New developments in nuclear supersymmetry*

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We discuss several new developments in nuclear supersymmetry, in particular the identification of a new supersymmetric quartet of nuclei in the $A \sim 190$ mass region consisting of the $^{192,198}$Os and $^{193,194}$Ir nuclei, and a study of correlations between different transfer reactions.

Keywords: Nuclear structure models and methods; supersymmetry; algebraic methods.

Se discuten varios avances en la supersimetría nuclear, pecialmente la identificación de un nuevo cuadruplete supersimétrico de núcleos con masa $A \sim 190$ que consiste en los núcleos $^{192,198}$Os y $^{193,194}$Ir, y un estudio de correlaciones entre reacciones distintas de transferencia de nucleones.

Descriptores: Modelos y métodos en la estructura nuclear; supersimetría; métodos algebraicos.

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1. Introduction

Nuclear structure physics has seen an impressive progress in the development of \textit{ab initio} methods (no-core shell model, Green’s Function Monte Carlo, Coupled Clusters,…), mean-field techniques and effective field theories for which the ultimate goal is \textit{an exact treatment of nuclei utilizing the fundamental interactions between nucleons} [1]. All involve large scale calculations and therefore rely heavily on the available computing power and the development of efficient algorithms to obtain the desired results.

A different, complementary, approach is that of symmetries and algebraic methods. Rather than trying to solve the complex nuclear many-body problem numerically, the aim is to identify effective degrees of freedom, develop schematic models based upon these degrees of freedom and study their solutions by means of symmetries, etc. Aside from their esthetic appeal, symmetries provide energy formula, selection rules and closed expressions for electromagnetic transition rates and transfer strengths which can be used as benchmarks to study and interpret the experimental data, even if these symmetries may be valid only approximately. Historically, symmetries have played an important role in nuclear physics. Examples are isospin symmetry, the Wigner multiplet theory, special solutions to the Bohr Hamiltonian, the Elliott model, pseudo-spin symmetries and the dynamical symmetries and supersymmetries of the IBM and its extensions.

The purpose of this contribution is to discuss several new developments in nuclear supersymmetry, in particular evidence for the existence of a new supersymmetric quartet in the $A \sim 190$ mass region, consisting of the $^{192,198}$Os and $^{193,194}$Ir nuclei, and correlations between different one- and two-nucleon transfer reactions.

2. Nuclear supersymmetry

Nuclear supersymmetry is a composite-particle phenomenon that should not be confused with fundamental supersymmetry, as used in particle physics and quantum field theory where it is postulated as a generalization of the Lorentz-Poincaré invariance as a fundamental symmetry of Nature and predicts the existence of supersymmetric particles, such as the photino and the selectron, for which experimental evidence is yet to be found. If experiments about to start at the LHC at CERN find evidence of supersymmetric particles, supersymmetry would be badly broken, as their masses must be much higher than those of their normal partners. In contrast to particle physics, nuclear supersymmetry has been verified experimentally.

Dynamical supersymmetries were introduced in nuclear physics in the context of the Interacting Boson Model (IBM) and its extensions [2]. The IBM describes collective excitations in even-even nuclei in terms of a system of interacting monopole ($s^l$) and quadrupole ($d^l$) bosons [3]. The bosons are associated with the number of correlated proton and neutron pairs, and hence the number of bosons $N$ is half the number of valence nucleons. For odd-mass nuclei the IBM was extended to include single-particle degrees of freedom [4]. The ensuing Interacting Boson-Fermion Model (IBFM) has as its building blocks $N$ bosons with $l = 0, 2$ and $M = 1$ fermion ($a_j^+ \otimes a_j^-$) with $j = j_1, j_2, \ldots$ [5]. The IBM and IBFM can be unified into a supersymmetry (SUSY) $U(6/\Omega) \supset U(6) \otimes U(\Omega)$ where $\Omega = \sum_j (2j + 1)$ is the dimension of the fermion space [2]. In this framework, even-even and odd-even nuclei form the members of a supermultiplet which is characterized by $N = N + M$, i.e. the total number of bosons and fermions. Supersymmetry distin-
guishes itself from other symmetries in that it includes, in addition to transformations among fermions and among bosons, also transformations that change a boson into a fermion and vice versa.

