Measurement and prediction of springback in bending beams

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Springback is the elastic recovery of a material when it is loaded into the plastic region and then unloaded. Springback is an implicit phenomenon in bending operations. A good knowledge of springback is essential in order to limit the number of iterations required in order to fabricate forming tools and dies within specified tolerances. This work presents the results of a systematic study of springback in beams using different methods and considering different materials and thicknesses. Springback was determined using experimental testing with strain gauges, and the results obtained are compared with analytical and finite element analysis (FEA) results. Very good agreement was obtained in all cases considered.

Keywords: Springback; strain gauges; finite element analysis

1. Introduction

Sheet metal forming is one of the most important manufacturing processes for mass production in industry. There are several kinds of sheet-forming operations and bending is involved in most of them. In the bending process, plastic deformation is followed by some elastic recovery upon unloading. This elastic recovery is called springback. Springback behavior depends on material properties, geometry and tooling dimensions. The springback effect occurs not only in flat sheets or plates, it is also present in rods, wires and beams with any cross-section.

For decades, the springback phenomenon has been analysed from different points of view. Correction curves based on empirical information have been developed [1, 2]. Many analytical solutions have been derived [3–6]; some reviews have appeared in recent works [7–9], and springback has been analysed using the finite element method [10, 11]. Many publications present the results of theoretical analysis or finite element method and the results of experimental work separately. In this work, the springback in bending of beams was measured and compared with results obtained from analytical and finite element methods.

2. Material

Beams with different thicknesses 6, 8, and 12.7 mm, and two different materials, EN8 and EN3B steels, were tested. All the specimens were annealed in order to relieve any previous residual stresses caused by processing and fabrication. The specimens were placed in a steel foil jacket in order to reduce oxidation problems and annealed in a furnace at 660°C for one hour. Due to the annealing process, residual stresses and the Bauschinger effect are not considered in this work. The elastic and plastic properties of all the materials considered are given in Table I. Strain gauges were attached to the specimens in order to obtain the load-strain data required to determine the radius of curvature before and after loading which shows the springback.

3. Experimental procedure

The beams were tested in a four point bending configuration as illustrated in Fig. 1 and a load was applied large enough to

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$(GPa)</th>
<th>$\sigma_y$(MPa)</th>
<th>$\varepsilon$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EN8</td>
<td>210</td>
<td>695</td>
<td>0.0033095</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>725</td>
<td>0.100000</td>
<td></td>
</tr>
<tr>
<td>EN3B-8</td>
<td>210</td>
<td>495</td>
<td>0.002357</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>550</td>
<td>0.100000</td>
<td></td>
</tr>
<tr>
<td>EN3B-6</td>
<td>210</td>
<td>535</td>
<td>0.002547</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>595</td>
<td>0.100000</td>
<td></td>
</tr>
</tbody>
</table>
produce a moment higher than the maximum elastic moment but lower than the complete plastic moment (see Fig. 2). After bending, the load was released and data were collected using a computer and a data logger in order to obtain the moment-strain curves as shown in Fig. 3. Although the radius of curvature could be obtained from the strain and beam thickness dimensions only, Vernier callipers were also used to collect the deflection data after loading and unloading. These data provided an additional means of verifying the experimental testing process and results.

4. Theory

4.1. Radius of curvature from strain gauge readings

It is possible to obtain the radius of curvature if the strain and thickness of the specimen are known [12]. The derivation of the relationship is summarised in what follows. Consider an element $ef$ as shown in Fig. 4a. From Fig. 4b it can be seen that $ef$ is given by Eq. (1) and the strain in $ef$ is given by Eq. (2)

$$
e f = (R + y)d\theta, \quad (1)$$

$$\varepsilon = \frac{ef - gh}{gh}, \quad (2)$$

From Fig. 4a it can be seen that $ef = gf$, and from Fig. 4b, we have $gh = R d\theta$ and $ef = (R + y)d\theta$ as indicated in Eq. (1), hence the strain in $ef$ is obtained as

$$\varepsilon = \frac{y}{R} \quad (3)$$

From Eq. (3), it is possible to calculate the initial and final
radii of curvature by using Eqs. (4) and (5) respectively.

\[
R_o = \frac{h}{2\varepsilon_o} \quad (4)
\]

\[
R_F = \frac{h}{2\varepsilon_f} \quad (5)
\]

where \( h \) is the depth of the beam and \( \varepsilon_o \) and \( \varepsilon_f \) are the surface strains on loading and unloading states respectively. By using the surface strain values in Eqs. (4) and (5), the initial and final radii of curvature can be calculated, and hence the springback is known. Radii of curvature can also be obtained from Vernier calliper measurements by using Eq. (6). The full derivation of the equation is given in Appendix I (see Fig. 5).

