Collective treatment of the isobaric analogue state

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We study the effects of isospin symmetry violation in the description of the Isobaric Analogue State (I.A.S.). The restoration of the symmetry is enforced at the collective level, in order to calculate physical isospin violating terms within an isospin conserving basis. The method is illustrated for a schematic model and for realistic single-particle model spaces.

Keywords: Symmetry of isospin; brake and restauration; width of the analog isobaric state.

In this presentation we consider two cases:

i) a Hamiltonian which conserves isospin for certain values of the parameters associated with the residual two-body interaction, and that may be straightforwardly treated within the present formalism;

ii) a realistic single-particle Hamiltonian for which the difference between neutrons and protons single-particle energies proceeds from both an isospin conserving interaction, the isovector Hamiltonian, and from a non-conserving isospin force, the Coulomb interaction.

2. The Hamiltonian

Let us split the Hamiltonian into single-particle and two-body terms,

$$ H = H_{sp} + V, $$

where at least $H_{sp}$ is not invariant against symmetry transformations associated with isospin symmetry. Thus the single-particle basis states, determined by $H_{sp}$, do not carry the quantum numbers corresponding to this symmetry. The symmetry may be restored at the collective level by raising the transformation parameters determining the orientation of an “intrinsic” or “moving” frame of reference, to the status of collective coordinates. The total Hilbert space is thus factorized into an intrinsic and a collective sector. This last one may be labelled by the quantum numbers associated with the original symmetry

$$ \Psi = \psi_{\text{intrinsic}} \times D_{MK}^{T}(\phi_{A}, \phi_{B}, \phi_{C}). $$

1. Introduction

In nuclear structure calculations, the treatment of the isospin degree of freedom is frequently hampered by use of single-particle bases which violate the isospin symmetry. This fact has produced ambiguities both in the description of Isobaric Analogue States (I.A.S.). A common source of violations is to only include terms of the type $T_0 \tau_0$ in the construction of the basis states, while isospin symmetry would require that the contributions $-T_\pm \tau_\mp$ should be taken into account as well. There are also legitimate sources of isospin mixtures, due the presence of physical isospin non-conserving terms in the Hamiltonian, such as Coulomb contributions to the proton single-particle Hamiltonian, or differences between the strength of the isovector proton-proton, neutron-neutron and proton-neutron pairing channels. Although the existence of this problem was recognized long time ago in studies concerning I.A.S. and isospin impurities [1, 2], the problem has remained largely unsolved. A global analysis of the energy and spreading width of the I.A.S., in the range $A = 110$ to 238 can be found in [3, 4].

A collective treatment has been proposed for the pairing case [5] and for the treatment of center of mass effects [6]. The perfect analogy to this treatment is the use of intrinsic and collective degrees of freedom in the description of nuclear deformations and space rotations [7]. Although such an analogy has been drawn long time ago [7–10], the microscopic theory of gauge and isospin collective phenomena was not discussed before in detail.
Here $\phi_\alpha, \phi_\beta$ and $\phi_\gamma$ represent the Euler angles in isospin space. The quantum numbers $M$ and $K$ are the eigenvalues of the isospin projection along the laboratory and intrinsic frames of reference, respectively.

We operate on the intrinsic system, where symmetry violations may still take place, since it is an artificial construction. Therefore any operator must first be transformed to the intrinsic frame, before acting on the product states (2). Obviously problems of over-completeness and of constraints should be properly taken into account.

If the total Hamiltonian does not conserve the symmetry, the same procedure may be carried out, yielding an appropriate isospin conserving base for the calculation of matrix elements of isomultipole operators. Thus we disentangle the physical effects due to non-conservation of isospin in the Hamiltonian, from the spurious effects arising as a consequence of isospin violations within the set of basis states.

