Phase time properties for an uncoupled multicomponent physical heterostructure

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The results of a calculation of the phase time as a function of different parameters of the semiconductor layered media to study are presented. Some general features are reported to be present in the tunneling of uncoupled holes for energies above and under the barrier height. Evidence similar to that observed for electrons and photons as Hartman predicted for the dependence of the tunneling time with the barrier thickness is also reported.

Keywords: Hole tunnelling; transfer matrix; scattering matrix; spin transitions.

Se presentan los resultados del cálculo del tiempo de fase, como función de diferentes parámetros de una heteroestructura semiconductor a capas. Reportamos algunas de las facetas en el tunelaje y la transmisión de huecos desacoplados, para energías por debajo y por arriba de la barrera, respectivamente. Presentamos evidencias similares a aquellas observadas para electrones y fonones, predichas por Hartman para la dependencia del tiempo de tunelaje, como función del espesor de la barrera.

Descriptores: Tunelaje de huecos; matriz de transferencia; matriz de dispersión; transiciones de espín.

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1. Preliminary comments

The question of “how much time does tunneling take” was formulated since the early days of the quantum theory. The striking results found by McColl [1], for Gaussian wave packet, were followed by a large period of debate and alternative proposals of tunneling time definitions [2–7], running from pure semiclassical to fully quantum mechanical models. Accurate experiments with photons and optical pulses [8, 9] became the cornerstones in the tunneling time issue. The excellent agreement of these results with precise phase-time calculations [10] upheld the relevance of the phase time model. Recently, its consistency with the predictions of Maxwell’s equations has been unambiguously shown [11].

Nowadays, the impressive number of low-dimensional semiconductor devices has brought a new urgency to the essential measurement and/or modelling of tunneling time for charge carriers motion. In the last years, attempts to indirectly measure the tunneling time of electrons and holes were reported [12, 13]. Most of the experimental set ups involve other times, such us the scape and decay time. For this reason, their results have become questionable. We are interested here in calculating the tunneling time of light and heavy holes through a single potential barrier. For this purpose, we will calculate the phase time within the Multicomponent Scattering Approach (MSA) [14].

From some theoretical studies [10,15,16] and experimental measurements [9,17,18] in quite different potentials and physical systems, it was suggested that the tunneling time could be independent from the barrier width. Such a disturbing behavior also drove us to disclose some general hole tunneling features related with this effect.

2. Phase transmission time

A detailed description of the quantum transport of heavy (hh) and light (lh) holes through semiconductor heterostructures has been given within the MSA in Ref. 14. This approach is based on the \((N \times N)\) Hamiltonians and the transfer matrix method. Thus, scattering amplitudes can be obtained straightforwardly. To study the hole’s transport, we considered a \((4 \times 4)\) Kohn-Luttinger Hamiltonian, and also simultaneously deal with the 4 accessible physical channels of the system: \(hh_{+3/2}, lh_{-1/2}, lh_{+1/2}, hh_{-3/2}\). Therefore, it allows us to evaluate the scattering amplitudes \(t_{ij}=t_{R,ij}+it_{I,ij}\) from the \(i\)-th incoming channel (incident propagating mode) to the \(j\)-th outgoing channel (reflected and transmitted propagating modes). We emphasize that none of the incoming amplitudes are required to be zeroed, as it is commonly assumed, and hence, a multicomponent description of hole transmission amplitudes is naturally achieved.

Given the transmission amplitudes \(t_{ij}\), we can evaluate the transmission amplitude phases for each of the 16 possible
paths \(ij\) and for the phase times

\[
\tau_{ij} = \frac{\hbar^2}{t_{R_{ij}}^2 + t_{I_{ij}}^2} \frac{\partial}{\partial E} \left[T_{ij}\right].
\]

Here \(E\) is the energy of the incident flux of \(hh\) and \(lh\); \(E\) is taken as positive by a proper choice of the energy origin.

