$N$ and $\Delta$ resonances in the rigid quark–di-quark vibrator

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A nearest-neighbor analysis of light flavor baryon mass spectra reveals striking degeneracy patterns – narrow mass bands populated by series of parity twins of steadily increasing spins. Each series terminates by an unpaired resonance of the highest spin in the group. We trace back such degenerate series of resonances (to be termed by us as “megastates”) to internal baryon structure dominated by a quark–(rigid di-quark) configuration. Nucleon and $\Delta$ spectra are, surprisingly enough, exact replicas to each other in the sense that each of them features three megastates of equal quantum numbers. We fit positions and splittings of the observed megastates by an algebraic Hamiltonian which translates into a potential that is a combination of Coulomb– and Morse-like potentials and predict few more such degenerate series to be observed by the TJNAF “missing resonance” program. Finally, we explore consequences of the model for some electrodynamic properties of the spectra.

Keywords: Rovibron; quark-di-quark model; baryon degeneracy.

Un análisis a primeros vecinos del espectro de masa de bariones con sabores ligeros revela una notable degeneración en estrechas bandas de masa se encuentran localizados una serie de estados con espines crecientes. Cada espín viene con dos paridades opuestas. la serie termina con un estado desapareado. Nosotros interpretamos dicha serie degenerada de resonancias (que llamaremos mega-estados) como una estructura intrínseca de los bariones dominada por una configuración quark-di-quark rígido. Los espectros del nucleon y de Delta son, sorpresivamente, replicas exactas uno del otro en sentido que cada uno de ellos contiene tres mega-estados con los mismos números cuánticos. Hicimos ajuste de las posiciones de los mega-estados reportados por medio de un hamiltoniano algebraico que se traduce en un potencial que es una combinación de un potencial de Coulomb– y Morse-like que esperamos sean observados por el programa “missing resonance” en TJNAF. Finalmente, exploramos las consecuencias del modelo en algunas propiedades electrodinámicas del espectro.

Descriptores: Rovibrón; modelo quark-diquark; degeneración bariónica.

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1. Order versus uniformity in light-quark baryon spectra

The importance of the spectrum of composite systems for unveiling dynamics and relevant degrees of freedom in the theories of the micro-world is hardly to be overestimated. Suffices to recall that quantum mechanics was established only after the successful description of the experimentally observed degeneracy patterns of the Balmer series in the excitations of the hydrogen atom. Furthermore, in solid state physics, a spectrum with a continuous and gap-less branch of the low–lying excitations like that of the infinite ferromagnet, has been decisive for unveiling the magnons as the relevant degrees of freedom and for establishing the mechanism of the spontaneous reduction of rotational symmetry. A further high impact example is a spectrum with a low lying gap, like the one of superconducting materials, that hints onto spontaneous breaking of local symmetries and signals massiveness of gauge fields. It is not exaggerated to notice that spectra are Nature messengers about the dynamical properties of internal structure.

For same reasons, search for degeneracy patterns of baryon excitations is interesting and instructive for uncovering the dynamical properties of the theory of strong interaction– the Quantum Chromo- Dynamics.

The structure of the nucleon spectrum is far from being settled despite its long history. The situation relates to the fact that the first facility to measure $N$ and $\Delta$ levels, the Los Alamos Meson Physics Facility (LAMPF), ceased to encounter all the states that were possible as excitations of three quarks. Later on, the Thomas Jefferson National Accelerator Facility (TJNAF) was designed to search (among others) for those “missing resonances”. At present, all data have been collected and are awaiting evaluation [1].

In the series of papers [2] I performed a nearest neighbor analysis of all available data [3] on mass distribution of nucleon resonances, and drew attention to the phenomenon of state–density increase in three particularly narrow mass bands and its almost exact replica in the $\Delta$ spectrum (see Fig. 1).

