Electronic transmission for a finite superlattice with a Pascal profile in the wells width

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We calculate the electronic transmission coefficient as a function of the electron energy in a finite semiconductor superlattice where the width of the wells is modulated by a numerical sequence taken from Pascal’s Triangle. This sequence is formed by the quantity of odd numbers in each of the Triangle’s rows and has the pattern 1-2-2-4-2-4-4-8-... The sequence has the property of self-similarity. Our superlattice is based on the AlAs and GaAs materials. The transmission spectrum is intermediate between that produced by a periodic finite superlattice and that produced by a disordered one.

Keywords: Superlattice; quasi-periodic.

1. Introduction

The electronic properties of artificially periodic semiconductor structures have been intensively studied since the work of Tsu and Esaki [1], and nowadays they are well understood. These studies have led to many extensively used devices. Besides, the pioneering work of Yablonovitch [2] opened a new area of research, which is the propagation of electromagnetic waves in periodic dielectric media. The interest lies in the development of structures possessing photonic band gaps. A complete photonic gap means the absence of photon propagation modes for a range of frequencies. The studies of photonic band structures have potential applications in optical and electronic devices. On the other hand, random media have also been explored [3,4]. Nonperiodic but deterministic media constitute a separate field of research. These quasiperiodic systems, intermediate between a periodic structure and a completely disordered one, have also been studied since the discovery of quasi-crystalline materials by Shechtman and co-workers [5]. There are many studies of propagation of electrons, electromagnetic waves and acoustic waves in quasi-periodic multi-layered one-dimensional structures with several profiles like those of Fibonacci, Cantor, Rudin-Shapiro, Thue-Morse, etc. [6-8]. The numerical sequence formed by the quantity of odd numbers in each row of Pascal’s Triangle has the property of self-similarity. In an article published formerly [9] a calculation was made of the electronic transmission for a finite superlattice where the barriers width is modulated by this sequence. Here we theoretically study the transmission of electrons in a finite superlattice made of the materials AlAs and GaAs, where the width of the wells is modulated by this quasi-periodic Pascal’s sequence. We use the one band effective mass framework and the transfer matrix technique.

2. Theoretical model and method of calculation

Pascal’s Triangle is an arithmetical triangle with many interesting mathematical properties [10,11]. We show a section of Pascal’s Triangle in Fig. 1. At the tip of the Triangle is the number 1, which makes up the zeroth row. The first row (1 1) contains two 1’s, both formed by adding the two numbers above them to the left and the right, in this case 1 and 0 (all numbers outside the Triangle are 0’s). We do the same to create the second row: 0 + 1 = 1; 1 + 1 = 2; 1 + 0 = 1. And the third one: 0 + 1 = 1; 1 + 2 = 3; 2 + 1 = 3; 1 + 0 = 1. In this way, the rows of the Triangle go to infinity. Also, the entries of the Triangle are the binomial coefficients. If we count the quantity of odd numbers in each row, we can form the sequence 1-2-2-4-2-4-4-8-... This sequence has the property of self-similarity: if we take the sequence by pairs (1-2)-(2-4)-(2-4)-(4-8)-..., this sequence also follows the original sequence; if we form a sequence of quartets (1-2-2-4)-(2-4-4-8)-..., we see that the quartets follow the original sequence, and also for octets and so on. We can construct the sequence...
3. Results

We consider AlAs-GaAs superlattices. For simplicity we consider a uniform effective mass throughout the structure, \( m^* = 0.067 \, m \), where \( m \) is the free electron mass. The height \( V_o = 0.35 \, \text{eV} \) is the same for all barriers. The width of the barriers is 7.5 \( \text{Å} \), a constant value for all barriers in all superlattices. We have calculated the electronic transmission coefficient for a quasi-periodic Pascal superlattice of 32 wells, corresponding to the sequence \( 1-2-2-4-2-4-4-8-2-4-4-8-4-8-8-16-2-4-4-8-4-8-8-16-4-8-8-16-8-16-16-32 \). In the calculation, the width of the wells is modulated by this Pascal sequence. The minimum width for wells is 7.5 \( \text{Å} \) (first well), corresponding to the number \( l \) of the sequence; the following widths of the wells are proportional to the following numbers of the sequence. We also calculate the transmission for a periodic superlattice of 32 wells. In this case, the width of the wells is 37.96875 \( \text{Å} \), which is the arithmetic mean of the width of the 32 wells of the Pascal superlattice. Moreover, we calculate the transmission for a random superlattice of 32 wells. In this case, the width of the wells follows a random sequence chosen from the Pascal sequence. For our random superlattice the sequence is \( 4-8-32-16-4-8-8-4-16-16-4-8-4-8-4-2-4-8-2-16 \).

Our results of the transmission spectra when we modulate the wells width are presented in Fig. 2 where (a), (b) and (c) plot the periodic, Pascal and random superlattice, respectively. For the purpose of comparison, we also present in plots (d), (e), and (f), taken from Ref. [9], the spectra transmission when the modulation is on the width barriers for periodic, Pascal and random superlattices, respectively. From (a) and (d) plots, we see that the periodic superlattices present the characteristic transmission bands separated by gaps without transmission. Inside the transmission bands, there are oscillations due to resonances of the incident electron energy with the energy eigenvalues of the superlattice. We see from (c) and (f) plots that the random superlattices do not present any regularity in its spectra; there are only peaks in the transmission coefficient, corresponding to the resonances. From plot (b) we see that the spectrum for the Pascal superlattice, when the wells widths are modulated by the Pascal sequence, shows some regularity with gaps centered about 0.13 eV and 0.55 eV, and external to these gaps there are irregular transmission bands with peaks due to the resonances. This behavior is intermediate between that of the periodic superlattice and that of the disordered one. If we compare it with the (e) plot, we notice that the intermediate behavior is more remarkable in the case when the Pascal sequence modulates the barriers width than when it modulates the wells width.

4. Conclusions

We have presented a calculation of the electronic transmission spectrum for a finite superlattice, when a quasi-periodic sequence taken from Pascal’s Triangle modulates the wells width. We compare this spectrum with that produced by a superlattice when the same Pascal sequence modulates the barriers width. In both cases the transmission spectra is intermediate between that of a periodic superlattice and that of a disordered one. The intermediate behavior is more noticeable when the Pascal sequence modulates the barriers width. We think that this fact is due to the asymmetry between the two superlattices. When we modulate the barriers width, the Pascal sequence of barriers is bounded by two wells of infi-
Figure 2.
nite width. On the other hand, for the modulated wells superlattice, the Pascal sequence of wells is bounded by two barriers of finite width, and before and after these first and last barriers, there are two wells of infinite width. We think that the symmetrical physical situation is a superlattice with a Pascal sequence of wells bounded by two barriers of infinite width. In this case, the transmission coefficient would be for unconfined (above barriers) electrons. We will consider this problem in the future.

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