The intrinsic generators of the isospin transformations in the intrinsic frame are the operators $\tau_q$ ($q = 0, 1$ and $-1 = 1$). The operators $T_0, T_1, T_2$ denote the corresponding collective momenta. The over-completeness of the basis poses a problem that is clarified through the observation that a system, described as above, possesses a "gauge" symmetry. It is expressed by the constraints

$$\tau_q - T_q = 0, \quad (3)$$

The set of basis states within the collective sector may be straightforwardly obtained in two limiting situations, according to whether the Coriolis type of interaction $\frac{1}{2}(-1)^q T_0 T_0$ or $1$ is included in the intrinsic single-particle spectrum (as in the cranking model).

The assumption of large values of $T$ suggests the use of the Marshalek generalization of the Holstein-Primakoff representation [11]. Thus, the rotational isospin sector in (2) may be expressed in terms of the boson creation operators $Y^+, \xi^+$ and $\varsigma^+$

$$|TMK\rangle = \frac{(\xi^+)^m (\varsigma^+)^k}{\sqrt{2T!} \sqrt{(m)!} \sqrt{(k)!}};$$

$$m, k = 0, 1, 2, \ldots, \quad (4)$$

where the quantum numbers $m = (1/2)(T + M)$ and $k = (1/2)(T + K)$ substitute the isospin projections $M$ and $K$, respectively. The collective components of the isospin operators may be written in terms of the bosons $\varsigma^+, \varsigma^-$

$$T_0 = -T + \varsigma^+ \varsigma^-;$$

$$T_1 = \varsigma^+ \sqrt{T - \frac{1}{2} \varsigma^+ \varsigma^-} \approx -\sqrt{T} \varsigma^-;$$

$$T_1 = -T_1^+ \approx \sqrt{T} \varsigma^+, \quad (5)$$

and the constraints (3) are written, to leading order in $1/T$, as

$$\tau_0 = -T + \varsigma^+ \varsigma^-; \quad \tau_1 = -\sqrt{T} \varsigma^+; \quad \tau_1 = \sqrt{T} \varsigma^+. \quad (6)$$

The Hamiltonian (1) displays single-particle and two-body contributions. Since the calculations are performed in the intrinsic system, any operator should be transformed to this frame. As usual [7], the transformation between laboratory (lab) and intrinsic (int) tensor operators is expressed by

$$C_{\lambda\mu}^{\text{lab}} = D^\lambda_{\mu\nu} C_{\lambda\nu}^{\text{int}}, \quad (7)$$

where $D^\lambda_{\mu\nu}$ are the rotational matrices, discussed in the previous section. (We use the Einstein convention that the repetition of an index on a given side of an equation implies a summation over that index.) The operators $C_{\lambda\mu}$ are irreducible tensor operators carrying isospin $\lambda$ and isospin-projection $\mu$.

The application of this transformation to the Hamiltonian is a trivial step in cases for which the Hamiltonian displays only scalar terms. In the present situation the Hamiltonian transforms as

$$H = H_0 + H_1 + H_2 \quad (8)$$

$$H_0 = H_{sp00} + V_{00}$$

$$H_1 = D_{0\nu} H_{sp1\nu} + D_{0\nu} V_{1\nu}$$

$$H_2 = D_{2\nu}^2 V_{2\nu}$$

The terms $H_1$ and $H_2$ of the Hamiltonian (8) are physical operators, because they commute with the constraints (3).

We replace the rotational functions $D_{0\nu}$ and $D_{2\nu}^2$ by the corresponding leading order terms

$$H = h_0 + \omega_\xi (1 + \varsigma^+ \varsigma^- + \xi^+ \varsigma^- - \beta^{-2} \varsigma^- \varsigma^- - \beta^2 \xi^+ \varsigma^+)$$

$$+ \frac{1}{\sqrt{T}} (H_{sp11} + V_{11} + \sqrt{3} V_{21}) (\varsigma^+ - \beta^{-2} \varsigma^-)$$

$$- \frac{1}{\sqrt{T}} (H_{sp11} + V_{11} + \sqrt{3} V_{21}) (\varsigma - \beta^2 \xi^+)$$

