Electromagnetic form factors and the hypercentral constituent quark model

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Recibido el 20 de enero de 2004; aceptado el 23 de mayo de 2004

We briefly report on results about the electromagnetic form factors of the nucleon obtained with different models and then we concentrate our attention on recent results obtained with the hypercentral constituent quark model (hCQM).

Keywords: Hyperon; pentaquarks, spin-flavor

Se presenta una reseña de los resultados para los factores de forma del nucleón obtenidos en varios modelos. Se discute en forma más detallada el modelo hipercentral de quarks constituyentes.

Descriptores: Hiperones; pentaquarks, espin-sabor.

PACS: 13.40 Gp

1. Introduction

The new data on the ratio of the electric and magnetic form factors of the proton [1, 2] showing an unexpected decrease with \( Q^2 \) have triggered again the interest in the description of the internal nucleon structure in terms of various effective models: bag models, chiral soliton models, quark-diquark, constituent quark models, etc. The proton has an excitation spectrum and a finite size: these two properties are strictly related and are both an indication of the composite character of the proton. Already in 1973 Iachello, Jackson and Landé [3] were able to obtain a good reproduction of all the existing nucleon form factors data using a Vector Meson Dominance (VMD) model introducing an intrinsic form factor to describe the internal structure of the nucleon. The results of the original fit, if one plots the ratio of \( G_E/G_M \), show not only a decrease with \( Q^2 \) but also a crossing of the zero at about \( 8 \text{ GeV}^2 \). In 1995 using a constituent quark model Cardarelli et al. [4] have calculated the e.m. form factors of the nucleon in a light front approach fitting the SLAC data [5, 6] by means of form factors for the constituent quarks. Frank et al. [7] have constructed a relativistic light cone constituent quark model and calculated the electric and magnetic form factors of the proton. If one plots their original results as a ratio of the electric and magnetic form factors one can see a strong decrease with \( Q^2 \) and the presence of a zero in the electric form factor at \( Q^2 = 6 \text{ GeV}^2 \). In 2002 Miller [8] with a refined version of the model has improved the reproduction of the decrease with \( Q^2 \) of the ratio. In 1999 [9] with a simple non relativistic quark model, the hCQM [10], boosting the initial and final state to the Breit Frame and considering relativistic corrections to the non relativistic current [11] we have shown explicitly that the decrease is a relativistic effect [9] and it disappears without these corrections [9, 12, 13]. This calculation makes use of the nucleon form factors previously determined [11]. Using a chiral CQM and a point form dynamics the Pavia-Graz group [14, 15] has shown a good reproduction of the form factors and of the ratio up to \( 4 \text{ GeV}^2 \). In 1996 Holzwarth [16] has shown that the simple Skyrme model, with vector meson corrections and with the nucleon initial and final states boosted to the Breit Frame, leads to \( G_E^p \), that decreases with \( Q^2 \) and crosses zero at \( 10 \text{ GeV}^2 \). In the MIT Bag model it is expected a sharp decrease and a zero at \( Q^2 = 1.5 \text{ GeV}^2 \) with a change of sign, but with a cloudy bag model it is shown that the inclusion of the pion cloud not only improves the static properties of the model and restore the chiral symmetry but also improves the behavior of the ratio \( G_E^p/G_M^p \) [17–19].

Finally we can say that the extended VMD model by Lomon [20], the soliton model calculation by Holzwarth [16], the calculation by Miller [8] and the relativistic quark spectator-diquark model calculation by Ma et al. [21] describe the new Jlab data quite well.

In the following we will concentrate our attention on the results obtained with a very simple CQM, the hypercentral constituent quark model [10]. We shall introduce the model and then we shall show the new results about the e.m. form factors obtained with a relativistic version of the model and a relativistic current.

2. The hypercentral model

The experimental 4 and 3 star non strange resonances can be arranged in \( SU(6) \) multiplets. This means that the quark dynamics has a dominant \( SU(6) - \) invariant part, which accounts for the average multiplet energies. In the hCQM it is assumed to be [10]

\[
V(x) = -\frac{\tau}{x} + \alpha x,
\]

where \( x \) is the hyperradius

\[
x = \sqrt{\rho^2 + \lambda^2},
\]
where $\tilde{r}$ and $\tilde{\lambda}$ are the Jacobi coordinates describing the internal quark motion. The dependence of the potential on the hyperangle $\xi = \text{arctg}(\xi')$ has been neglected.

Interactions of the type linear plus Coulomb-like have been used since long time for the meson sector, e.g. the Cornell potential. This form has been supported by recent Lattice QCD calculations [22].

