Progress in understanding the nuclear equation of state at the quark level

A.W. Thomas\textsuperscript{a,b} and P.A.M. Guichon\textsuperscript{c}
\textsuperscript{a}Thomas Jefferson National Accelerator Facility, 12000 Jefferson Ave., Newport News, VA 23606, USA.
\textsuperscript{b}College of William and Mary, Williamsburg, VA 23187, USA.
\textsuperscript{c}SPhN-DAPNIA, CEA Saclay, F91191 Gif sur Yvette, France.

Recibido el 31 de enero de 2007; aceptado el 13 de abril de 2007

At the present time there is a lively debate within the nuclear community concerning the relevance of quark degrees of freedom in understanding nuclear structure. We outline the key issues and review the impressive progress made recently within the framework of the quark-meson coupling model. In particular, we explain in quite general terms how the modification of the internal structure of hadrons in-medium leads naturally to three- and four-body forces, or equivalently, to density dependent effective interactions.

Keywords: Nuclear forces; phenomenological quark models; QCD in nuclei; nuclear matter; neutron stars.

Hoy en día, en la comunidad de física nuclear hay un debate animado sobre la relevancia de los grados de libertad de cuarks en la estructura nuclear. Se presentan los aspectos claves y se discuten los avances impresionantes obtenidos recientemente en el marco del modelo de acoplamiento cuark-meson. En particular, se explica en términos generales cómo la modificación de la estructura interna de los hadrones en el medio lleva de manera natural a interacciones entre tres y cuatro cuerpos, y a interacciones efectivas dependiente de la densidad.

Descriptores: Interacciones nucleares; modelos fenomenológicos de cuarks; QCD en núcleos; materia nuclear; estrellas de neutrones.

PACS: 21.30.-x; 12.39.-x; 24.85.+p; 26.60.+c; 97.60.Jd

1. Introduction

We have a fundamental theory of the strong interaction, namely Quantum Chromodynamics (QCD), the mathematical beauty of which, in combination with its phenomenological successes, has convinced most physicists that it must be correct. Its predictions have been accurately confirmed in the short-distance (or high energy) regime of “asymptotic freedom”. In the opposite limit of long distances (corresponding to quark confinement), it will be extensively tested over the next decade through advances in lattice QCD combined with new experimental facilities, such as the 12 GeV upgrade at Jefferson Lab and the new hadronic capabilities at J-PARC and FAIR. One of the most compelling challenges in modern nuclear physics is to relate the properties of nuclear matter, from finite nuclei to neutron stars, to the underlying quark and gluon structure of matter and QCD itself.

Over the past 50 years the standard approach to understanding nuclear structure has involved the application of non-relativistic many-body theory, based upon nucleon-nucleon (NN) potentials fit to experimental two-nucleon scattering data. Amongst the potentials used at various stages we mention the Paris potential [1], with its intermediate range structure determined through dispersion relations and hence almost model independent. Other approaches which are still widely used are the one-boson-exchange forces, such as the various Bonn [2] and Nijmegen [3] potentials. In its modern form this approach is best represented by the Argonne 18 potential [4], supplemented by a phenomenological three-body force [5]. This has been used in combination with Green function monte-carlo methods to calculate the energy levels of light nuclei.

These models of the nuclear forces have no direct connection to QCD, in that they operate at the purely hadronic level. Another recent approach, which preserves the chiral symmetry of QCD [6], is based on effective field theory (EFT). Such calculations are just now achieving the phenomenological success in fitting NN data that is characteristic of the Bonn and Nijmegen potentials mentioned above – with a similar number of fitting parameters, typically between 20 and 30. The apparatus of EFT is also being exploited to generate the corresponding three-body forces [7], so that one can tackle the energy levels of light nuclei. While the use of EFT permits one to preserve the chiral symmetry of QCD, this is a pale reflection of the full power of QCD itself. Indeed, given the extensive knowledge of chiral symmetry at the time, one could have carried out the EFT program for the NN force in the 1960s, before the discovery of QCD.