$$+ O \left( T^{-\frac{1}{2}} \right) \quad (9)$$

where

$$h_0 \equiv H_{sp00} + V_{00} + H_{sp10} + V_{10} + V_{20}$$

$$\omega_\xi \equiv - \frac{1}{T} \left( \langle H_{sp10} \rangle + \langle V_{10} \rangle + 3 \langle V_{20} \rangle \right) \quad (10)$$

and,

$$h_1 = -[\tau_1, h_0] = H_{sp11} + V_{11} + \sqrt{3} V_{21}$$

$$h_1 = [\tau_1, h_0] = H_{sp11} + V_{11} + \sqrt{3} V_{21} \quad (11)$$

If we also replace the operators $\varsigma^+, \varsigma$ by their equivalent values obtained from the constraints (6), we obtain

$$H = W + \omega_\xi \left( \xi^+ \varsigma^+ + \frac{3}{2} \right) + H_{\text{coup}} + O \left( T^{-\frac{1}{2}} \right)$$

$$H_{\text{coup}} = - \frac{1}{\sqrt{T}} \left( \beta^{-2} \xi^+ \varsigma^- + \beta^2 \xi^+ \varsigma^- \right) \quad (12)$$

where
\[ W \equiv h_0 - \frac{\omega_q}{2T}[\tau_1, \tau_1] + \frac{1}{T}(h_1 \tau_1 + h_1 \tau_1) \]
\[ \Xi^+ \equiv h_1 - \omega_\xi \tau_1 \]
\[ \Xi \equiv -h_1 + \omega_\xi \tau_1. \quad (13) \]

Here \( W \) is an effective Hamiltonian, yielding the normal modes \( q \), carrying energy \( \omega_q \) and isospin \( T - 1 \), through the TDA or RPA.

The coupling term \( H_{\text{coup}} \) mixes the I.A.S. with the \( q \) modes. It creates (annihilates) the modes \( q \) simultaneously with the annihilation (creation) of the I.A.S. mode and the lowering (rising) of the isospin \( T \) by one unit.

The transition (Fermi) operator is written
\[ \beta^{(F-)} = \sqrt{2} \tau_1. \quad (14) \]

As we proceeded in the case of the Hamiltonian, we must transform the operator \( \beta^{(F-)} \) to the intrinsic frame
\[ \beta^{(F-)} \rightarrow \sqrt{2} \left(D_1 \tau_1 + D_{10} \tau_0 + D_{1(-1)} \tau_{-1}\right) \]
\[ = \sqrt{2}D_{10}(\tau_0) + O(T^{-1/2}) \]
\[ = -\sqrt{2T} \xi + O(T^{-1/2}). \quad (15) \]

Thus the Fermi operator would only populate the I.A.S., but for the presence of the coupling term \( H_{\text{coup}} \). We diagonalize this term following the procedure advocated in Ref. [12], i.e., we neglect the residual matrix element between the I.A.S. itself and between the \( q \)-modes. The energies \( E_k \) are given by the dispersion relation
\[ (\omega_\xi - E_k) = \sum_q \frac{\langle q | H_{\text{coup}} | \xi \rangle^2}{\omega_q - E_k}, \quad (16) \]
while the eigenstates
\[ |k\rangle = c_k(\xi)|\xi\rangle + \sum_q c_k(q)|q\rangle \quad (17) \]
display the amplitudes
\[ c_k(\xi) = \left(1 + \sum_q \frac{\langle q | H_{\text{coup}} | \xi \rangle^2}{(E_k - \omega_q)^2}\right)^{-1/2} \]
\[ c_k(q) = c_k(\xi) \frac{\langle q | H_{\text{coup}} | \xi \rangle}{E_k - \omega_q} \quad (18) \]

The matrix elements of the Fermi operator between the ground state and any state \( k \) is given by the product
\[ \langle k | \beta^{(F-)} | g.s. \rangle = -\sqrt{2T} c_k(\xi) \quad (19) \]
The strength of this transition is centered at an energy \( \bar{E} \) and has a spread \( \sigma \) given by
\[ \bar{E} = \frac{\sum_k E_k |c_k(\xi)|^2}{\sum_k |c_k(\xi)|^2} \]
\[ \sigma = \left(\frac{\sum_k (E_k - \bar{E})^2 |c_k(\xi)|^2}{\sum_k |c_k(\xi)|^2}\right)^{1/2} \quad (20) \]

