Chaos in hadrons

R. Fossion and R. Bijker
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, México, 04510, Distrito Federal, México.

Recibido el 13 de febrero de 2009; aceptado el 1 de junio de 2009

Classical chaos looks for patterns of order in the properties of non-linear dynamical systems in terms of (regular and strange) attractors of many types. Also local fluctuation in the excitation spectra of quantum systems have been conjectured to obey to universal laws, a phenomenon called by the fancy name of quantum chaos. These universal laws have been confirmed both experimentally and theoretically in many different quantum systems, such as atoms, nuclei and quantum dots. Recently, also the mass spectra of hadrons have been studied in this context, but leading to a contradiction between the experimental results (chaos) and the theoretical results (regularity or integrability), which has been explained as due to an inherent incapacity of quark models to reproduce the hadron mass-spectrum. In this contribution, we show the sensitivity of the statistical results on the process of unfolding that is used to separate the global from the local density variation of the hadron masses with excitation energy. We find that the apparent contradiction between experiment and theory is more probable to be an artifact of the unfolding process than an unsuitability of quark models to describe the lower part of the hadron mass-spectrum.

Keywords: Properties of baryons; quantum chaos; semiclassical models; relativistic quark model.

1. Introduction

Classical Chaos is the study of the extreme sensitivity of the dynamics of non-linear systems to the initial conditions. It can be more intuitive however to think of chaos as searching for patterns of order in what at first sight would seem to be only disorder. Poincaré discovered that there can be orbits which are nonperiodic, and yet not forever increasing nor approaching certain fixed points, or specific patterns of points, that later became called (regular) attractors, such as the stable point or the limit circle [1]. Lorenz was the first to discover also strange attractors in non-linear systems [2]. Quantum chaos, or chaos in quantum systems is less well defined Bohigas et al., in a seminal paper [3], conjectured as a tentative definition for quantum chaos to look for universal laws to which the fluctuation in excited states in the spectra of quantum systems should obey. These universal laws are described by Wigner’s Random Matrix Theory [4]. More recently, Relaño et al. [5] conjectured that $1/f$ noise in the power spectrum of a time series, or in the power spectrum of an excitation spectrum interpreted as a time series, might be a characteristic of both classical and quantum chaos, as observed on the one hand e.g. in the dripping faucet [6] and in heartbeat dynamics [7], and on the other hand in specific atomic nuclei, such as Ca-48 [8]. A bridge between the classical and the quantum world might thus be provided by the study of chaos. Regularity (integrability), on the contrary, would be characterised by $1/f^2$ noise.

These universal laws for fluctuation in the excitation spectra of quantum spectra have been verified in an impressive variety of systems, from simple two-dimensional one-body systems such as the Sinai billiard [3], up to complicated many-body systems such as quantum dots [9], atoms [10], and atomic nuclei (see Refs. 11 to 13 for recent reviews). Very recently, also the hadronic mass spectrum has been studied in relation with quantum chaos. In Ref. 14, the experimental hadron masses were studied, and in Ref. 15 the corresponding theoretical hadron masses as calculated by different quark models. The experimental and the theoretical studies however reached contradictory results, out of which was concluded that quark models might not be able to describe well the hadron mass-spectrum [15].

In this contribution, we will carry out a statistical analysis of the baryon mass-spectrum, as obtained from the Bonn instanton model [16]. En particular, we will pay attention to the sensitivity of the results to the process of unfolding and
to the cutoff in the mass spectrum. The organisation of the present contribution is as follows: in Sec 2, we give a brief overview of Wigner’s Random Matrix Theory (RMT) an of the GOE and GDE matrix ensembles [4], in Sec. 3, the results of the previous studies on the experimental [14] and theoretical hadron mass-spectrum [15] are discussed, in Sec. 4 we show the strong dependence of the analysis of the hadron mass-spectrum on the process of unfolding, and finally, in Sec. 5 we present a summary of the conclusions.