Neither the usual treatments of nuclear structure outlined above, nor the usual EFT approach, have yet incorporated the requirements of special relativity. We mentioned earlier the results of the Paris group’s dispersion relation analysis of the nature of the intermediate range NN force. The intermediate range NN force is unambiguously dominated by two-pion exchange with a Lorentz scalar and isoscalar character. Quantum Hadrodynamics (QHD) exploited this by formulating a description of nuclear matter in which Lorentz scalar attraction (represented by a $\sigma$ meson) competed with Lorentz vector repulsion (represented by an $\omega$ meson) [8]. At the saturation density of nuclear matter, $\rho_0$, the mean scalar field strength in QHD was of order 400 MeV. A similar result was found in a relativistic Brueckner Hartree-Fock calculation using a boson exchange potential [9]. Given such a huge scalar field, with a strength almost half of the mass of the...
nucleon itself, one must anticipate that it will have a significant effect on the internal structure of the nucleons making up the nuclear matter. In particular, we recall that the typical energy associated with the excitation of internal degrees of freedom in the nucleon is only 300 MeV (for spin excitations) to 500 MeV (for orbital excitations).

Let us briefly summarize. Any serious treatment of the model independent Lorentz scalar nature of the intermediate range NN force leads to the conclusion that the bound nucleons experience a scalar potential of a strength up to one half of the mass of the nucleon itself – a strength comparable with the typical internal excitation energy of the nucleon. This immediately suggests that the internal structure of the nucleon should indeed play a crucial role in nuclear structure. The popular prejudice which leads many to neglect this role is generally based on the observation that the nucleon binding energy is very small in comparison with its mass. However this small binding actually arises from a cancellation between the scalar and the vector potential in-medium, while the structure modification is expected to arise primarily through the scalar potential.

In order to anticipate the possible consequences of this very general observation, we turn to atomic and molecular systems where we have textbook experience. We know that when an atom is subjected to a strong electric field, its electron structure will rearrange in order to oppose the applied field. This change in the internal structure of the atom, at least if one is concerned solely with describing the energy of the system, can be described in terms of an electric polarizability. In particular, the energy of the system has a term quadratic in the applied electric field, with the coefficient being (one half of) the electric polarizability. Exactly the same thing happens if we apply a magnetic field, with the coefficient of the term quadratic in the applied magnetic field involving the magnetic polarizability.

Turning to the nucleon itself, we know that applied electric and magnetic fields alter its internal structure, giving rise to electric and magnetic polarizabilities and even, in sophisticated electron scattering experiments, the so-called generalized polarizabilities [10, 11]. Given this background it is remarkable that it has taken so long for nuclear physicists to pay serious attention to the response of the nucleon to an applied scalar field, especially its scalar polarizability.

As important as the results of Ref. 21 were, many important issues were left unaddressed:

In the first examination of nuclear matter from this point of view, using the MIT bag model to describe the quark structure of the bound nucleons, Guichon found exactly such a behavior, with \( d = +0.22 R \) and \( R \) the bag radius [12]. Since that time, this model, which is known as the quark-meson coupling (QMC) model, has been extended to describe finite nuclei [13]. It has also been widely applied to the extremely interesting problem of how the structure of hadrons in general might be expected to change in-medium [14, 15] – applications which have included the possibility of \( \eta \) - and \( \omega \)-nucleus bound states [16], the structure of hypernuclei [17] and the modification of the electric and magnetic form factors of bound protons [18], as well as their structure functions [19].

Rather than reviewing these many applications, which have recently been the subject of a major review [14], we return to the atomic and molecular analogy for further insights which, as we shall see, are crucial to understanding the structure of atomic nuclei. When two atoms approach each other they mutually modify their internal structure. This implies that when a third atom approaches it does not see a pair of bare atoms. Consequently, the total energy of the system will not be the sum of the individual pair-wise interactions (potentials) – that is:

\[
V_{\text{tot}} \neq V_{12} + V_{23} + V_{13}
\]  

and the difference is, by definition, a “three-body force”, \( V_{123} \). By analogy, we anticipate that the scalar polarizability of the nucleon will naturally lead to many-body forces in-medium.