The first bunch of nucleon states consists of the parity twins $[P_{11}(1440)–S_{11}(1535)]$, and the edge $D_{13}(1520)$ resonance. It is paralleled in the $\Delta$ spectrum by $[P_{13}(1750)–S_{13}(1620)]$ and $D_{33}(1700)$, respectively.

The second series of $\Delta$ resonances starts with the three parity twins $[P_{31}(1910)–S_{31}(1900)], [P_{33}(1920)–D_{33}(1940)]$, and $[F_{35}(1905)–D_{35}(1930)]$, with spins ranging from $(1/2)^+$ to $(5/2)^+$, and terminates with the edge spin–$7^+$ resonance, $F_{37}(1950)$. In the nucleon spectrum this series is paralleled by $[P_{11}(1710)–S_{11}(1650)], [P_{13}(1720)–$
D_{13}(1700)] and [F_{15}(1680)−D_{15}(1675)], while the counterpart to F_{37} is “missing”.

Finally, the third series consists of five parity twins with spins ranging from (1/2)\(\pm\) to (9/2)\(\pm\) (see Refs. [3], [4]) and terminates by the edge \(H_{3,11}(2420)\) resonance. A comparison between the \(N\) and \(\Delta\) spectra shows that the third \(N\) and \(\Delta\) series are identical up to four more unoccupied resonances. These are: (i) the nucleon counterpart to four-star \(\Delta\) resonances \(H_{3,11}(2420)\), and (ii) the \(\Delta\) counterparts to the \(N\) (one–and two star) resonances \(P_{11}(2100)\), \(P_{13}(1900)\), and \(D_{13}(2080)\), respectively. Above analysis suggests to characterize the series of (quasi)degenerate resonances by the number (here denoted by \(K\)), of the parity twins contained there. In the following we shall refer to such series as \(K\)-megastates.

The \(\Delta(1600)\) resonance in being most probably an independent hybrid state, is the only state that at present drops out of our systematics. Compared to resonance classification schemes based upon \(SU(6)_{sym} \times O(3)_L\), the light flavor baryon spectra clearly reveal regularities rather than the expected uniformity.

The existence of identical \(N\) and \(\Delta\) degeneracy patterns, raises the question as to what extent are we here facing a new type of symmetry which was not anticipated by anyone of the market models or theories. The next Section devotes itself to answering this question.

2. Spectroscopy of megastates

2.1. QCD motivated necessity for a quark–di-quark configuration. A preliminary

To the extent QCD prescribes baryons to be constituted of three quarks in a color singlet state, one can exploit for the description of baryonic systems algebraic models developed for the purposes of triatomic molecules, a path pursued by Refs. 5. An interesting dynamical limit of the three quark system is \(U(7) \rightarrow U(3) \times U(4)\), when two of the quarks act as an independent entity, a di-quark (Dq), while the third quark (q) remains spectator. While the di-quark approximation [6] turned out to be rather convenient for describing various properties of the ground state baryons [7], [8], applications to excitation spectra are still lacking. It is one of our goals to partly fill this gap.

The usefulness, even necessity, for having a quark–di-quark configuration within the nucleon is independently supported by arguments related to spin in QCD. In particular, the meaning of spin in QCD was revisited in Refs. [9], and [10] in connection with the proton spin puzzle.

As is well known, the spin degrees of freedom of the valence quarks alone are not sufficient to explain the spin-\(\frac{3}{2}\) of the nucleon. Rather, one needs to account for the orbital angular momentum of the quarks (here denoted by \(L_{QCD}\)) and the angular momentum carried by the gluons (so-called field angular momentum, \(G_{QCD}\)).