\section{Model I}
This model is analytically tractable but non trivial. We expect to learn from it how the above formalism works, rather than to extract, at this point, physical predictions. The single-particle levels are labelled by the isospin component \((p, n)\) and the angular momentum \((j, m)\) quantum numbers
\[ \tau_0 = \sum_j \tau_{0j}; \quad \tau_{0j} = \frac{1}{2} \left( c_{pj} c_{njm} - c_{nj} c_{pjm} \right) \]
\[ \tau_1 = \sum_j \tau_{1j}; \quad \tau_{1j} = -\frac{1}{\sqrt{2}} \left( c_{pj} c_{njm} \right) \]
\[ \tau_1 = -\tau_1^+ \quad (21) \]

We assume that every single-particle state is completely filled with neutrons and completely empty of protons. Therefore
\[ \langle \tau_{0j} \rangle = -\bar{j}^2; \quad T = \bar{j}^2; \quad j \equiv \sqrt{j^2 + \frac{1}{2}} \quad (22) \]
The Hamiltonian of this model is
\[ H = H_{\text{sp}} + H_{TD} + H_{sc} \quad (23) \]
\[ H_{sp} = (\epsilon_{aj} + \epsilon_{oj}) \tau_{0j} \]
\[ H_{TD} = \alpha \langle k | \bar{V} | j \rangle \tau_{1k} \tau_{1j} \]
\[ H_{sc} = -\beta \frac{\langle k | \bar{V} | j \rangle}{2} \tau_{10k} \tau_{0j} \]
where
\[ \epsilon_{0j} \equiv s_j + r_j; \quad s_j \equiv \langle j | \bar{V} | k \rangle \bar{k}^2; \quad r_j \equiv \frac{1}{2} \langle j | \bar{V} | j \rangle \quad (24) \]

This Hamiltonian is an isoscalar provided \( \alpha = \beta = 1 \), since
\[ [\tau_1, H] = -[(1 - \alpha)s_j + (1 - \beta)r_j] \tau_{1j} \]
\[ -[\alpha - \beta] \langle k | \bar{V} | j \rangle \tau_{1k} \tau_{0j} \quad (25) \]

We also define the large quantities of the problem,
\[ S \equiv \bar{j}^2 s_j; \quad R \equiv \bar{j}^2 r_j \quad (26) \]
and we assume that \( S \) is of order \( T \). Thus \( R \) is an order of magnitude smaller, in powers of \( T^{-1} \). The matrix elements \( \langle k | \bar{V} | j \rangle \) are of \( \mathcal{O}(T^{-1}) \). From here on we keep only the leading order terms of an expansion in powers of \( T^{-1} \).
The Hamiltonian displays in general isoscalar, isovector and isoquadrupole terms. The transformation to the intrinsic system yields

\[ h_0 = \epsilon_0 \tau_0 + \alpha \langle k | \hat{V} | j \rangle \tau_1 k \tau_1 j - \frac{1}{2} \langle k | \hat{V} | j \rangle \tau_0 k \tau_0 j + \mathcal{O} (T^{-1}) \]

\[ h_1 = (\epsilon_0 - \alpha s_j) \tau_1 j - \beta \langle k | \hat{V} | j \rangle \tau_1 k \tau_0 j + \mathcal{O} (T^{-2}) \]

\[ h_2 = (\epsilon_0 - \alpha s_j) \tau_1 j - \beta \langle k | \hat{V} | j \rangle \tau_0 k \tau_1 j + \mathcal{O} (T^{-2}) \]

\[ \omega_\xi = \frac{S(1 - \alpha)}{T} + \mathcal{O} (T^{-1}) \] (27)