\[
R = \frac{(X^2 + \delta^2)}{2\delta} \quad (6)
\]

### 4.2. Analytical equation for springback

Consider a beam with a rectangular cross section made from an elastic perfectly plastic material having a yield strength, \( Y \), loaded to a moment \( M \) which is greater than the yield moment \( M_E \) but less than the plastic moment, \( M_P \), i.e. \( M_E < M < M_P \). Let the yield surface be at a distance \( e \) from the neutral axis as illustrated in Fig. 6. The moment \( M \) is given by Eq. (7) and in this case simplifies to give Eq. (8)

\[
M = \int_{-h/2}^{h/2} b\sigma y dy, \quad (7)
\]

\[
M = \frac{Yb(3h^2 - 4e^2)}{2}. \quad (8)
\]

At the elastic plastic interface we have \( Y = Ee/R \) and hence substituting for \( e \) in Eq. (8) gives

\[
M = \frac{bh^3}{12} \left( \frac{1}{R} - \frac{1}{R_F} \right). \quad (9)
\]

The moment due to springback is obtained from the relationship \( M = EI(1/R_E) \), where \( E \) is the Young’s modulus of the material and \( (1/R_E) \) is the change in curvature, hence

\[
M = E\frac{bh^3}{12} \left( \frac{1}{R} - \frac{1}{R_F} \right). \quad (10)
\]

Equating Eqs. (9) and (10) and simplifying gives the ratio of the initial radius, \( R \), to the final springback radius as

\[
\frac{R}{R_F} = 4 \left( \frac{RY}{Eh} \right)^3 - 3 \left( \frac{RY}{Eh} \right) + 1 \quad (11)
\]
Eq. (11), together with the knowledge of the dimensions and the properties of the material, i.e. \( h, E \) and \( Y \) and the initial radius of curvature, \( R \), can be used to find the final radius \( R_F \).

5. Finite element analysis

In this work, the ABAQUS FEA [13] package was used to predict the springback in the model of the beams that were analysed experimentally and analytically. Plane stress elements were used in order to be consistent with the same underlying assumptions of Eq. (11). The elements were of a quadratic order and the properties of the materials utilized are given in Table I. As with the experimental testing conditions, the beam was analysed in a four point bending configuration as illustrated in Fig. 8.

The analysis was divided into two steps. The load was applied in the first step and removed in the second step. The strain data were collected under loading and after unloading. This allows the radius of curvature to be calculated under load and after unloading using Eqs. (4) and (5). The initial radius of curvature was set to be the same as in the experimental tests, so that the result required is the final radius \( R_F \).

6. Results

As indicated in the introduction section, the main aim of this paper is to provide a complete set of experimental, analytical and numerical analyses of springback in beams. Three sets of tests were carried out and three specimens were tested in each case. Specimens with thicknesses 12.7, 8, and 6 mm are identified as EN8-12.7, EN3B-8 and EN3B-6 respectively.

Experimental tests were first carried out. The radius of curvature obtained using Eqs. (4) and (5), based on strain gauge readings after loading and unloading, are given in Tables II. Those results obtained using Eq. (6), based on Vernier calliper readings, are given in Table III. Table IV shows the initially loaded curvature \( R \) and a comparison of the springback radius \( R_F \), results obtained from experiment and from analytical and finite element methods.
It can be seen from the result in Tables II–IV that very good agreement is obtained from the results of strain gauges and calliper. It can also be seen from the results in the Tables that the springback radii $R_F$ agree very well for all methods considered. As previously reported by other authors [7,11], results obtained in this study also show that springback increases with increasing yield strength, thickness and curvature.

7. Conclusions

The effect of springback was analysed in beams with different thicknesses and materials. Experimental, analytical and numerical methods were used for the analysis. Good agreement was obtained for the results from all methods considered. The paper has provided a systematic study using different methods.

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Appendix I

Determination of radius of curvature from coordinates data

The radius of curvature can be determined from deflection data by using simple trigonometric relationships as follows. From Fig. 4 in the body of the paper and by application of the Pythagorean theorem, we can write

$$\frac{R^2}{2} = \frac{Y_c^2}{2} + \left(\frac{L}{2}\right)^2 \quad (A1)$$

also from the Figure, we have $R = Y + \delta$, hence we can write

$$R^2 = (Y_c + \delta)^2 \quad (A2)$$

By subtracting Eq. (A1) from (A2), and simplifying, we have Eq. (3)

$$Y_c = \frac{L^2}{8\delta} - \frac{\delta}{2} \quad (A3)$$

Since $L = 2X_c$, from Fig. 4, and $R = Y + \delta$, Eq. (A3) can be written as

$$R = \frac{(X_c^2 + \delta^2)}{2\delta}. \quad (A4)$$

4. R.G. Sturm and B.J. Fletcher, Product Engineering (1941) 590.