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_{QCD} + G_{QCD}
\]

\[
= \int d^4x \left\{ \frac{1}{2} \bar{\psi} \gamma_\mu \gamma_5 \psi + \bar{\psi} \left[ i \vec{x} \times (-\vec{B}) \right] \psi \right. 
\]

\[
\left. + \bar{\psi} \times \left( \vec{E}^a \times \vec{B}^a \right) \right\}
\]

In so doing one encounters the problem that neither \(L_{QCD}\), nor \(G_{QCD}\) satisfies the spin \(su(2)\) algebra. If at least \((L_{QCD} + G_{QCD})\) is to be given the interpretation of angular momentum, then one has to require validity of the \(su(2)\) commutator

\[
\left[ (L_{QCD}^i + G_{QCD}^i), (L_{QCD}^j + G_{QCD}^j) \right] = i\epsilon^{ijk} (L_{QCD}^k + G_{QCD}^k) \quad (1)
\]

Equation (1) restricts \(\vec{E}^{i,a}\) to a chromo-electric charge,

\[
E^{i,a} = \frac{g}{\sqrt{3}} T^a,
\]
while $\tilde{B}^{ia}$ has to be a chromo-magnetic dipole according to,

$$B^{ia} = \left(3x^i \sum \frac{x^j m^l}{r^5} - \frac{m^i}{r^5}\right) T^a,$$

where $x^i = x^i - R^i$. Above color fields correspond to the perturbative one-gluon approximation typical for a di-quark-quark structure. The di-quark and the quark are in turn the perturbative one-gluon approximation typical for a di-quark–di-quark form, the nucleon wave function does indeed have the required quark–di-quark form

$$|p_1\rangle = \frac{\epsilon_{ijk}}{\sqrt{18}} \left[ u^+_i d^+_j - u^+_i d^+_j \right] u^+_k |0\rangle.$$  

A similar situation appears when looking up the covariant QCD solutions in form of a membrane with the three open ends being associated with the valence quarks. When such a membrane stretches to a string (see Fig. 2), so that a linear action (so called gonihedric string) can be considered, one again encounters same $K$-megastate degeneracies in the excitations spectra of the baryons, this time as a part of an infinite $K$ tower. The result was reported by Savvidy in Ref. 11. Thus not only the covariant spin-description provides an independent argument in favor of a dominant quark-di-quark configuration in the nucleon structure, but also search for covariant resonant QCD solutions leads once again to same picture. Within the context of the quark–di-quark (q-Dq) model, the ideas of the vibron model, known from the spectroscopy of diatomic molecules [12] acquires importance as a tool for the description of the excitations of the (q-Dq) vibrator.

2.2. The quark vibrator

In the rigid vibrator model (RVM) the relative (q-Dq) motion is described by means of four types of boson creation operators $s^+, p^+_1, p^+_{-1}$, and $p^-_{-1}$ (see Refs. [12], [13]). The operators $s^+$ and $p^+_{m}$ in turn transform as rank-0, and rank-1 spherical tensors, i.e. the magnetic quantum number $m$ takes in turn the values $m = 1, 0, \text{and} -1$. In order to construct boson-annihilation operators that also transform as spherical tensors, one introduces the four operators $\tilde{s} = s$, and $\tilde{p}_m = (-1)^m p_{-m}$. Constructing rank-$k$ tensor product of any rank-$k_1$ and rank-$k_2$ tensors, say, $A^{k_1}_{m_1}$ and $A^{k_2}_{m_2}$, is standard and given by

$$[A^{k_1} \otimes A^{k_2}]_m = \sum_{m_1, m_2} (k_1 m_1 k_2 m_2 | k m) A^{k_1}_{m_1} A^{k_2}_{m_2}. \quad (5)$$

Here, $(k_1 m_1 k_2 m_2 | k m)$ are the standard $O(3)$ Clebsch-Gordan coefficients.