The concept of nuclear SUSY was extended in 1985 to include the neutron-proton degree of freedom [6]. In this case, a supermultiplet consists of an even-even, an odd-proton, an odd-neutron and an odd-odd nucleus. The first experimental evidence of a supersymmetric quartet was found in the \( A \sim 190 \) mass region in the \(^{184,195}\)Pt and \(^{195,196}\)Au nuclei as an example of the \( U(6/12)\) supersymmetry [7–11], in which the odd neutron is allowed to occupy the \( 3p_{1/2}, 3p_{3/2} \) and \( 2f_{5/2} \) orbits of the \( 82-126 \) shell, and the odd proton the \( 2d_{3/2} \) orbit of the \( 50-82 \) shell. This mass region is a particularly complex one, displaying transitional behavior such as prolate-oblate deformed shapes, \( \gamma \)-unstability, triaxial deformation and/or coexistence of different configurations which present a daunting challenge to nuclear structure models. Nevertheless, despite its complexity, the \( A \sim 190 \) mass region has been a rich source of empirical evidence for the existence of dynamical symmetries in nuclei both for even-even, odd-proton, odd-neutron and odd-odd nuclei, as well as supersymmetric pairs [2, 12] and quartets of nuclei [6, 7].

Recently, the structure of the odd-odd nucleus \(^{194}\)Ir was investigated by a series of transfer and neutron capture reactions [13]. The odd-odd nucleus \(^{194}\)Ir differs from \(^{196}\)Au by two protons, the number of neutrons being the same. The latter is crucial, since the dominant interaction between the odd neutron and the core nucleus is of quadrupole type, which arises from a more general interaction in the IBFM for very special values of the occupation probabilities of the \( 3p_{1/2}, 3p_{3/2} \) and \( 2f_{5/2} \) orbits, i.e. the location of the Fermi surface for the neutron orbits [14]. This situation is satisfiable to a good approximation by the \(^{195}\)Pt and \(^{196}\)Au nuclei which both have the 117 neutrons. The same is expected to hold for the isotones \(^{193}\)Os and \(^{194}\)Ir. For this reason, it is reasonable to expect the odd-odd nucleus \(^{194}\)Ir to provide another example of a dynamical symmetry in odd-odd nuclei. Strictly speaking, a dynamical supersymmetry involves the simultaneous study of pairs or quartets of nuclei that make up a supermultiplet.

2.1. Energies

In general, a dynamical (super)symmetry arises whenever the Hamiltonian is expressed in terms of the Casimir invariants of the subgroups in a group chain. The relevant subgroup chain of the \( U(6/12)\) supersymmetry is given by [6]

\[
\begin{align*}
U(6/12) & \supseteq U(6/4) & \supseteq U^B(6) \otimes U^{F_2}(6) \otimes U^{F_4}(6) \otimes U^{F_3}(4) \\
& \supseteq U^B(6) \otimes U^{F_2}(6) \otimes U^{F_4}(2) \otimes U^{F_3}(4) \\
& \supseteq U^{B, F_2}(6) \otimes U^{F_2}(2) \otimes SU^{F_3}(4) \\
& \supseteq SU^{B, F_2}(6) \otimes SU^{F_2}(2) \otimes SU^{F_3}(4)
\end{align*}
\]

In this case, the Hamiltonian

\[
H = A C_{2U^{B, F_2}(6)} + B C_{SU^{B, F_3}(4)} + B' C_{2SU^{F_2}(4)} + C C_{2SU^{F_2}(4)} + E C_{2SU^{F_2}(4)} + F C_{2SU^{F_2}(4)}
\]

describes the excitation spectra of the quartet of nuclei. Here we have neglected terms that only contribute to binding energies. The energy spectra of the four nuclei belonging to the supersymmetric quartet are described simultaneously by a single energy formula in terms of the eigenvalues of the Casimir operators

\[
\begin{align*}
E &= A [N_1(N_1 + 1) + N_2(N_2 + 3) + N_1(N_1 + 1)] \\
& + B [\Sigma_1 (\Sigma_1 + 4) + \Sigma_2 (\Sigma_2 + 2) + \Sigma_3^2] \\
& + B' [\sigma_1 (\sigma_1 + 4) + \sigma_2 (\sigma_2 + 2) + \sigma_3^2] \\
& + C [\tau_1 (\tau_1 + 3) + \tau_2 (\tau_2 + 1)] \\
& + D (L + 1) + E J (J + 1).
\end{align*}
\]

The coefficient \( A, B, B', C, D, E \) can be determined in a fit of the experimental excitation energies.