In the present work, we will deal with \(\kappa_T \approx 0\), which excludes the channels' coupling. In this particular case, the transmission amplitude matrix is diagonal, and is given by

\[
t = \frac{1}{\alpha} \text{ with } \alpha_{ii} = \alpha_{R_{ii}} + i\alpha_{I_{ii}}
\]

with \(\alpha\) being the upper left block of the transfer matrix which relates the outgoing propagating modes (at the right hand side) with the incoming ones (at the left hand side). Thus, the phase time can be cast in the form

\[
\tau_{ii} = \hbar T_{ii} \left(\alpha_{R_{ii}} \frac{\partial \alpha_{I_{ii}}}{\partial E} - \alpha_{I_{ii}} \frac{\partial \alpha_{R_{ii}}}{\partial E}\right).
\]

It is worth noticing that in this case, like in the 1D one-channel approximation, the phase time \(\tau_{ii}\) is proportional to the transmission coefficient \(T_{ii}\).

3. Numerical results and discussion

We will assume that our heterostructure \((GaAs/AlAs/GaAs)\) is a perfect-interface layered media with one barrier of width \(L_b\) in the \(10 - 80\) Årange. The barrier height will be taken as \(V_b = 0.498\) eV. As mentioned before, the in-plane momentum \(\kappa_T\) is fixed as \(\approx 0\) (uncoupled holes). Hence, we expect that in only 4 of the 16 possible paths the transition probabilities are different from zero, i.e. only direct transition probabilities \((T_{ii})\) are allowed.
those obtained in the experiment by Dragoman’s et al. compare the order of magnitude of our results in Fig. 1b with idate our numerical simulation with experimental data, we (solid line), although the difference is not remarkable. To val-

light holes (dashed line) propagate faster than the heavy holes $hh$

$\kappa$

$E \approx 0.4$ eV, $\tau_{\text{ii}}$ is sensitive to $L_b$ and presents an oscillating behavior. The resonances are shifted to lower energies and increase their intensities, as $L_b$ increases. This behavior is consistent with the familiar lowering in energy of virtual well quasi-stationary states under similar circumstances. For $E < 0.4$ eV (panel (b)), the values of $\tau_{\text{ii}}$ become autonomous from the barrier width, as Hartman predicted for electrons [15]. These features of hole tunneling qualitatively agree with measurements of transit time for optical pulses through opaque barriers (which transmit about $10^{-4}$ of the incident radiation) [9].

In Fig. 3 we show results for barrier thicknesses $0 < L_b \leq 150$ Å and energies above but close to the barrier height. There are some experimental results by Heberle et al. for $L_b < 80$ Å, in biased double asymmetric QWs, where the recombining electrons and holes first have to stay some time within a narrow quantum well (the scape time $\tau_s$) tunnel, then the barrier towards the neighbor wide well (spending the true tunneling time $\tau_t$), and finally, decay from the excited state after a dwelling time $\tau_d$ (also named the decay time). Since the measured times $\tau_{\text{exp}}$ are equal $\tau_s + \tau_t + \tau_d$, it is clear that $\tau_{\text{exp}}$ may be orders of magnitude larger that the true tunneling time $\tau_t$, and we cannot compare our results with them. The circles in Fig. 3 are the MSA’s numerical results for the tunneling time of direct paths (indicated in the legend) that should be compared with $\tau_t$. Here we must notice that the oscillating behavior is an expected effect due to the barrier’s interference with the continuum states.

We have obtained negative dips of the phase time (see panel (b) of Fig. 1), which might be a consequence of channel thresholds, channel interference, or an artifact of the numerical calculations. A careful investigation of this subject is required.

Summarizing up: The main features revealed for the holes phase time within the selected range of parameters point to, (i) the reduction of this quantity with the incident energy, (ii) an oscillating behavior with the barrier thickness for unbounded continuum states, and finally (iii) at under-barrier energies, the phase time of heavy and light holes is found to become autonomous of the barrier thickness, as was exactly predicted for electron tunneling in Hartman’s classic work [15].