In the next section we briefly review the exploration of the origin of many-body forces (or equivalently density dependent effective interactions), arising as a consequence of the modification of the internal structure of the nucleon in-medium, within the framework of QMC. We shall see that it is indeed possible, starting from the quark level, to generate realistic effective interactions of the Skyrme type [20].

### 2. Effective Forces of the Skyrme Type Derived from the Quark Level

In 2004 Guichon and Thomas [21] showed that by expanding about \( \langle \sigma \rangle = 0 \) one could derive an effective force of the Skyrme type (widely used in nuclear structure calculations) in which the local two-body effective interaction was supplemented by three- and four-body forces, proportional to \( d \) and \( d^2 \), respectively. A comparison between the various coefficients in the SkIII force with those derived from the QMC model showed agreement at typically the 10% level. This remarkable result provided the first, direct microscopic connection between a commonly used effective nuclear force and the underlying degrees of freedom of QCD.

As important as the results of Ref. 21 were, many important issues were left unaddressed:
The relatively large scalar field found in nuclear matter means that one would really like to remove the need to expand about $\sigma = 0$.

Modern Skyrme forces tend to include density dependent coefficients (fit to appropriate nuclear data), rather than being density independent with an explicit three-body term. Hence one would really like to rewrite the energy functional for QMC in terms of a density dependent effective force.

A crucial issue at higher densities, when for example one is calculating the equation of state (EoS) for application to neutron stars, concerns the density at which hyperons appear in matter in $\beta$-equilibrium. For this purpose one would very much like to derive effective, density dependent effective hyperon-nucleon and hyperon-hyperon forces. Because one starts from the quark level, QMC is ideally suited for this purpose.

All of these issues were successfully resolved by the recent formulation of Guichon et al. [22]. The resulting energy functional in its full relativistic form was too difficult to use in finite nuclei. However, in that case a non relativistic expansion was allowed and that provided an effective force with finite nuclei. However, in that case a non relativistic expansion was allowed and that provided an effective force with finite nuclei. However, in that case a non relativistic expansion was allowed and that provided an effective force with finite nuclei.

In order to make a quantitative comparison between these two forms of effective potential the SkM form was fit to the QMC form over the density range $(0, \rho_0)$. The comparison between the usual phenomenological values of the constants appearing in the SkM force and those fitted to QMC showed a level of agreement at the 10-20% level. This is reasonable but by no means impressive. On the other hand, since the functional forms are quite different it seemed appropriate to compare the predictions of the two effective forces for real nuclear properties. Accordingly the effective force derived from the QMC model was employed within the Hartree-Fock framework to calculate the properties of closed shell, finite nuclei. The result was extremely gratifying, with the level of agreement between theory and data (where available and phenomenological Skyrme-type forces where data was unavailable) being very good. In Fig. 1 we show a comparison between the charge densities of selected closed shell nuclei calculated within QMC in comparison with experimental data, as well as the widely used Sly4 force [23]. Clearly the level of agreement between the two theories is remarkably good. Given that the QMC calculation had no parameters adjusted to reproduce the properties of finite nuclei, the agreement with the experimental charge densities is also very satisfactory.

As there is no model independent experimental determination of the neutron densities we show, in Fig. 2, a comparison between the neutron densities produced by Sly4 and by the QMC model. Once again, it is reassuring in terms of assessing the quality of the effective interaction derived from the QMC model that they are so close.