Figure 1. The non-diagonal elements $m_{ij} = m_{ji}$ ($i \neq j$) of a random symmetrical matrix of the Gaussian Orthogonal Ensemble (GOE) are chosen from a Gaussian distribution with mean $\mu = 0$ and width $\sigma^2 = 1$. The diagonal elements $m_{ii}$ are chosen from a Gaussian distribution of the double width $\sigma^2 = 2$, to ensure that they have the same weight as the non-diagonal matrix elements. The matrices of the Gaussian Diagonal Ensemble (GDE) only have diagonal elements, also chosen from a Gaussian distribution.

Figure 2. An RMT ensemble of 100 matrices of the GOE type, each matrix of the dimension $N \times N = 50 \times 50$. After diagonalisation, the eigenvalues of the GOE ensemble are correlated, and a histogram of the level density of the whole ensemble has a semi-circular shape (exact for large dimensions $N \to \infty$). The histogram of this figure contains 5000 eigenvalues.

Figure 3. Same as Fig. 3, but for an RMT ensemble of 100 matrices of the GDE type. The eigenvalues of the GDE ensemble are not correlated, and a histogram of the level density by construction has a Gaussian shape (exact for large dimensions $N \to \infty$). The histogram contains 5000 eigenvalues.

2. Random matrix theory (RMT) and quantum chaos

2.1. Random-matrix ensembles: GOE and GDE

The excited states in the lower part of the spectrum of atomic nuclei (below the nucleon separation-energy of about $\sim 8$ MeV) can be explained by different models in terms of properties of the single protons and neutrons. The nuclear shell model, e.g., describes the nuclear excitation spectrum of a specific nucleus by writing out in a matrix the interaction Hamiltonian $\hat{H}$ in a basis of single-particle states. After a diagonalisation, the resulting eigenvalues give the energies of the subsequent excited nuclear states. At higher excitation energies, the density of states becomes too large for a description of individual states, and only statistical studies make sense. E. Wigner was the first to successfully describe the physics of neutron resonances [17,18], with Random Matrix Theory (RMT) [4], using ensembles of matrices with random elements that only have to obey to the specific symmetries of the problem in study. The atomic nucleus is invariant under time reversal and has rotational symmetry. In the language of RMT, these nuclear symmetries translate into symmetrical orthogonal matrices with elements randomly chosen from a Gaussian distribution (see Fig. 1), leading to a Gaussian Orthogonal Ensemble (GOE) of matrices. Because of the diagonalisation process, the eigenvalues of each matrix will be correlated, and the level density of the GOE ensemble has a typical semicircular shape (see Fig. 2). A second limiting case in RMT is the Gaussian Diagonal Ensemble (GDE) of matrices with only diagonal elements, chosen at random from a Gaussian distribution. The eigenvalues of the GDE ensemble are not correlated and the level density has a typical Gaussian shape (see Fig. 3). The observation that the GOE ensemble not only describes very well complex many-body systems such as the atomic nucleus, but also much simpler
one-body problems, such as the two-dimensional Sinai quantum billiard, led O. Bohigas et al. to conjecture that spectra of time-reversal invariant quantum systems whose classical analogues are chaotic show the same fluctuation properties as predicted by GOE [3]. The other extreme, regularity or integrability, would be described with the GDE ensemble [19].

Statistical studies concentrate not on the absolute excitation energies of a quantum system, but on the local fluctuations of these energies around the global level-density evolution. The level-density variation with excitation energy, \( \rho(E) \), can be split in a smooth or global part, \( \bar{\rho}(E) \), and a locally fluctuating part, \( \tilde{\rho}(E) \), with \( \rho(E) = \bar{\rho}(E) + \tilde{\rho}(E) \). The process of separating the global from the local behaviour is called unfolding. In some cases, the global density function \( \bar{\rho}(E) \) is analytically known. In the limit for large matrices, \( N \to \infty \), in the case of GOE, this global density function is a semicircle (see Fig. 2), and in the case of GDE a Gaussian (see Fig. 3), and this analytical function can be used to perform the unfolding in an exact way [20]. In many practical cases, the unfolding must be performed numerically with a polynomial. In the following, we will see that the statistical results can depend in an important way on the unfolding applied. This fact has been stressed repeatedly in the literature [21].

