Dispersion effects on the ray tracing and reflectivity in a hybrid nematic cell under an electric field

C.I. Mendoza* and R. de la Teja

Instituto de Investigaciones en Materiales, Universidad Nacional Autónoma de México, Apartado Postal 70-360, 04510 México, D.F., Mexico,

J.A. Olivares

J.A. Reyes
Instituto de Física, Universidad Nacional Autónoma de México, Apdo. Postal 20-364, 01000 México, D.F., Mexico.

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In this work, we present calculations for the trajectories of an optical beam propagating in a planar-homeotropic hybrid nematic crystal cell. The cell is under a low-frequency electric field applied perpendicular to it and we take into consideration the dependence of the refractive index of the liquid crystal on the wavelength of the optical beam. The presence of the electric field gives rise to trajectories showing a non-trivial dependence of the beam’s range on the intensity of the applied electric field and on the wavelength of the beam. We also calculate the reflectivity curves for this cell and show that the electric field increases the reflectivity for the low beam’s incidence angle and decreases it for a high incidence angle.

Keywords: Liquid crystals; nematic; hybrid; rays; reflectance; dispersion.

En este trabajo mostramos cálculos para las trayectorias de un rayo de luz que se propaga en una celda híbrida planar-homeotrópica de cristal líquido nemático. La celda se somete a un campo eléctrico aplicado de baja frecuencia perpendicular a ella y tomamos en cuenta la dependencia del índice de refracción del cristal líquido en la longitud de onda del rayo luminoso. La presencia del campo eléctrico origina trayectorias que muestran una dependencia no trivial del rango del rayo con la intensidad del campo eléctrico aplicado y con la longitud de onda del rayo luminoso. También calculamos las curvas de reflectividad para esta celda y mostramos que el campo eléctrico incrementa la reflectividad para ángulos de incidencia pequeños del rayo luminoso y la disminuye para ángulos de incidencia grandes.

Descriptores: Cristales líquidos; nemático; híbrida; rayos; reflectancia; dispersión.

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1. Introduction

Application of the total internal reflection (TIR) techniques for studying optical properties of nematic cells with an inhomogeneous distribution of the director field are not common and their use in determining the reflectivity associated with bending ray trajectories in nematic cells is even scarcer. These measurements are of interest since orientational configurations in hybrid cells may be characterized and identified in terms of the behavior of the reflectivity $R$ versus angle of incidence $i$ curves [1]. The motivation of this work is to understand, from the theoretical point of view, the physical origin of the curves $R$ vs. $i$ and its relation to the orientational configuration of the hybrid cell. The nematic orientational configuration is varied using a low-frequency electric field perpendicular to the cell and we use a general theoretical approach developed previously for modeling the propagation of an optical beam in this situation [2]. We consider the optical limit approximation and we take into consideration the dispersion phenomena arising from the wavelength dependence of the refractive indexes of the nematic. Then, the reflectivity curves are obtained for a Gaussian optical beam in the total internal reflection regime. The fact that the beam trajectories depend on the wavelength of the light beam could be useful for the design of optical instruments that separate white light into colors with the characteristic that all the emerging beams are parallel to each other, facilitating their coupling with a CCD detector. The resolution of the instrument could be controlled by varying the electric field applied, because the spread of the light beam is modified by the field.

2. Calculation

We consider a nematic crystal layer of thickness $l$ measured along the $z$ axis and contained between two parallel isotropic dielectric media with refraction index $N_t$ and $N_b$, respectively, as depicted in Fig. 1. The thickness, $l$, is small compared to the transversal dimension, $L$, of the cell plates. We consider a hybrid configuration of the director

$$\mathbf{n} = [\sin \theta(z), 0, \cos \theta(z)],$$

that satisfies the hybrid boundary conditions

$$\theta(z = 0) = 0,$$

$$\theta(z = l) = \frac{\pi}{2},$$

(1) (2)
The dielectric tensor of the nematic configuration of the director’s field are specified by minimizing the free energy functional as shown in Ref. 3. The stationary configuration is then obtained by the corresponding Euler-Lagrange equation, which is obtained by minimizing the free energy functional. This solution reads

\[
\frac{d^2\theta}{d\zeta^2} - q \sin 2\theta(\zeta) = 0,
\]

where we have used the dimensionless variable \( \zeta \equiv z/l \) and the parameter \( q \equiv \varepsilon_a V^2/8\pi K \) measures the coupling between the electric field and the nematic. Here \( \varepsilon_a \) is the low frequency dielectric anisotropy and \( V \equiv E_0 l \) is the applied voltage.

A low intensity incident light beam with \( P \)-polarization \( (P\text{-wave}) \), that is, with the electric field contained in the incidence plane \( x-z \), impinges the nematic with an incident angle \( i \) as shown in Fig. 1. The dynamics of this optical field is described by the corresponding Maxwell’s equations which contain the dielectric tensor \( \varepsilon_{ij} \), Eq. (3), and therefore depend on \( \theta \). The procedure for solving them has been carried out in detail for a hybrid cell similar to the one considered here [4], and it is found that there is a regime for the incidence angle \( i \) where the ray trajectory exhibits a caustic; that is, where it bends and remains inside the cell until it reaches a maximum penetration, \( \zeta_c \) and then returns back towards the incidence substrate (see Fig. 1). This trajectory is given by [4]:

\[
v = \chi - \int_0^\zeta \frac{\varepsilon_{zz} \mp p\sqrt{\varepsilon_{zz} - \varepsilon_{\parallel}}}{\varepsilon_{zz}} d\eta.
\]

In this equation \( \chi \equiv x/l \) and \( p \equiv N_b \sin i \) is the ray component in the \( x \) direction. \( v \) is a constant that is determined by the incident point of the beam on the cell. The \( \pm \) sign in Eq. (5) corresponds to a ray traveling with \( k \) in the \( \pm \) direction, that is, going from \( A \) to \( B \) and from \( B \) to \( C \), respectively (see Fig. 1).