In the case of baryons a so called hypercentral approximation has been introduced [23, 24], this approximation amounts to average any two-body potential for the three quark system over the hyperangle $\xi$ and works quite well, specially for the lower part of the spectrum [25]. In this respect, the hypercentral potential Eq.1 can be considered as the hypercentral approximation of the Lattice QCD potential. On the other hand, the hyperradius $x$ is a collective coordinate and therefore the hypercentral potential contains also three-body effects.

The hypercoulomb term $1/x$ has important properties [10, 26]: it can be solved analytically and the resulting form factors have a power-law behaviour, at variance with the widely used harmonic oscillator; moreover, the negative parity states are exactly degenerate with the first positive parity excitation, providing a good starting point for the description of the spectrum.

The splittings within the multiplets are produced by a perturbative term breaking the SU(6) symmetry, which, as a first approximation, can be assumed to be the standard hyperfine approximation [23, 24], this approximation has been introduced [23, 24], this approximation has been published three years in advance with respect to the recent TJNAF data [35]. We have also calculated the longitudinal amplitudes in the Breit frame [30]. The hCQM results obtained in [39] in order to reproduce the $\Delta N$ splitting within the multiplets is:

$$H = \frac{p^2}{2m} + \frac{p_\rho^2}{2m} - \frac{x}{x} + \alpha x + H_{h\text{yp}}, \quad (3)$$

where $m$ is the quark mass (taken equal to $1/3$ of the nucleon mass). The strength of the hyperfine interaction is determined in order to reproduce the $\Delta - N$ mass difference, the remaining two free parameters are fitted to the spectrum, reported in Fig. 1, leading to the following values $\alpha = 1.61 \, fm^{-2}$, $\tau = 4.59$.

Keeping these parameters fixed, the model has been applied to calculate various physical quantities of interest: the

\begin{itemize}
  \item the electromagnetic transition amplitudes [30], the elastic nucleon form factors [11] and the ratio between the electric and magnetic form factors of the proton [13].
\end{itemize}

### 3. The results

The electromagnetic transition amplitudes are defined as the matrix elements of the electromagnetic interaction, between the nucleon, $N$, and the resonance, $B$, states:

$$A_{1/2} = \langle B, J', J'' \rangle = \frac{1}{2} \langle H_{em}^* | N, J = \frac{1}{2}, J_z = \frac{1}{2} \rangle \xi$$

$$A_{3/2} = \langle B, J', J'' \rangle = \frac{3}{2} \langle H_{em}^* | N, J = \frac{1}{2}, J_z = \frac{3}{2} \rangle \xi$$

$$S_{1/2} = \langle B, J', J'' \rangle = \frac{1}{2} \langle H_{em}^* | N, J = \frac{1}{2}, J_z = \frac{1}{2} \rangle \xi \quad (4)$$

where $\xi$ is the sign of the $N\pi$ amplitude.

The proton photocouplings of the hCQM [29] (Eq. 4) calculated at the photon point, in comparison with other calculations [32–34], have the same overall behaviour, having the same SU(6) structure in common, and they all show a lack of strength: considering the square modulus of the helicity amplitudes these discrepancies, on average, range from 30 to 50%.

Taking into account the $Q^2$ behaviour of the transition matrix elements of Eq. (4), one can calculate the hCQM helicity amplitudes in the Breit frame [30]. The hCQM results for the $D_{13}(1520)$ and the $S_{11}(1535)$ resonances [30] are given in Fig. 2 and 3, respectively. The agreement in the case of the $S_{11}$ is remarkable, the more so since the hCQM curve has been published three years in advance with respect to the recent TJNAF data [35]. We have also calculated the longitudinal helicity amplitudes [37]. In general the $Q^2$ behaviour is reproduced, except for discrepancies at small $Q^2$, especially in the $A_{1/2}^\ell$ amplitude of the transition to the $D_{13}(1520)$ state. The kinematical relativistic corrections at the level of boosting the nucleon and the resonance states to a common frame are not responsible for these discrepancies, as we have demonstrated in Ref. [38]. These discrepancies, as the ones observed in the photocouplings, can be ascribed to the lack of explicit quark-antiquark configurations, which may be important at low $Q^2$: in reference [40] is shown that pion loops are important in this $Q^2$ region. Similar results are obtained for the other negative parity resonances [30]. It should be mentioned that the r.m.s. radius of the proton corresponding to the parameters of the hypercentral potential determined in the previous section is 0.48 fm, which is the same value obtained in [39] in order to reproduce the $D_{13}$ photocoupling.

For example, for the Delta resonance the contribution of the pion cloud is very important [40]. For the transverse amplitudes $A_{1/2}$ and $A_{3/2}$ it is about 50% at low $Q^2$ and for the longitudinal amplitude as well as for the electric amplitude the pion cloud is absolutely dominant.