For the sake of simplicity, we further assume that the interaction matrix elements can be factorized

\[ \langle k | \hat{V} | j \rangle = v_k v_j \] (28)

Under this approximation, we write

\[ \epsilon_{0j} = s_j = V v_j; \quad S = V^2 \] (29)

where

\[ \Lambda_q = \lambda_{qj} \hat{j}^2 = \Lambda_q \frac{(1 - \alpha) V}{T} \left( X_1 - \frac{V}{T} X_0 \right) + \Xi_q \left( \alpha X_1 + \frac{(1 - \alpha) V}{T} X_0 \right) \]

\[ \Xi_q = \lambda_{qj} v_j \hat{j} = \Lambda_q \frac{(1 - \alpha) V}{T} \left( X_2 - \frac{V}{T} X_1 \right) + \Xi_q \left( \alpha X_2 + \frac{(1 - \alpha) V}{T} X_1 \right) \]

\[ X_\nu = \frac{v_j \hat{j}^2}{\epsilon_{0j} - \omega_q} \]

Eqs. (34) constitute a homogeneous system of equations in the amplitudes \( \Lambda_q, \Xi_q \). The vanishing of the determinant of the coefficients of this system yields the roots \( \omega_q \).

As a check, we verify that there is a root \( (q = 0) \) corresponding to the eigenvalue \( \omega_0 = 0 \), and for it

\[ X_0 = \frac{\hat{j}^2}{V v_j}; \quad X_1 = \frac{T}{V}; \quad X_2 = 1 \] (35)

Using these values, the second eq. (34) becomes an identity. Thus the root \( \omega_0 = 0 \) is always present. The first eq. (34) determines the ratio

\[ \Xi_q / \Lambda_q = -V / T \] (36)

According to eq. (12), the matrix elements of the Hamiltonian between the I.A.S. state \( \xi^+ | j \rangle \) and the states \( \Gamma_q^+ | j \rangle \) are given by

\[ \langle q | H_{\text{coupl}} | \xi \rangle = \frac{1}{\sqrt{T}} \lambda_{qj} \hat{j} (\epsilon_{0j} - \alpha s_j - \omega_\xi) \] (37)

We remind that this schematic model does not represent quantitatively any physical situation. We rather use it as a qualitative check of the formalism in situations where isospin invariance may be invoked.

We have solved the system of eqs. (34) using the single-particle states \( v_j \) and factors \( v_j \) in matrix elements, listed in Table I. There are \( \Omega = 6 \) unperturbed configurations, and the isospin of the g.s. has the value \( T = 22 \) (as in \(^{208}\text{Pb}\)).

<table>
<thead>
<tr>
<th>#</th>
<th>( j )</th>
<th>( \epsilon_{0j} )</th>
<th>( v_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9/2</td>
<td>0.630</td>
<td>0.100</td>
</tr>
<tr>
<td>2</td>
<td>7/2</td>
<td>1.260</td>
<td>0.200</td>
</tr>
<tr>
<td>3</td>
<td>13/2</td>
<td>1.890</td>
<td>0.300</td>
</tr>
<tr>
<td>4</td>
<td>3/2</td>
<td>2.520</td>
<td>0.400</td>
</tr>
<tr>
<td>5</td>
<td>5/2</td>
<td>3.150</td>
<td>0.500</td>
</tr>
<tr>
<td>6</td>
<td>1/2</td>
<td>3.780</td>
<td>0.600</td>
</tr>
</tbody>
</table>

The parameter $\alpha$ is varied within the interval (-1,1). The isospin symmetry is fully restored for $\alpha = 1$. The TDA intrinsic, $\Omega - 1 = 5$, roots $\omega_q$ are shown in Fig. 1 as a function of $\alpha$, together with the energy $\omega_\xi$ of the collective I.A.S. The root $\omega_0 = 0$ is present for any value of $\alpha$. It must not be confused with the root $\omega_\xi$, which vanishes only for $\alpha = 1$. This behavior clearly illustrates the well known but frequently ignored fact that the presence of a zero frequency eigenvalue does not imply the restoration of the symmetry.