2.2. Statistical analysis

Many statistical tools are available to study the nature of the excitation spectrum of quantum systems.

(i) Long-range correlations can be studied with the \( \Delta_s \) statistics [4], the DFA method [8, 22], or the power spectrum of the excitation spectrum interpreted as a time series [5, 8]. The study of long-range correlations however requires spectra with long sequences of levels, that are not always available experimentally (see e.g. the limited experimental baryonic mass spectrum of Sec. 3).

(ii) Somewhat less demanding is the study of short-range correlations between subsequent excited states, that can be studied with Nearest-Neighbour Distribution (NND) diagrams [4], where the distribution of fluctuations in the level spacings between neighbouring excited states are compared with the predictions of GOE and GDE.

The NND of a GOE ensemble (paradigm of a correlated or chaotic spectrum) behaves like,

\[
P_W(s) = \frac{\pi}{2} s e^{-\frac{\pi}{2} s^2}, \quad (1)
\]
a formula coined the Wigner surmise, that has been derived analytically for a GOE ensemble of \( 2 \times 2 \) matrices. The other limiting case, the NND of a GDE ensemble (paradigm for an uncorrelated or regular spectrum) is given by a Poisson distribution,

\[
P_S(s) = e^{-s}. \quad (2)
\]

In Fig. 4, NND diagrams are presented for the energy fluctuation of the specific GDE and GOE ensembles introduced in the Figs 2 and 3. The unfolding of the GDE and GOE excitation spectra has been done numerically with a polynomial, and the dependence of the statistical results on the degree of the polynomial is shown. It can be seen that for the GDE ensemble (upper panels), for an unfolding with a polynomial with degree 1 up to 10, the NND indeed is well fit by the Poissonian of Eq. (2). For higher degrees however, the order of the polynomial becomes of the order of the data fit (remember that the dimension of the matrices used is \( 50 \times 50 \)), and apart from the global density evolution the polynomial starts to fit local correlations as well. We see a similar behaviour also for the GOE ensemble (lower panels). For an unfolding with a polynomial of degree 1 (a straight line), the NND is not yet the perfect Wigner distribution of Eq. (1). The histogram seems to be best fitted with a polynomial of degree 10. For higher degrees, similar as in the GDE case, the data become over-fit and the correspondence with the Wigner distribution is lost.

In Refs. 14 and 15, that studied the hadron mass-spectrum for the first time (see Sec. 3 for a brief discussion), additional statistical tools are used to study the short-range correlations between subsequent levels. For small spacings, the difference between the Poisson and Wigner distribution in the NND histograms (Fig. 4) is obvious. For larger spacings, the Wigner and Poisson distributions decay both exponentially [see Eq. (1) and (2)], the former one with \( s^2 \) and the latter one with \( s \). The accumulated spacing-distribution function, \( F(x) \) [15],

\[
F(x) = 1 - \int_0^x P(s)ds = \int_x^\infty P(s)ds, \quad (3)
\]

allows to differentiate more easily between the tails of the two limiting cases of the Poisson and the Wigner distribution. When the logarithm of \( F(x) \) is plotted, as in the insets of Fig. 4, the Poisson distribution follows a straight line, whereas \( F(x) \) of the Wigner distribution a curved line. The results of \( F(x) \) for the GDE and GOE level spacings depend on the unfolding performed. In the case of GDE (upper panels), for a degree 1 polynomial, \( F(x) \) is slightly over-Poisson. For a degree 10 polynomial, \( F(x) \) is Poissonian. For higher degrees (degree 25-50), \( F(x) \) becomes intermediate between Poisson and Wigner. We find similar results for the GOE ensemble (lower panels). For too low degrees of the fit polynomial (1 up to below 10), \( F(x) \) is in between Poisson and Wigner. For a degree 10 polynomial, \( F(x) \) behaves well like the Wigner distribution. For still higher degrees, \( F(x) \) falls below the expected Wigner function.

