$ assignments, compare also Fig. 4. Their number and the safe or restricted assignments are in agreement with the predictions from the dynamical $U_u(6/12) \otimes U_e(6/4)$ supersymmetric scheme.

The remaining step will be the comparison of the spectroscopic factors with model predictions. If we include our ($\alpha$,d) data too, we have spectroscopic factors for four different transfer channels. When model predictions of spectroscopic factors become available, these data will provide a further critical test to what extent this symmetry is realized in nature.

### 3. Summary and conclusions

Based on the data of the high resolution $^{197}$Au(p,d)$^{196}$Au spectrum we achieve rather complete information about excitation energies in $^{196}$Au. From the polarized $^{197}$Au(d,d)$^{196}$Au and $^{198}$Hg(d, $\alpha$)$^{196}$Au transfer data, and the $^{195}$Pt($^3$He,d)$^{196}$Au measurement definite assignments or at least restrictions of $J^\pi$ were derived. Especially the polarized two nucleon transfer (d, $\alpha$)$^{196}$Au was important to assign $J^\pi$ to excited states in the odd odd nucleus $^{196}$Au. Combined with the $\gamma - \gamma$ and conversion electron spectroscopy we thus obtain a rather complete and definite set of experimental information of all known negative parity energy levels of $^{196}$Au up to $E_x = 490$ keV. These are 26 states to compare with a $U_u(6/12) \otimes U_e(6/4)$ supersymmetric dynamical symmetry scheme (SUSY). The supersymmetric dynamical symmetry scheme derives as a prediction from a fit to the spectra of the related even-even and even-odd nuclei $^{194}$Pt, $^{195}$Pt and $^{196}$Au.

With respect to the energy range and the $J^\pi$ values we observe full agreement with respect to the 20 safe and 6 tentatively assigned states. The excitation energies of the individual states differ to a minor extent, for a model as schematic as SUSY this has to be expected.

From the observed energy dependence of the $3s_{1/2}$ proton transfer strength in ($^3$He,d) we conclude, that in this energy range near and below $E_x = 490$ keV the $3s_{1/2}$ proton orbital, neglected in the supersymmetric dynamical symmetry scheme, will not create intruder states, that are additional states outside the supersymmetry scheme, they only will cause some relatively weak admixtures to these states, which will modify to some extent the excitation energies of the observed states.

A remaining test of the SUSY scheme, especially of the relation of the experimental states to SUSY bands with specific quantum numbers, is the comparison of the experimental spectroscopic factors - and also of gamma transition probabilities - with model predictions. We studied all accessible transfer channels: neutron pick up, proton stripping, deuteron pick up and deuteron stripping. When these respective SUSY model calculations become available, these data will provide a further critical test, to what an extent this symmetry as a scheme of order in an otherwise very complex situation is realized in nature.

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**Figure 4.** Level scheme of low lying negative parity states in $^{196}$Au: The prediction of extended supersymmetry and respective quantum numbers [10] (left side) is compared with experimental data (right side). Experimental $J^\pi$ values are given in parentheses, if the data are consistent with this assignment but a deviating value cannot be excluded. Below each band the quantum numbers $<\sigma_1,\sigma_2,\sigma_3>,[N_1,N_2] <\Sigma_1,\Sigma_2>$, and on the left the quantum numbers ($\tau_1,\tau_2$) are given (only states lower than $5^-$ are shown).

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