The steady-state orientational configuration is found by solving Eq. (4) subject to the boundary conditions Eqs. (2). This is done in Ref. 2.

As explained in Ref. 3, there are two regimes for \( i \). The first one corresponds to \( i < i_c < 0 \), with \( i_c \) a critical angle, where all the rays always reach the top substrate and part of the ray is transmitted to the top plate. On the other hand, the second regime corresponds to \( i > i_c > 0 \), namely, when the ray does not reach the top substrate and is reflected back to the inside of the cell as depicted in Fig. 1. Besides \( i_c \), there is a second critical angle, \( i_c \), for which the beam no longer penetrates the liquid crystal cell, and at which it is reflected back to the lower substrate. Here we shall consider only angles \( i_c < i < i_2 \) for which the ray penetrates the cell and is reflected back.
The director’s angle at the returning point, $\theta_c$, is given by

$$\theta_c = \arccos \left( \frac{p^2 - \varepsilon_\perp}{\varepsilon_a} \right),$$

from which the critical angles, $i_c$ and $i_o$, can be obtained by substituting $\theta_c = 90^\circ$ and $\theta_c = 0^\circ$, respectively. The other parameters used in the figures were $N_b = N_t = 1.81$.

The range of a bending ray, $\chi_{\text{max}}$, may be calculated from Eq. (5) with $\theta = 0^\circ$. This is shown in Fig. 3, where we show the range as a function of $q$ and $\lambda$. We observe a complex behavior: For $q$ close to $0$ the range is larger for longer wavelengths and smaller for shorter wavelengths. As we increase the magnitude of the field, this is no longer true and for large $q$ the situation is reversed, i.e. the range is larger for shorter wavelengths and smaller for longer wavelengths. For intermediate values of $q$, there is a crossover between these two cases and a region appears where there is a superposition of waves with different wavelengths. The key feature of our system is that, in contrast with the case of the prism, all the outgoing rays with different wavelength are parallel to each other. This is a convenient characteristic that could simplify the design of a multiplexor.

It is possible to extend the model to consider a Gaussian beam instead of an incident plane wave as shown in Ref. 5. Assuming that the set of rays that are part of the incident beam are parallel to each other and that it has a waist $\omega_0$, then, all the reflected beams will also have a Gaussian profile $\exp(-r^2/\omega_0^2)$. Taking a system of axis in $\Sigma$ (see Fig. 4) with the origin in the center of the first reflection, the reflectivity at the central part of the primary reflection is

$$R = |E_R|^2,$$

where the total reflected field, $E_R$, is given by

$$E_R = r_{12} \exp(-t^2 r^2) + (1 - r_{12}^2) \sum_{n=1}^{N_R(i)} \left[ (-r_{12})^{n-1} \exp(jn\Gamma) \exp (-t^2 (r-n\lambda)^2) \right],$$

with

$$r_{12} = \frac{n_\perp n_\parallel \cos i - N_b \sqrt{n_\parallel^2 - N_b^2 \sin^2 i}}{n_\perp n_\parallel \cos i + N_b \sqrt{n_\parallel^2 - N_b^2 \sin^2 i}},$$

the Fresnel coefficient for the substrate-nematic interface. In this equation, $r$ is the radial component in $\Sigma$ (normalized by $l$), and $N_R$ is the total number of reflections in the cell and is defined by

$$N_R = \text{Integer}\left( \frac{L}{\chi_{\text{max}} l} \right),$$

where the right-hand side denotes the integer part of $N_R$. $\Gamma$ in Eq. (8) denotes the phase shift between the ray propagating in the liquid crystal and the one reflected back to the lower substrate, and is given by

$$\Gamma = -2k_0 l \int_{0}^{\frac{\varepsilon_\parallel}{\varepsilon_\perp}} \frac{\varepsilon_\parallel}{\varepsilon_\perp} \left( \frac{\varepsilon_\parallel - \varepsilon_{zz}}{\varepsilon_{zz}} \right) d\eta.$$

The parameter $t$ in Eq. (8) measures the ratio between the cell thickness and the beam waist

$$t \equiv \frac{l}{\omega_0},$$

and the distance $d$ shown in Fig. 4 is given by

$$d = \chi_{\text{max}} \cos i.$$

In Fig. 5 we plot $R$ vs. $i$ for different values of the electric field and wavelength. We can see that the larger the electric field, the more the reflectivity increases for low angles and decreases for larger angles. By changing the wavelength of the incoming beam one changes the critical angles.
3. Conclusions

We have analysed the ray tracing and reflectivity of a $P$-polarized light beam impinging on a nematic hybrid cell under an applied electric field, taking into consideration the wavelength dependence of the refractive indices of the nematic liquid crystal. Our results show that the range and the penetration length of the trajectory of the light beam depends on the color of the beam and that these parameters can be controlled by varying the intensity of the applied electric field. The reflectivity curves for incident beams with a Gaussian profile show a clear influence of both the electric field and the color of the beam. As we increase the value of the electric field, the reflectivity increases for low incidence angles and decreases for large incidence angles. Also, as we increase the wavelength of the beam, the angles of incidence for which there are total internal reflection in the cell shift towards smaller values while keeping the overall shape of the reflectivity curve almost unchanged. These results could be useful for the design of field-controlled wavelength selection devices or for multiplexer applications due to the fact that all the outgoing rays with different wavelength are parallel to each other.

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* Corresponding author: Phone: +52 55 56224644; Fax: +52 55 56161201; e-mail: cmendoza@iim.unam.mx