Histograms like those of Fig. 4 can be sensitive to the choice of the energy grid. It can be instructive to also consider some integrated statistics, that does not depend on the choice of the histogram bin-width, such as the moments of
the spacing distribution [14]. The moments of a discrete sequence of level spacings are defined as,

\[ \langle s_{\text{discr}}^g \rangle = \frac{1}{N} \sum_{i=1}^{N} s_i^g, \] (4)

whereas the spacing moments for the continuous Poisson and Wigner distributions can be written as,

\[ \langle s_{\text{cont}}^g \rangle = \int_0^\infty s^g P(s) ds. \] (5)

Computing the first ten moments \( g = 1 \) up to 10 of the relevant distributions (GDE and GOE data, Poisson and Wigner distributions), the results of Fig. 5 are obtained. We see a similar dependence on the degree of the unfolding polynomial as we already have seen for the NND diagrams and the function \( F(x) \). For the GDE ensemble (upper panels), for a degree 1 of the polynomial, the results are over-Poisson, whereas for a degree 10, we are already in an intermediate regime in between Poisson and Wigner. For the GOE ensemble (lower panels), for an unfolding with a straight line, the moments look like intermediate in between Poisson and Wigner. For a degree 10 unfolding polynomial, the GOE moments fall well on the moments for the Wigner distribution, whereas for higher order, the GOE moments fall under the Wigner distribution.

![Figure 4](image-url)

**Figure 4.** (Upper panels) Comparison between histograms for nearest-neighbour spacings NND for an ensemble of 100 GDE matrices of dimensions 50 \times 50, in comparison with Poisson (black solid line) and Wigner (grey dashed line) distributions, after an unfolding with polynomials of different degrees (1 up to 50). In the inset, the tail of the histogram is studied (crosses) in comparison with the distributions Poisson (straight line) and Wigner (curved line), using the accumulative spacing-distribution function \( F(x) \) of Eq. (3). (Lower panels) The same, but for an ensemble of 100 GOE matrices of the same dimensions.

![Figure 5](image-url)

**Figure 5.** (Upper panels) Comparison between the moments distribution of Eq. (4) for the ensemble of 100 GDE matrices of dimensions 50 \times 50 (crosses), with the moments for the Poisson (full line) and Wigner distributions (dashed line), after an unfolding with polynomials of different degrees (1 up to 50). (Lower panels) The same, but for an ensemble of 100 GOE matrices of the same dimensions.
3. Quantum chaos in hadrons: contradiction between experimental and theoretical results

3.1. Hadrons: baryons and mesons

Hadrons are particles that respond to the strong interaction (from the ancient Greek word ἄρσιος which means 'strong'). Hadrons are formed of quarks, that are elementary particles, and that are 'glued' together by gluons. There are two subsets of hadrons, baryons (from βαρύς which means 'heavy'), a heavy system formed of three quarks, and mesons (from μεσος which means 'middle'), a medium-heavy system formed by a quark and an anti-quark. The quark model was the first model to successfully explain many of the properties of the hadrons in terms of properties of its constituents quarks. Not less importantly, the quark model was able to put order in the confusing panoply of observed hadrons using symmetry principles, and arranging them into multiplets (singlets, octets and decuplets). An early success of the quark model was the theoretical prediction of the \( \Omega^- \) particle, that was discovered experimentally only a few years later [23].