Now, the lowest states of the two-body system are identified with $N$ boson states and are characterized by the ket-vectors $| n_s n_p l m \rangle$ (or, a linear combination of them) within a properly defined Fock space. The constant $N = n_s + n_p$ stands for the total number of $s$- and $p$-bosons and plays the rôle of a parameter of the theory. In molecular physics, the parameter $N$ is usually associated with the number of molecular bound states. The group symmetry of the vibrator is well known to be $U(4)$. The fifteen generators of the associated $su(4)$ algebra are determined as the following set of bilinears

$$A_{00} = s^+ \tilde{s}, \quad A_{0m} = s^+ \tilde{p}_m, \quad A_{m0} = p^+_m \tilde{s}, \quad A_{mm'} = p^+_m \tilde{p}_{m'}.$$  

(6)

The $u(4)$ algebra is then recovered by the following commutation relations

$$[A_{\alpha \beta}, A_{\gamma \delta}]_+ = \delta_{\beta \gamma} A_{\alpha \delta} - \delta_{\alpha \delta} A_{\gamma \beta}.$$  

(7)

The operators associated with physical observables can then be expressed as combinations of the $u(4)$ generators. To be specific, the three-dimensional angular momentum takes the form

$$L_m = \sqrt{2} [p^+ \otimes \tilde{p}^+_m]. \quad (8)$$

Further operators are $(D_m)$ and $(D'_m)$ defined as

$$D_m = [p^+ \otimes s^+ \otimes \tilde{p}^+_m], \quad (9)$$

$$D'_m = [i p^+ \otimes s^+ \otimes \tilde{p}^+_m], \quad (10)$$

respectively. Here, $\vec{D}$ plays the rôle of the electric dipole operator.

Finally, a quadrupole operator $Q_m$ can be constructed as

$$Q_m = [p^+ \otimes \tilde{p}^+_m], \quad \text{with} \ m = -2, \ldots, +2. \quad (11)$$

2.3. Degeneracy of the rigid vibrator

The $u(4)$ algebra has the two algebras $su(3)$, and $so(4)$, as respective sub-algebras. The $so(4)$ sub-algebra of interest here, is constituted by the three components of the angular momentum operator $L_m$, on the one side, and the three components of the operator $D'_m$, on the other side. The chain of reducing $U(4)$ down to $O(2)$

$$U(4) \supset O(4) \supset O(3) \supset O(2), \quad (12)$$

is

$$N \quad K \quad l \quad m,$$
corresponds to an exactly soluble rigid vibrator model (RVM) limit. In the second row of Eq. (12) we indicate the quantum numbers associated with the respective group of the chain. Here, \( N \) stands for the principle quantum number of the four dimensional harmonic oscillator associated with \( U(4) \). \( K \) refers to the four dimensional angular momentum in \( O(4) \), while \( l \) and \( m \) are in turn ordinary three– and two angular momenta.

In order to demonstrate how this model applies to baryon spectroscopy, let us consider the case of (q-Dq) states associated with \( N = 5 \). It is of common knowledge that the totally symmetric irreps of the \( u(4) \) algebra with the Young scheme \([N]\) contain the \( SO(4) \) irreps \((\frac{l}{2}, \frac{m}{2})\) with

\[
K = N, N - 2, \ldots, 1 \quad \text{or} \quad 0. \tag{13}
\]

Each one of the \( K \)- irreps contains \( SO(3) \) multiplets with three dimensional angular momentum

\[
l = K, K - 1, K - 2, \ldots, 1, 0. \tag{14}
\]

In applying the branching rules in Eqs. (13), (14) to the case \( N = 5 \), one encounters the three series of levels

\[
K = 1: \quad l = 0, 1;
K = 3: \quad l = 0, 1, 2, 3;
K = 5: \quad l = 0, 1, 2, 3, 4, 5. \tag{15}
\]

The parity carried by these levels is \( \eta(-1)^l \) where \( \eta \) is the parity of the di-quark. In coupling now the angular momentum in Eq. (15) to the spin-\( \frac{1}{2} \) of the three quarks in the nucleon, the following sequence of states is obtained:

\[
K = 1: \quad \eta J^p = \frac{1}{2} \begin{pmatrix} 1 & 1 & -3 & -3 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 \end{pmatrix};
K = 3: \quad \eta J^p = \frac{1}{2} \begin{pmatrix} 1 & 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 \end{pmatrix} ;
K = 5: \quad \eta J^p = \frac{1}{2} \begin{pmatrix} 1 & 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 7 & 9 & 11 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 \end{pmatrix}. \tag{16}
\]