Another key test of the model is its ability to describe the dependence of the spin-orbit splittings on both mass number and isospin. At first glance this looks like an area in which QMC might fail, in view of the strong isospin dependence of the effective force derived from QMC. However, as we see in Table I (taken from Ref. 22), this worry is in fact misplaced. Once the interaction is used, within the Hartree-Fock framework, to self-consistently determine the nuclear structure there is agreement between the predictions of QMC and

\[
\langle H(\mathbf{r}) \rangle = \rho M + \frac{\tau}{2M} + H_0 + H_3 + H_{eff} + H_{fin} + H_{SO} \quad (3)
\]

took the form:

\[
H_0 + H_3 = \rho^2 \left[ -\frac{3G_\rho}{32} + \frac{G_\sigma}{8(1 + dG_\sigma)} - \frac{G_\sigma}{2(1 + dG_\sigma)} + \frac{3G_\omega}{8} \right] + (\rho_n - \rho_p)^2 \left[ \frac{5G_\rho}{32} + \frac{G_\sigma}{8(1 + dG_\sigma)} - \frac{G_\omega}{8} \right]. \quad (4)
\]

This appears rather different from commonly used density dependent Skyrme forces such as SkM, which includes a dependence on a fractional power of the density. For comparison, we show for SkM the central pieces corresponding to those that were given for QMC in Eq. (4):

\[
H_0 + H_3 = \rho^{1/6} t_3 \left( 2\rho^2 - \rho_n^2 - \rho_p^2 \right) \frac{24}{24} + t_0 \left( \rho^2(2 + x_0) - \left( 1 + 2x_0 \right) \rho_n^2 + \rho_p^2 \right) \frac{4}{4}. \quad (5)
\]

In order to make a quantitative comparison between these two forms of effective potential the SkM form was fit to the QMC form over the density range $(0, \rho_0)$. The comparison between the usual phenomenological values of the constants appearing in the SkM force and those fitted to QMC showed a level of agreement at the 10-20% level. This is reasonable but by no means impressive. On the other hand, since the functional forms are quite different it seemed appropriate to compare the predictions of the two effective forces for real nuclear properties. Accordingly the effective force derived from the QMC model was employed within the Hartree-Fock framework to calculate the properties of closed shell, finite nuclei. The result was extremely gratifying, with the level of agreement between theory and data (where available and phenomenological Skyrme-type forces where data was unavailable) being very good. In Fig. 1 we show a comparison between the charge densities of selected closed shell nuclei calculated within QMC in comparison with experimental data, as well as the widely used Sly4 force [23]. Clearly the level of agreement between the two theories is remarkably good. Given that the QMC calculation had no parameters adjusted to reproduce the properties of finite nuclei, the agreement with the experimental charge densities is also very satisfactory.

As there is no model independent experimental determination of the neutron densities we show, in Fig. 2, a comparison between the neutron densities produced by Sly4 and by the QMC model. Once again, it is reassuring in terms of assessing the quality of the effective interaction derived from the QMC model that they are so close.

Another key test of the model is its ability to describe the dependence of the spin-orbit splittings on both mass number and isospin. At first glance this looks like an area in which QMC might fail, in view of the strong isospin dependence of the effective force derived from QMC. However, as we see in Table I (taken from Ref. 22), this worry is in fact misplaced. Once the interaction is used, within the Hartree-Fock framework, to self-consistently determine the nuclear structure there is agreement between the predictions of QMC and

\[\text{FIGURE 1. Proton densities calculated using the density dependent effective interaction derived from the QMC model compared with experiment and with the predictions of the Skyrme Sly4 force -- from Ref. 22.}\]
be expected to be reliable much beyond the non-relativistic approximation, which cannot produce just the main ideas of these modern QMC based calculations. The results obtained with the density dependent effective force are particularly impressive. However, they are well known in the case of \(^{40}\text{Ca}\) and \(^{208}\text{Pb}\), there we give the values corresponding to the Skyrme Sly4 force.