However, the admixture between the I.A.S. and the states $q$ cannot be inferred from the results displayed in Fig. 1. One must diagonalize the matrix (37). The results are shown in Fig. 2, where the physical roots $E_k$ (16) are represented as a function of $\alpha$. For comparison, we have included also the value of $\omega_\xi$, which has no meaning now, since it has been included already in the diagonalization. Several consequences are extracted from the results shown in Fig. 2, namely:

i) there is no $E_k = 0$ root, except for $\alpha = 1$;

ii) in contrast with the smooth behavior shown in Fig. 1, every energy eigenvalue is affected by the coupling between the collective and intrinsic states;

iii) the number of roots is again the original number of $j$-states, $\Omega = 6$;

iv) in the symmetry limit $\alpha = 1$ the lowest root corresponds to the collective I.A.S.

Figure 3 shows the energy centroid $\bar{E}$ of the states populated by the Fermi transition from the ground state and the spread $\sigma$ (20), as a function of the parameter $\alpha$. Note that $\bar{E} \approx \omega_\xi$. The distribution narrows as $\alpha$ approaches the symmetry point ($\alpha = 1$).

4. Model II

Since model II is supposed to be realistic, we start discussing with some detail the single-particle states. They are created by the operators $b_{p\omega jm}^\dagger$ for protons and by the $c_{n\nu jm}^\dagger$ for neutrons. Here $\omega, \nu$ stand for the sequential number of times that the state with the same $j$ and $l$ appears (the label $j$ includes the orbital angular momentum $l$). We allow for the fact that the proton and the neutron bases may be different. For instance, they may be obtained from Woods-Saxon potentials with different parameters. We denote by $c_{p\nu jm}^\dagger$ the proton creation operator that is obtained from the $c_{n\nu jm}^\dagger$ through the
We add counterterms that may introduce significant errors in the population of the appearance of a zero-energy root, may introduce significant errors in the population of the appearance of a zero-energy root.

charge conjugation. It may be expanded in terms of the $b^{+}_{pωjm}$ operators with the same values of $j, m$

$$c_{pωjm}^{+} = x_{νωj} b^{+}_{pωjm} \quad (38)$$

For the Hamiltonian, we assume total and Coulomb single-particle Hamiltonians

$$H^{(c)}_{sp} = e_{pωj} b^{+}_{pωjm} b_{pωjm} + e_{νωj} c^{+}_{nνjm} c_{nνjm} \quad (39)$$

$$H^{(q)}_{sp} = (pωj | ν_{ωj} | ψ j) b^{+}_{pωjm} b_{pσjm} \quad (40)$$

If the Coulomb term is the only isospin symmetry breaking contribution, the difference

$$H^{(c)}_{sp} = H^{(c)}_{sp} - H^{(q)}_{sp} \quad (41)$$

should be attributed to the Hartree-Fock contribution of an (unknown) isospin conserving two-body interaction. We may replace this interaction with a contribution that restores the isospin symmetry. To do so, we follow the treatment of the motion of the center of mass developed in Ref. 13. (The commonly used procedure of adjusting constants to ensure the appearance of a zero-energy root, may introduce significant errors in the population of the $q$-modes through the operator $τ_1$, and thus in the calculation of the width of the I.A.S.) Thus we add counterterms

$$H^{(c)} = H^{(c)}_{sp} - T_1 τ_1 - T_1 τ_1 \quad (42)$$

The operators $T_{±1}$ are determined by the requirement that $H^{(c)}$ is invariant under isospin transformations.