Therefore, states of half-integer spin emerging from the underlying vibrator modes transform according to \((\frac{1}{2}, \frac{1}{2}) \otimes \{(1, 0) \oplus (0, \frac{1}{2})\}\) representations of \( SO(4) \), a result due to [13]. There, we accounted pragmatically for isospin structure through attaching to the \( K \)-megastates an isospin spinor \( \chi^I \) with \( I \) taking the values \( I = \frac{3}{2} \) and \( I = \frac{1}{2} \) for the nucleon, and the \( \Delta \) states, respectively. Notice, that as long as compositeness of the di-quark has been completely ignored here (the di-quark is rather treated as a fundamental scalar/pseudoscalar), the question about anti–symmetrization of the baryon wave function does not stay.

As illustrated by Fig. 1, the above quantum numbers completely cover both the nucleon and the \( \Delta \) excitations. The states in Eq. (16) are degenerate and the dynamical symmetry is \( O(4) \).

Above considerations apply to the rest frame. In order to describe megastates in flight one needs to subject the \( O(4) \) degenerate resonance states to a Lorentz boost. One can achieve this task either boosting them spin by spin, or boosting the \( K \)-megastate as a whole. In favoring the latter option one gains the possibility to map (up to form factors) \( K \)-megastates onto totally symmetric rank-\( K \) Lorentz tensors with Dirac spinor components, \( \psi_{\mu_1 \ldots \mu_K} \). [See Ref. [14] for details on a totally neutral \( \psi_{\mu} \), associated with the massive gravitino, and Ref. 15 for a discussion on why \( \psi_{\mu_1 \ldots \mu_K} \) (for any \( K \)) qualify also for the description of charged baryon megastates.]

### 2.4. Observed and "missing" \( K \) megastates

The comparison of the states in Eq. (16) with the reported ones in Fig. 1 shows that the predicted quantum numbers of the resonance series (with masses below \( \sim 2500 \) MeV) would be in agreement with the quantum numbers of the reported non-strange baryon excitations only if the rigid di-quark changes from scalar \( (\eta = 1) \) for the \( K = 1 \), to pseudoscalar \( (\eta = -1) \) for the \( K = 3, 5 \) series. That is why we assign unnatural parities to the resonances from the second and third \( K \) megastates.

A pseudoscalar di-quark can be modeled in terms of an excited composite di-quark carrying an internal angular momentum \( L = 1 \) and maximal spin \( S = 1 \). In one of the possibilities the total spin of such a system can be \( |L - S| = 0 \). To explain the ground state, one has to consider separately even \( N \) values, such as, say, \( N' = 4 \). In that case another branch of excitations, with \( K = 4, 2 \), and 0 will emerge. The \( K = 0 \) value characterizes the ground state, \( K = 2 \) corresponds to \((1, 1) \otimes [(1/2, 0) \oplus (0, 1/2)]\), while \( K = 4 \) corresponds to \((2, 2) \otimes [(1/2, 0) \oplus (0, 1/2)]\). These are the multiplets that we shall associate with the “missing” resonances predicted by the rigid vibrator model. In this manner, reported and “missing” resonances fall apart and populate distinct \( U(4) \)-, and therefore distinct \( SO(4) \) representations. In making observed and “missing” resonances distinguishable, reasons for their absence or, presence in the spectra are easier to be searched for. In accordance with Ref. [16] we here will treat the \( N = 4 \) states to be all of natural parities and identify them with the nucleon \( (K = 0) \), the natural parity \( K = 2 \), and the natural parity \( K = 4 \)-megastates. We shall refer to the latter as “missing” \( K \)-megastates.