Another challenge faced by nuclear theory, as the density increases, is the possible entry of hyperons into the nuclear stew. Hyperon-nucleon forces are unfortunately poorly constrained by data, even in the case of \(\Lambda N\), while in the case of \(\Sigma N\) and \(\Xi N\) the situation is worse. Typical phenomenological forces rely on SU(3) symmetry, a hard path to tread when cancellations between various components of the force are so important. When it comes to using hypernuclear data to constrain the parameters of Skyrme-type forces we note that only a single \(\Sigma\)-hypernucleus has been confirmed and no \(\Xi\)-hypernuclei. For \(\Lambda\)-hypernuclei the situation is a little better. It is little wonder that there is no consensus on the threshold density at which hyperons appear, nor even which one will appear first. As the appearance of hyperons will soften the nuclear equation of state and, for example, raise the critical density for the transition to quark matter, this is a vital issue.

Because the QMC model starts at the quark level, it can be used to calculate density dependent, effective hyperon-nucleon or even hyperon-hyperon forces – with no additional parameters. The only hypothesis needed is that the nuclear fields do not couple to the strange quark. This is certainly a good approximation if the meson fields simulate multi-pion exchanges. It is also supported by the absence of spin-orbit splitting in \(\Lambda\) hypernuclei. As shown in Fig. 4, a striking prediction of the model is that the thresholds for the appearance of strange particles do not follow the usual pattern where the \(\Sigma^-\) and \(\Lambda\) appear first. Because of the scalar polarizability of the baryons and the use of the Hartree-Fock approximation [24] (rather than the mean field approximation), the \(\Xi\) and \(\Lambda\) appear first and at a rather high density (\(\sim 3\rho_0\)). The \(\Sigma\) does not show up in the density domain pertinent to neutron stars. The resulting equation of state is relatively stiff and produces a maximum neutron star mass of about \(2.1M_\odot\), with a rather small central density (\(\sim 6\rho_0\)).
Finally, we note that the QMC model is built upon the MIT bag model. This is a fairly ancient and unsophisticated quark model and although all indications are that the details of the model are not so important one would like to do better. In particular, one would ideally like to have a quark model describing the structure of the hadrons which is covariant, in order to address the problem of nuclear structure functions – the famous EMC effect [26, 27] – with some confidence.

In parallel with the developments concerning QMC, Bentz, Thomas and collaborators have made considerable progress with just such a model. They took the NJL model, modified using proper time regularization in order to simulate confinement. The model is therefore covariant, confining and respects chiral symmetry. Exactly as in QMC, the hadron structure in-medium was self-consistently modified (by solving the corresponding Faddeev equations). Also as in the QMC model the scalar polarizability led to the saturation (by solving the corresponding Faddeev equations). This model respects chiral symmetry. Exactly as in QMC, the scalar polarizability led to the saturation of nuclear matter and, incidentally, solved the long-standing problem of chiral collapse suffered by the NJL model [28].

In application to the nuclear EMC effect the model is able to reproduce the data for unpolarized structure functions [29, 30]. However, even more important, it makes a prediction characteristic of models of the QMC type in which the mean scalar field enhances the lower Dirac components of the confined quark wave functions. (We note that a similar calculation within the chiral quark soliton model [31] produces a qualitatively similar result.) That is, it predicts a polarized EMC effect roughly twice as big as the unpolarized effect. This will provide a critical test of such models.

A final success of this approach has been to use the same underlying quark model to describe matter made of “nucleons” (albeit with modified internal structure), as well as matter made of quarks [32, 33]. Indeed a major discovery of Lawley and collaborators [33] was that, once the effect of the pion cloud of the nucleon [34] is taken into account, one can use the same underlying, quark level Hamiltonian to consistently describe the hadronic, quark matter and superconducting quark matter phases of dense matter. This is a major step forward in a field where one is usually forced to employ completely different models for the different phases. Such calculations are of great interest in view of the difficulties in simulating supernova explosions, as well as because of the very different mass-radius relations that one typically finds for stars containing quark matter.

Acknowledgements

It is a pleasure to acknowledge the collaboration with W. Bentz, I. Cloët, S. Lawley, H. Matevosyan, J. Riskova Stone, K. Saito, N. Sandulescu and K. Tsushima, who made crucial contributions to the work described here. This work was supported by DOE contract DE-AC05-06OR23177, under which Jefferson Science Associates operates Jefferson Lab.