3.2. Experimental baryon masses up to 2.2 GeV

The experimentally measured hadron (baryon and meson) mass-spectrum was studied for its statistical properties in Ref. 14, with data taken from the Particle Data Group (PDG) Summary Tables [24]. In the following, we will concentrate on the baryon masses. More in particular, we will take into account the \( N, \Delta, \Lambda \) and \( \Sigma \) baryons up to \( N(2200) \). The poorly known 'one-star baryons' were excluded. The baryons masses were ordered in different sequences, each sequence having the same quantum numbers of isospin, spin, parity, strangeness and baryon number. In doing so, 24 mass spacings \( \Delta m_i (i = 1, 24) \) were obtained, distributed in 17 sequences. The experimental data on the baryon masses is very limited. The different sequences exist of only one or two mass-spacings and so it is impossible to perform a proper unfolding to separate the global evolution of the mass-level density from the local variations that interest us for our statistical analysis. Instead, in [14] a pseudo-unfolding was suggested, dividing all 24 baryon mass-spacings by their mean spacing \( \langle \Delta m \rangle = 290 \) MeV, after which the unfolded and now dimensionless spacings \( s_i = \Delta m_i / \langle \Delta m \rangle (i = 1, 24) \) can be joined in one single array with unit mean spacing \( \langle s \rangle = 1 \) to be studied for their statistical properties. In doing so, it is assumed that the experimental mean level density is constant, \( \rho = 1 / \langle s \rangle \). This approximation holds well in the mass region between 1 and 2 GeV [14]. A histogram of the unfolded baryon spacings is shown in a NND in Fig. 6 (left-hand panel), in comparison with the Poisson (GDE) and Wigner (GOE) distributions. All distributions (experimental mass spacings, Poisson and Wigner) are properly normalized, so that the mean spacing is unitary, \( \langle s \rangle = 1 \). Despite the low statistics of only 24 spacings, it can be appreciated that the spacings distribution is more like the Wigner distribution. In the inset of Fig. 6 (left panel), the behaviour of the nearest-neighbour distribution is shown for the tail of the histogram, according to the function \( F(x) \) of Eq. (3). It can be seen that the experimental data follow very well the Wigner distribution. Studying the moments of the experimental spacings with Eq. (4), the results of Fig. 6 (right-hand panel) are obtained. It can be seen that the Wigner distribution fit the data very well, also for this integrated statistics.

3.3. Quark-model baryon mass-spectrum up to 2.2 GeV

In Ref. 15, the statistical properties of several quark models that fit the physical baryon spectrum have been studied. One of these models is the Bonn Instanton Quark Model [16]. Up to a cutoff energy of 2.2 GeV, the baryon masses of model \( A \) from the Bonn Instanton model can be arranged in 25 different sequences with the same quantum numbers, giving 127 mass differences. A slightly different unfolding process is followed in Ref. 15, every sequence of mass differences is now divided by the mean mass-difference of the same sequence. The drawback of this method is that only sequences of more than three masses can be taken into account (a sequence of two masses, or one mass difference would give an artificial unfolded unitary spacing). Physically speaking, this way of unfolding is more correct, but in the experimental case the statistics is too low to sacrifice the sequences with only two masses. We have checked that in the theoretical case these two different ways of unfolding do not make a significant difference in the statistical results. Results of the statistical analysis for the theoretical data are shown in Fig. 7. In the left-hand panel, it can be seen that the NND of the mass spacings follows more the Poisson distribution. The function \( F(x) \) confirm that this is true also for the tail of the distribution. In the right-hand panel, it can be seen that also the integrated statistics of the moments distribution of the spacings follows the Poisson distribution.

3.4. Conclusions on the contradiction between experimental and theoretical results on baryon masses up to a cutoff energy of 2.2 GeV

The number of baryons predicted by quark models is substantially larger than what is observed in experiments (compare the 127 mass differences in model \( A \) of the Bonn Instanton model, with the 24 mass differences in the experimental data, both up to the same cutoff energy of 2.2 GeV). One possible reason can be the missing resonances, that for their unnatural parity are very hard to produce experimentally. It would be tempting to relate the contradiction in the statistical results for the experimental and the theoretical data to these missing resonances. It has been shown however [25] that removing at random levels from a correlated spectrum produces a spectrum which is less correlated, translating into a shift from Wigner to Poisson in a study of the spectrum's statistical properties in an NND. This is, however, incompatible...
with the present results of an experimental baryon spectrum which results to be more correlated than the theoretical one. In Ref. 15, it is concluded that quark models might be not suitable to reproduce the low-lying baryon mass-spectrum, or to predict the existence of missing resonances. In the following (see part 4.), we will show that a statistical analysis of the theoretical baryon mass-spectrum is extremely dependent on the unfolding procedure applied.