The RVM Hamiltonian is constructed in the standard way as a properly chosen function of the Casimir operators of the algebras of the subgroups entering the chain. For example, in case one approaches \( O(3) \) via \( O(4) \), the algebraic Hamiltonian of a dynamical \( SO(4) \) symmetry can be cast into the form [13]:

\[
H_{RVM} = H_0 - f_1 \left( 4C_2 \left( so(4) \right) + 1 \right)^{-1} + f_2 C_2 \left( so(4) \right). \tag{17}
\]
The Casimir operator $C_2(\text{so}(4))$ is defined accordingly as
\begin{equation}
C_2(\text{so}(4)) = \frac{1}{4} \left( \hat{L}^2 + \hat{D'}^2 \right)
\end{equation}
and has an eigenvalue of $(K/2)[(K/2) + 1]$. The parameters of the Hamiltonian that fits masses of the observed $K$-megastates are:
\begin{align}
H_0 &= M_{N/\Delta} + f_1, \\
f_1 &= 600 \text{ MeV}, \\
f_2^N &= 70 \text{ MeV}, \\
f_2^\Delta &= 40 \text{ MeV}.
\end{align}
Thus, the $SO(4)$ dynamical symmetry limit of the RVM picture of baryon structure explains observation of (quasidegenerate resonance series in both the $N$- and $\Delta$ baryon spectra. In Table I we list the masses of the $K$-megastates concluded from Eqs. (17), and (19).

In order to translate the Hamiltonian in Eq. (19) into coordinate space,
\begin{equation}
\mathcal{H} = -\frac{\hbar^2}{2\mu} \nabla^2 + V(r),
\end{equation}
one can consider a central potential $V(r)$ that is the following combination of Coulomb- and Morse-like potentials
\begin{equation}
V(r) = -\frac{\alpha^2}{r} + V_0(1 - e^{-\beta r})^2,
\end{equation}
\begin{equation}
\alpha^2 = \frac{2f_1}{\mu}, \quad f_2 = -\frac{\beta^2}{\mu}, \quad V_0 = \frac{(N + 2)^2 \beta}{8\mu}.
\end{equation}
The $\sim 1/r$ potential reproduces the $1/2(K+1)^2$ contribution to the mass, $M_K$, of the excited $K$ megastates coming from the $\{4C_2[\text{so}(4)] + 1\}^{-1}$ term in Eq. (17), while the Morse potential describes the $K/2(K/2 + 1)$ contribution to $M_K$ coming from the $C_2[\text{so}(4)]$ term in Eq. (17) (see Ref. [12] for more details on the latter point). Here $\mu$ stands for the reduced mass of the quark and the di-quark. Equation (21) provides the ground for the calculation of the wave RVM wave functions and thereby for a more profound analysis of the decay properties of the $K$-megastates, a subject that is currently under investigation.

### Table I. Mass distribution of observed (obs), and missing (miss) vibrator $K$-megastates (in MeV) concluded from Eqs. (9,11).

<table>
<thead>
<tr>
<th>$K$</th>
<th>sign $\eta$</th>
<th>$N^{\text{obs}}$</th>
<th>$N^{\text{exp}}$</th>
<th>$\Delta^{\text{obs}}$</th>
<th>$\Delta^{\text{exp}}$</th>
<th>$\Delta^{\text{inexp}}$</th>
<th>$\Delta^{\text{inexp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>939</td>
<td>939</td>
<td>1232</td>
<td>1232</td>
<td>1232</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>+</td>
<td>1441</td>
<td>1498</td>
<td>1712</td>
<td>1690</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>1612</td>
<td>1689</td>
<td>1944</td>
<td>1922</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>1764</td>
<td>1689</td>
<td>1944</td>
<td>1922</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>2135</td>
<td>2102</td>
<td>2165</td>
<td>2276</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.5. Electromagnetic gross properties of $K$ megastates.