We perform the calculations indicated in the previous section for this Hamiltonian and we obtain the expressions

$$\omega_κ = -\frac{1}{T} \left( ⟨H^{(c)}_{sp10}⟩ - ⟨H^{(q)}_{sp10}⟩ \right)$$

$$h_0 = H^{(c)}_{sp} - (T_1 τ_{-1} + T_1 τ_1)$$

$$h_{±1} = H^{(c)}_{sp1t} - H^{(q)}_{sp1t}$$

$$W = H^{(c)}_{sp} + \frac{1}{T} \left( H^{(c)}_{sp1τ_{-1}} + H^{(c)}_{sp1τ_{+1}} \right)$$

$$\Xi^+ = H^{(q)}_{sp1τ_{1}} + \frac{1}{T} ⟨H^{(q)}_{sp10}⟩ τ_1$$ \quad (43)

The unperturbed particle-hole creation operators $γ_{taj}^{±}$ include a quantum number $t = ±1$, according to whether they increase/decrease the isospin projection and a label $a = a(ω, ν) = 1, 2, \ldots$ specifying the proton and neutron configuration

$$γ^{+}_{taj} = \frac{1}{j} b^{+}_{pωjm} c_{nωjm}$$

$$γ^{+}_{taj} = \frac{1}{j} n_{νωjm} b_{pωjm} \quad (44)$$

The RPA expressions for the isospin components and for the single-particle Hamiltonian components carrying $t = ±1$ are

$$τ_1 = -j \left( η_{taj} γ^{+}_{taj} + η_{taj} γ_{taj} \right)$$

$$H^{(c)}_{sp1τ_{1}} = -j \left( Δ_{taj} γ_{taj}^{+} - Δ_{taj} η_{taj} γ_{taj} \right) \quad (45)$$

The uncoupled boson creation operators $Γ^{+}_{tq}$ carry also the quantum number $q$ and they are also labelled by the sequential number $q = 1, 2, \ldots$. They are written as the linear combination

$$Γ^{+}_{tq} = λ_{tqaj} γ^{+}_{taj} - μ_{tqaj} γ_{taj} \quad (46)$$

As usual, the linearization equation $[W, Γ^{+}_{tq}] = ω_{tq} Γ^{+}_{tq}$ determines both the finite frequencies $ω_{tq}$ and the amplitudes $λ_{tqaj}, μ_{tqaj}$. The coupling between the I.A.S. and the intrinsic phonons $⟨q⟩$ is obtained by means of expressions

$$h_t = -j \left( η_{taj} γ_{taj}^{+} + η_{taj} γ_{taj} \right)$$

$$Q_{tq} = -j \left( η_{taj} λ_{tqaj} - η_{taj} μ_{tqaj} \right) \quad (47)$$

where

$$Q_{tq} = -j \left( Q_{tq} λ_{tqaj} - Q_{tq} μ_{tqaj} \right) \quad (48)$$

and therefore
\[ \langle q | H_{\text{coup}} | \xi \rangle = Q_{1q} \]  
(49)