4. Studying the complete theoretical baryon mass-spectrum

The study of the experimental baryon mass-spectrum cannot be done in another way because of the scarcity of experimental data. There is however no reason to limit oneself to only the lowest masses of the theoretical hadronic mass spectrum, if quark models (e.g. the Bonn Instanton quark model of Ref. 16) are able to produce a more complete spectrum. In Fig. 8, a histogram is shown for the mass density of all 22161 masses for the \( N, \Delta, \Lambda, \Sigma \) and \( \Omega \) baryons. It can be seen that the baryon level density has quite a different shape than the level density of the GOE (see Fig. 2) or GDE ensemble (see Fig. 3). The baryon masses have to be separated in sequences with the same quantum numbers. We find thus 84 different sequences for the 5 different types of particles, with \( J^\pi \) ranging from

\[
\frac{1}{2}^+, \frac{1}{2}^-, \text{ up to } \frac{18}{2}^+, \frac{18}{2}^-.
\]

We are now able to study (i) the dependence of the statistics on the mass cutoff (whether we study the complete mass spectrum or only parts from it) and, (ii) the dependence on the degree of the polynomial used for the unfolding. In Fig. 9, the process of numerical unfolding is demonstrated for the lower 5\% of the masses of the nucleon with \( J^\pi = \frac{1}{2}^+ \), using a polynomial of the order 3. The polynomial represents the global variation in mass density, whereas the difference between the polynomial and the theoretical masses are the...
fluctuation that will be studied statistically. In a similar way, all 84 sequences are unfolded numerically with polynomials of different degrees to study the dependence.

After unfolding, the mass spacings all together can be studied for their statistical properties. In Fig. 10, the mass spacings are represented in NND diagrams for two different

\[\text{Figure 10.} \text{ (Upper panels) NND of the lower 5\% of the baryon mass spectrum (histogram), in comparison with the Poisson (full line) and Wigner (dashed line) distributions. Results for different degrees of the unfolding polynomial are shown (degrees 1, 3 and 5). The maximum hadron mass included is 3.05GeV, the histogram includes about a 1000 mass spacings, distributed over 84 different sequences with dimensions ranging from 2 up to 23. (Lower panels) Similar, but for the lower 30\% of the baryon mass spectrum, and for degrees 1, 10 and 30 of the unfolding polynomial. The maximum hadron mass included is 3.76GeV, the histogram includes over 6000 mass spacings, distributed over 84 different sequences with dimensions ranging from 15 up to 141.} \]

\[\text{Figure 11.} \text{ (Upper panels) Spacing moments of the lower 5\% of the baryon mass-spectrum calculated with Eq. 5 and based on the histograms of Fig. 10 (crosses), in comparison with the Poisson (full line) and Wigner (dashed line) distributions. Results for different degrees of the unfolding polynomial are shown (degrees 1, 3 and 5). (Lower panels) Similar, but for the lower 30\% of the baryon mass-spectrum.} \]
mass cutoffs, the lower 5% (upper panels) and the lower 30% of the baryon masses (lower panels). Further, the dependence of the results on the degree of the polynomial used for the unfolding are shown. It can be seen that for a low degree of the unfolding polynomial (with a straight line for the lower 5 or 30% window), the spacing histogram looks like a Poisson distribution, whereas for a high degree (starting from degree 3 for the lower 5% window, and starting from a degree 10 for the lower 30%) the histogram looks more like a Wigner distribution. It can also be appreciated that the larger the window is of the mass spectrum under study, the higher the degree of the unfolding polynomial needs to be to obtain a result similar to the Wigner distribution. In conclusion, we see that the NND is very dependent not only on the degree of the unfolding polynomial, but also on the size of the window we consider of the baryon mass spectrum (i.e. the mass cutoff).