The four dimensional Racah algebra that allows to calculate transition probabilities for electromagnetic de-excitations of the vibrator levels was presented in Ref. [13]. The interested reader is invited to consult this article for details. Here I restrict myself to reporting on the following two results:

1. All resonances from a $K$-megastate have same widths.
2. As compared to the natural parity $K = 1$ states, the electromagnetic de-excitations of the unnatural parity $K = 3$ and $K = 5$ RVM states appear strongly suppressed.

To see how above predictions match with experiment, I compiled in Table II below data on the measured total widths of resonances belonging to $K = 3$, and $K = 5$.

### Table II. Measured total widths of $K$-megastates.

<table>
<thead>
<tr>
<th>$K$</th>
<th>Resonance</th>
<th>width [in GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$N \left( \frac{1}{2}^+ ; 1650 \right)$</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>$N \left( \frac{1}{2}^- ; 1710 \right)$</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>$N \left( \frac{3}{2}^+ ; 1720 \right)$</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>$N \left( \frac{5}{2}^+ ; 1700 \right)$</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>$N \left( \frac{5}{2}^- ; 1675 \right)$</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>$N \left( \frac{5}{2}^- ; 1680 \right)$</td>
<td>0.13</td>
</tr>
<tr>
<td>5</td>
<td>$N \left( \frac{3}{2}^+ ; 1900 \right)$</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>$N \left( \frac{5}{2}^+ ; 2000 \right)$</td>
<td>0.49</td>
</tr>
</tbody>
</table>

### Table III. Measured electromagnetic helicity amplitudes of resonances.

<table>
<thead>
<tr>
<th>$K$</th>
<th>parity of the spin-0 di-quark</th>
<th>Resonance</th>
<th>$A_{1/2}^{\pm}$</th>
<th>$A_{3/2}^{\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-</td>
<td>$N \left( \frac{1}{2}^- ; 1710 \right)$</td>
<td>9±22</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>$N \left( \frac{3}{2}^+ ; 1720 \right)$</td>
<td>18±30</td>
<td>-19±20</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>$N \left( \frac{5}{2}^- ; 1700 \right)$</td>
<td>-18±30</td>
<td>-2±24</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>$N \left( \frac{5}{2}^- ; 1675 \right)$</td>
<td>19±8</td>
<td>15±9</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>$N \left( \frac{5}{2}^+ ; 1680 \right)$</td>
<td>-15±6</td>
<td>133±12</td>
</tr>
<tr>
<td>1</td>
<td>+</td>
<td>$N \left( \frac{3}{2}^- ; 1520 \right)$</td>
<td>-24±9</td>
<td>166±5</td>
</tr>
</tbody>
</table>

In order to give an interpretation of significantly non-vanishing electromagnetic widths within the advocated model, one may entertain the idea that an enhanced helicity amplitude may signal small admixtures from natural parity states of same spins belonging to megastates of even $K$ that belong to the “missing” modes. For example, the significant value of $A_p^3$ for $N[(5/2)^+; 1680]$ from $K = 3$ may appear as an effect of mixing with the edge $N[(5/2)^+; 1612]$ state from the natural parity “missing” megastates with $K = 2$. This gives one the idea to use helicity amplitudes to extract “missing” states.

3. Conclusions

The degeneracy phenomenon in the spectra of the light quark baryons was successfully explained in terms of $K$-megastates, i.e. $(K/2, K/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$, taking their origin from the quark version of the rigid vibrator with a dynamical O(4) symmetry. We predicted series of degenerate “missing” resonances with $K$ even. To conclude, the $N$, or, $\Delta$ excitation modes seem to fit better into ply-spectra composed by nearly equidistant $K$-megastates as displayed in Fig. 3, but into any other patterns. To account for the relatively small crumbling of the megastates one needs to weaken the dominance of the quark–(rigid di–quark) configuration and consider mixing with three quark states. It is an intriguing task to build up baryon spectra from the rigid vibrator and upon subjecting it to small symmetry violations, to account for the realistic mass distribution of baryon resonances. The advocated model captures better but any other quark model the essentials of QCD string solutions.

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