We have calculated the expressions obtained from Model II, starting from the separate diagonalization of a Woods-Saxon potential for protons and neutrons. The parameters have been taken from Ref. 7. The central part of the potential has the strength
\[ V_0^{(p)} = -51 \text{MeV}; \quad V_0^{(n)} = \left( -51 + 33 \frac{N-Z}{A} \right) \text{MeV}, \]  
(50)
while the spin orbit strength is
\[ V_{so}^{(p)} = -0.44 V_0^{(q)}. \]  
(51)
The radius \( R_0 \) and diffuseness \( a_0 \) are fixed at the values 1.27\( A^{1/3} \text{ fm} \) and 0.67 fm, respectively. We have parametrized the Coulomb potential at the interior as
\[ V_c(r) = \left( 2.16 \frac{Z}{R_0} - 0.72 \frac{Z}{R_0^3} r^2 \right) \text{MeV fm}. \]  
(52)
The coefficients \( x_{\nu a j} \) in eq. (38) have been obtained as follows: firstly, we have performed a diagonalization of the W.S. neutron potential in a harmonic oscillator basis, for each value of \((l, j)\). The radial part of the states is
\[ \phi_{\alpha, lj}^{(n)}(r) = c_{\alpha, Nlj}^{(n)} \psi_{Nlj}(r), \]  
(53)
where \( N \) is the principal quantum number, and \( \psi_{Nlj} \), harmonic oscillator wave functions. Similarly for protons
\[ \phi_{\beta, lj}^{(p)}(r) = a_{\beta, Nlj}^{(p)} \psi_{Nlj}(r). \]  
(54)
The inclusion of the Coulomb interaction requires an additional diagonalization in the basis of eigenstates \( \phi_{\beta, lj}^{(p)}(r) \). The resulting eigenstates may be expressed in terms of the \( \phi_{\beta, lj}^{(p)}(r) \)’s
\[ \Phi_{\gamma, lj}^{(p)}(r) = b_{\beta, lj}^{(p)} \phi_{\beta, lj}^{(p)}(r) \]  
(55)
Replacing the eigenstates of the W-S potential (protons without Coulomb) and using the completeness of the harmonic oscillator basis, the proton states may be expressed as linear combinations (38) of neutron states, where
\[ \eta_{\gamma, \alpha, lj} = b_{\beta, lj}^{(\gamma, p)} a_{Nlj}^{(\beta, p)} c_{\alpha, Nlj}^{(\alpha, n)}. \]  
(56)
The calculations have been performed for \( A = 208 \) \((0^+)\) states in \( ^{208}\text{Bi} \) as proton (particle) - neutron (hole) excitations on the ground state of \( ^{208}\text{Pb} \). We have included seven major harmonic oscillator shells in the calculation. This is the space which exhausts all possible transitions, connecting with the neutron excess region, to be included in the description of the isospin dependent monopole excitation.

For the considered single-particle basis we have obtained the value \( \omega_{\text{c}} = 18.17 \text{ MeV} \), a value which is indeed already quite comparable with the experimental value \( E_{\text{I.A.S}} = 17.852 \text{ MeV} \). Note that we are given the value of the I.A.S energy respect to the ground state of \( ^{208}\text{Pb} \). The experimental value \( E_{\text{I.A.S}} = 15.172 \text{ MeV} \), which is the energy measured from the ground state of \( ^{208}\text{Bi} \), is obtained after substraction of the neutron-proton mass difference (1.29 MeV) and the difference in the energies of the ground state of \( ^{208}\text{Bi} \) and \( ^{208}\text{Pb} \) (1.39 MeV). As said in the previous section, the RPA spectrum contains negative energy eigenvalues, a zero energy root and the non-zero energy eigenvalues, which include the excitations involving the neutron excess as well as the ones with relates single particle states two shells above and below the active ones. Like it has been done, for the case of the schematic Model I, we have solved the system of equations which couples the I.A.S with the RPA excitations. The resulting spectrum is shown in Fig. 4. The average excitation energy, determined from the eigenvalues and matrix elements for transitions between the perturbed states and the ground state, is equal to \( E_{\text{average}} = 18.17 \text{ MeV} \) and the spreading width is of the order of 84 keV. Again, these results agree rather nicely with the measured values [3].

\[ \text{FIGURE 4. Strength distribution, in percentage, for Fermi transitions from the ground state of } ^{208}\text{Pb}(\text{g.s}) \text{ to } ^{208}\text{Bi}, \text{ see Eq.(23).} \]

The values are the square of the coefficients \( c_{\nu}(\xi) \), since the total strength is equal to 2\( T \). The energies are measured from the ground state of \( ^{208}\text{Pb} \). The largest contribution corresponds to the excitation of the I.A.S. The results correspond to the realistic calculation described in section 4.
5. Conclusions

In this paper we have studied the effects of isospin symmetry violation in the description of the I.A.S. The formalism, based on the introduction of collective and intrinsic variables, allows for the treatment of both spurious and physical isospin symmetry violations. The formalism has been applied to two model situations, to illustrate the steps which we have followed in constructing the theory. The results of realistic calculations, performed for the case of $A = 208$, show that the predictions of the theory are in excellent agreement with data. Work is in progress concerning the application of the present formalism to the calculation of the structure of Fermi beta decay transitions between mirror nuclei, and on the comparison between the present method and other approaches predicting both the displacement and the width of the I.A.S. [14].

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