![Figure 1. The spectrum obtained with the hypercentral model Eq.(3) and the parameters Eq. (4) (full lines), compared with the experimental data of PDG [28] (grey boxes).](image-url)
where $V(x)$ is the linear plus hypercoulomb SU(6)-invariant potential of Eq. 1, while the remaining terms are the residual SU(6)-breaking interactions, responsible for the splittings within the multiplets. $H_S$ is a smeared standard hyperfine term, $H_I$ is isospin dependent and $H_{SI}$ spin-isospin dependent. The resulting spectrum for the 3 and 4 star resonances is shown in Fig. 4 [43]. The contribution of the hyperfine interaction to the $N-\Delta$ mass difference is in this case only about 35%, while the remaining splitting comes from the spin-isospin term, (50%), and from the isospin one, (15%). It should be noted that the position of the Roper and the negative parity states is well reproduced.

5. Relativity

The relativistic effects that one can introduce starting from a non relativistic quark model are: a) the relativistic kinetic energy; b) the boosts from the rest frames of the initial and final baryon to a common (say the Breit) frame; c) a relativistic quark current. All these features are not equivalent to a fully relativistic dynamics, which is still beyond the present capabilities of the various models.

The potential of Eq.1 has been refitted using the correct relativistic kinetic energy [44]

$$H_{rel} = \sum_{i=1}^{3} \sqrt{p_i^2 + m^2} - \tau x + \alpha x + H_{hyp}. \quad (6)$$

The resulting spectrum is not much different from the non relativistic one and the parameters of the potential are only slightly modified.

The boosts and a relativistic quark current expanded up to lowest order in the quark momenta has been used both for the elastic form factors of the nucleon [11] and the helicity amplitudes [38]. For the elastic form factors, the relativistic effects are quite strong and bring the theoretical curves much closer to the data; in any case they are responsible for the decrease of the ratio between the electric and magnetic proton form factors, as it has been shown in Ref. [13], in qualitative agreement with the recent Jlab data [45, 46].

4. The isospin dependence

In the chiral Constituent Quark Model [41, 42], the non confining part of the potential is provided by the interaction with the Goldstone bosons, giving rise to a spin- and flavour-dependent term, which is crucial in this approach for the description of the lower part of the spectrum. More generally, one can expect that the quark-antiquark pair production can lead to an effective residual quark interaction containing an isospin (flavour) dependent term.

Therefore, we have introduced isospin dependent terms in the hCQM hamiltonian. The complete interaction used is given by [43]

$$H_{int} = V(x) + H_S + H_I + H_{SI}, \quad (5)$$

FIGURE 2. The helicity amplitudes for the $D_{13}(1520)$ resonance, calculated with the hCQM of Eqs. (3) and (4) (full curve, [30]). The dashed curve is obtained with the analytical version of the hCQM ([26]), where the behaviour of the quark wave function is determined mainly by the hypercoulomb potential. The data are from the compilation of ref. [31].

FIGURE 3. The helicity amplitudes for the $S_{11}(1535)$ resonance, calculated with the hCQM of Eqs. (3) and (4) (dotted curve, [30]) and the model of ref. [33] (full curve).

FIGURE 4. The spectrum obtained with the hypercentral model containing isospin dependent terms Eq. (7) [43] (full lines), compared with the experimental data of PDG [28] (grey boxes).
A relativistic quark current, with no expansion in the quark momenta, and the boosts to the Breit frame have been applied to the calculation of the elastic form factors in the relativistic version of the hCQM Eq. (6) [44]. The resulting theoretical form factors of the proton, calculated without free parameters and using small quark form factors, are good and the result for the ratio $G_E/G_M$ is reported in Fig. 5 [44,47].

6. Conclusions

The hCQM is a generalization to the baryon sector of the widely used quark-antiquark potential containing a coulomb plus a linear confining term. The three free parameters have been adjusted to fit the spectrum [10] and then the model has been used for a systematic calculation of various physical quantities: the photocouplings [29], the helicity amplitudes for the electromagnetic excitation of negative parity baryon resonances [30, 37, 38], the elastic form factors of the nucleon [11,44] and the ratio between the electric and magnetic proton form factors [13,44,47]. The agreement with data is quite good, specially for the helicity amplitudes, which are reproduced in the medium-high $Q^2$ behaviour, leaving some discrepancies at low (or zero) $Q^2$, where the lacking quark-antiquark contributions are expected to be effective.

31. V.D. Burkert, private communication.
37. M.M. Giannini, E. Santopinto, and A. Vassallo, to be published.
44. M. De Sanctis, M.M. Giannini, E. Santopinto, and A. Vassallo, to be published.