The new data from the polarized \((d, \alpha)\) transfer reaction has provided crucial new information about and insight into the structure of the spectrum of \(^{194}\)Ir which led to significant changes in the assignment of levels as compared to previous work [15]. The main change is that the ground state of \(^{194}\)Ir is now assigned to the band \([N + 1, 0], (N + 3/2, 1/2, 1/2)\) instead of to \([N, 1], (N + 1/2, 1/2, -1/2)\) as in Ref. 15. In this notation, \( N \) is the number of bosons in the odd-odd nucleus \((N = 6 \text{ for } ^{194}\text{Ir})\). The new assignment agrees with that of the neighboring odd-odd nucleus \(^{196}\)Au [7–9]. Figure 2 shows the negative parity levels of \(^{194}\)Ir in comparison with the theoretical spectrum in which it is assumed that these levels originate from the \( \nu^3p_{1/2}, \nu^3p_{3/2}, \nu^2f_{5/2} \otimes \pi^2d_{3/2} \) configuration. The theoretical energy spectrum is calculated using the energy formula of Eq. (3) with \( A + B = 35.0, B' = -33.6, C = 35.1, D = 6.3, \) and \( E = 4.5 \) (all in keV). This parameter set is a lot closer to the parameter values used for \(^{196}\)Au [8] than the ones in Ref. 15, indicating systematics in this zone of the nuclear chart. Given the complex nature of the spectrum of heavy odd-odd nuclei, the agreement is remarkable. There is an almost one-to-one correlation between the experimental and theoretical level schemes [13].

The successful description of the odd-odd nucleus \(^{194}\)Ir opens the possibility of identifying a second quartet of nuclei in the \( A \sim 190 \) mass region with \( U(6/12)\) supersymmetry. The new quartet consists of the nuclei \(^{192,193}\)Os and \(^{193,194}\)Ir and is characterized by \( N' = 3 \) and \( N'' = 5 \). Whereas the \(^{192}\)Os and \(^{193,194}\)Ir nuclei are well-known experimentally, the available data for \(^{193}\)Os is rather scarce. In Fig. 2 we show the predicted spectrum for \(^{193}\)Os obtained
The ground state of $^{193}$ from Eq. (3) using the same parameter set as for $^{192}$, is the ground state band, rather than $^{194}$ Ir. $^{193}$ Os for the $^{192}$, $^{193}$ Ir may be interpreted in terms of a quartet of nuclei with $U(6/12) \otimes U(6/4)$ supersymmetry. A more detailed study of the energy spectra of these nuclei is in progress [18].

2.2. Two-nucleon transfer reactions

Two-nucleon transfer reactions probe the structure of the final nucleus through the exploration of two-nucleon correlations that may be present. The spectroscopic strengths not only depend on the similarity between the states in the initial and final nucleus, but also on the correlation of the transferred pair of nucleons.

In this section, the recent data on the $^{196}$ Pt $(d, \alpha)^{194}$ Ir reaction [13] are compared with the predictions from the $U_L(6/12) \otimes U_\nu(6/4)$ supersymmetry. This reaction involves the transfer of a proton-neutron pair, and hence measures the neutron-proton correlation in the odd-odd nucleus. The spectroscopic strengths $G_{L,J}$

$$G_{L,J} = \sum_{j_\nu,j_\pi} g_{L,j_\nu,j_\pi}^{194} \left| \langle a_{j_\nu}^\dagger a_{j_\pi}^\dagger \rangle^{(194)\mathrm{Ir}} \right| \langle 196\mathrm{Pt} \rangle |^2,$$

depend on the reaction mechanism via the coefficient $g_{L,j_\nu,j_\pi}^{194}$ and on the nuclear structure part via the reduced matrix elements.

In order to compare with experimental data we calculate the relative strengths $R_{L,J} = G_{L,J}/G_{L,J}^{\text{ref}}$, where $G_{L,J}^{\text{ref}}$ is the spectroscopic strength of the reference state. The ratios of spectroscopic strengths to final states with $(\tau_1, \tau_2) = (3/2, 1/2)$ provide a direct test of the nuclear wave functions, since they can only be excited by a single tensor operator [11]. In Table I we show the ratios for different final states with $(\tau_1, \tau_2) = (3/2, 1/2)$.
The nuclei belonging to a supersymmetric quartet are described by a single Hamiltonian, and hence the wave functions, transition and transfer rates are strongly correlated. As an example of these correlations, we consider here the transfer reactions between the $^{194,195}$Pt and $^{192,193}$Os nuclei. The Pt and Os nuclei are connected by one-neutron transfer reactions within the same supersymmetric quartet $^{194}$Pt $\rightarrow$ $^{195}$Pt and $^{192}$Os $\leftrightarrow$ $^{193}$Os, whereas the transitions between the Pt and Os nuclei involve the transfer of a proton pair between different quartets $^{194}$Pt $\leftrightarrow$ $^{192}$Os and $^{195}$Pt $\leftrightarrow$ $^{193}$Os.