In the same Fig. 10, in the insets of the NND histogram is represented with the function \( F(x) \) of Eq. (3). We see a similar dependence of the results for \( F(x) \) on the degree of the unfolding polynomial, and on the mass cutoff, as we already saw in the case of the NND diagrams. For low degrees of the unfolding polynomial, the baryon masses follow the Poisson distribution, whereas for higher degrees the masses follow an intermediate regime or the Wigner distribution. We can draw a similar conclusion also for the integrated statistics of the spacing momentos, see Fig. 11, for the same mass cutoffs, the lower 5% of the baryon masses (upper panels), and the lower 30% of the baryon masses (lower panels). Also in the case of the spacing moments, we see an important dependence on the degree of the polynomial used for the unfolding. For an unfolding with a straight line, the mass moments follow the Poisson distribution (lower 5% of the baryon masses) or are even over-Poisson (lower 30% of the baryon mass-spectrum). For higher degrees of the unfolding polynomial, the mass moments are in between the Poisson and Wigner distributions.

5. Conclusions and outlook

In Ref. 14, the statistical properties of the experimental hadron (baryon and meson) mass-spectrum up to 2.2 GeV has been studied, and it has been found the spectrum matches well the signature of quantum chaos of the Wigner distribution. In Ref. 15, the statistical properties of theoretical baryon mass-spectra up to the same mass cutoff has been studied for different quark models, and it has been found that they match with the signature of regularity of the Poisson distribution. The number of baryon masses up to the cutoff of 2.2 GeV is relatively low, and therefore a (pseudo) unfolding has been applied to separate the global density variation of baryon mass-density from the local variations, dividing each sequence of baryon masses with the same quantum numbers by the mean mass of the sequence. The conclusion of the apparent contradiction between the experimental and theoretical results was that quark models in their present state might be unable to describe the baryon mass-spectrum. A study on the experimental masses cannot be done in another way, because of the scarcity of data. In a theoretical study, however, there is no reason to limit oneself to only a small part of the mass spectrum. In this contribution, in Sec. 4, we studied the baryon mass-spectrum of the Bonn Instanton Quark model [16] up to different mass cutoffs. The number of baryon masses is now sufficient large to perform a (real) numerical unfolding with a polynomial. We found a strong dependence of the statistical results not only on the mass window of the baryon spectrum studied, but also on the degree of the polynomial used for the unfolding. In particular, depending on the window and on the degree of the polynomial, both the limiting cases of the Poisson and the Wigner distribution can be obtained, not only for the NND histogram (see Fig. 10), but also for the extra statistical tools of the function \( F(x) \) (see the insets of Fig. 10) and the spacing moments (see Fig. 11). In conclusion, the apparent contradiction between the results on the experimental baryon mass-spectrum of Ref. 14 and the theoretical baryon mass-spectrum of Ref. 15 would seem an artifact of the unfolding used, rather than an incapacity of the quark models to describe the baryon masses. Until now, short-range correlations have been studied in the experimental and theoretical baryon mass spectrum. A statistical study of long-range correlations in the baryon masses will be for a forthcoming paper [26].

Acknowledgements

The authors wish to thank Dr. B. Metsch for communicating to us the complete baryon mass-spectrum of the Bonn Instanton Model [16]. The authors wish to thank Dr. T. Van Cauteren for his comments. We are grateful to Dr. M. Ploszajczak, Dr. W.M. Snow and Dr. S. Wilburn for their many suggestions during the XXXII Symposium on Nuclear Physics. This work was supported in part by PAPIIT-UNAM grant IN113808.

---