### 3. Correlations

The correlations between different transfer reactions can be derived in an elegant and explicit way by a generalization of the concept of $F$-spin which was introduced in the neutron-proton IBM [19] in order to distinguish between proton and neutron bosons.

The eigenstates of the $U(6/12)\nu \otimes U(6/4)\pi$ supersymmetry are characterized by the irreducible representations $[N_1, N_2, N_3]$ of $U^{BF\nu}(6)$ which arise from the coupling of three different $U(6)$ representations, $[N_\nu]$ for the neutron bosons, $[N_\pi]$ for the proton bosons and $[N_\rho]$ for the pseudo-orbital angular momentum of the odd neutron ($N_\rho = 0$ for the even-even and odd-proton nucleus of the quartet, and $N_\rho = 1$ for the odd-neutron and the odd-odd nucleus). In analogy with the three quark flavors in the quark model ($u$, $d$ and $s$), also here we have three different types of identical objects ($\pi$, $\nu$ and $\rho$), which can be distinguished by $F$-spin and hypercharge $Y$. The two kinds of bosons form an $F$-spin doublet, $F = 1/2$, with charge states $F_\pi = 1/2$ for protons ($\pi$) and $F_\pi = -1/2$ for neutrons ($\nu$) [19]. In the framework of the generalized $F$-spin, we assign in addition a hypercharge quantum number to the bosons $Y = 1/3$. The pseudo-orbital part ($\rho$) has $F = F_\rho = 0$ and $Y = -2/3$.

Group theoretically, the generalized $F$-spin is defined by the reduction

\[
U(18) \supset U(6) \otimes U(3) \quad [N] \downarrow [N_1, N_2, N_3] \downarrow [N_1, N_2, N_3].
\]

Here $U(6)$ is to be identified with the $U^{BF\nu}(6)$ of the group reduction of Eq. (1), which is the result of first coupling the bosons at the level of $U(6)$ followed by coupling the orbital part

\[
[[N_\nu], [N_\pi]]; [N_\nu + N_\pi - i, i], [N_\rho]; [N_1, N_2, N_3].
\]

![Figure 3. Comparison of theoretical (left panels) and experimental values (right panels) of ratios $R_{LJ}$ of spectroscopic strengths.](image-url)
### 3.2. One-neutron transfer

In a study of the $^{194}\text{Pt} \rightarrow ^{195}\text{Pt}$ stripping reaction it was found [20] that one-neutron $j = 3/2$, 5/2 transfer reactions can be described by the operator

$$P^{(j)}_{\nu} = \frac{\alpha_j}{\sqrt{2}} \left( \hat{s}_\nu \times a_{\nu,j}^{\dagger} \right) - \left( \hat{d}_{\nu} \times a_{\nu,j}^{\dagger} \right). \quad (9)$$

It is convenient to take ratios of intensities, since they do not depend on the value of the coefficient $\alpha_j$ and hence provide a direct test of the wave functions. For the stripping reaction $^{194}\text{Pt} \rightarrow ^{195}\text{Pt}$ (ee → on) the ratio of intensities for the excitation of the $(\tau_1, \tau_2) = (1, 0)$, $L = 2$ doublet with $J = 3/2, 5/2$ belonging to the first excited band with $[N + 1, 1], (N + 1, 1, 0)$ relative to that of the ground state band $[N + 2], (N + 2, 0, 0)$ is given by [20]

$$R(\text{ee} \rightarrow \text{on}) = \frac{(N + 1)(N + 3)(N + 6)}{2(N + 4)}, \quad (10)$$

which gives $R = 29.3$ for $^{194}\text{Pt} \rightarrow ^{195}\text{Pt} (N = 5)$, to be compared to the experimental value of 19.0 for $j = 5/2$, and $R = 37.8$ for $^{192}\text{Os} \rightarrow ^{193}\text{Os} (N = 6)$. The equivalent ratio for the inverse pick-up reaction is given by

$$R(\text{on} \rightarrow \text{ee}) = \frac{(N_x + 1)}{(N_x + 2)}, \quad (11)$$

which gives $R = 1.96$ for $^{195}\text{Pt} \rightarrow ^{195}\text{Pt} (N_x = 1$ and $N_x = 4$) and $R = 3.24$ for $^{193}\text{Os} \rightarrow ^{194}\text{Os} (N_x = 2$ and $N_x = 4$). This means that the mixed symmetry $L = 2$ state is predicted to be excited more strongly than the first excited $L = 2$ state.

This correlation between pick-up and stripping reactions has been derived in a general way only using the symmetry relations that exist between the wave functions of the even-even and odd-neutron nuclei of the supersymmetric quartet. The factor in the right-hand side of Eq. (11) is the result of a ratio of two $SU(3)$ isoscalar factors. It is important to emphasize, that Eqs. (10) and (11) are parameter-independent predictions which are a direct consequence of nuclear SUSY and which can be tested experimentally.

### 3.3. Two-proton transfer

The two supersymmetric quartets in the mass $A \sim 190$ region differ by two protons. In principle, the connection between the two quartets can be studied by two-proton transfer reactions. In the IBM, two-proton transfer operator is, in first order, given by

$$P^{(j)}_{\tau} = \alpha s_{\tau}^{\dagger}, \quad P_{\tau} = \alpha s_{\tau}. \quad (12)$$

Whereas the operator $s_{\tau}$ only excites the ground state of the final nucleus, $s_{\tau}^{\dagger}$ can also populate excited states.

In Table II, we show the results for ratios of spectroscopic strengths between even-even nuclei. The selection rules of the operator $s_{\tau}^{\dagger}$ allow the excitation of states with $(\tau_1, \tau_2) = (0, 0)$ and $L = 0$ belonging to the ground band $(\Sigma_1, \Sigma_2, \Sigma_3) = (N + 3, 0, 0)$ and excited bands with $(N + 1, 0, 0)$. The corresponding ratios for the odd-neutron
nuclei are strongly correlated to those of the even-even nuclei (see Tables II and III)

\[
S_{2}(\text{on } \rightarrow \text{on}) = R_{2}(ee \rightarrow ee),
\]

\[
S_{3O}(\text{on } \rightarrow \text{on}) = R_{3}(ee \rightarrow ee) \frac{N_{\pi} + 2}{(N_{\nu} + 1)(N + 2)},
\]

\[
S_{3P}(\text{on } \rightarrow \text{on}) = R_{3}(ee \rightarrow ee) \frac{N_{\nu}(N + 3)}{(N_{\nu} + 1)(N + 2)}. \tag{13}
\]

As before, the coefficient in the right-hand side correspond to the ratio of two SU(3) Clebsch-Gordan coefficients

4. Summary and conclusions

In conclusion, in this contribution we have presented evidence for the existence of a second quartet of nuclei in the \(A \sim 190\) region with \(U(6/12)_{\nu} \otimes U(6/4)_{\pi}\) supersymmetry, consisting of the \(^{192,193}\text{Os}\) and \(^{193,194}\text{Ir}\) nuclei. The analysis is based on new experimental information on \(^{194}\text{Ir}\). In particular, the \((\vec{d}, \alpha)\) reaction is important to establish the spin and parity assignments of the energy levels, and to provide insight into the structure of the spectrum of \(^{194}\text{Ir}\). Given the complexity of the \(A \sim 190\) mass region, the simple yet detailed description of \(^{194}\text{Ir}\) in a supersymmetry scheme is remarkable.

Nuclear supersymmetry establishes precise links among the spectroscopic properties of different nuclei. This relation has been used to predict the energies of \(^{193}\text{Os}\). Since the wave functions of the members of a supermultiplet are connected by symmetry, there exists a high degree of correlation between different one- and two-nucleon transfer reactions not only between nuclei belonging to the same quartet, but also for nuclei from different multiplets. As an example, we studied the correlations between one-neutron transfer reactions and two-proton transfer reactions. In the former case, nuclear supersymmetry predicts that the \(L = 2\) mixed symmetry states in the even-even nuclei \(^{194}\text{Pt}\) and \(^{192}\text{Os}\) are excited much stronger (two to three times as strong) than the first excited \(L = 2\) state.

In order to establish the existence of a second supersymmetric quartet of nuclei in the \(A \sim 190\) mass region, it is crucial that the nucleus \(^{193}\text{Os}\) be studied in more detail experimentally. The predictions for correlations between one-neutron transfer reactions in Pt and Os can be tested experimentally by combining for example \((\vec{d}, p)\) stripping and \((p, d)\) pick-up reactions.

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1. See e.g. RIA Theory Bluebook: A Road Map (September 2005).