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Magnetic Diffusion using Lattice-Boltzmann

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We have implemented a lattice-Boltzmann model *(LBM)* to simulate the magnetic diffusion *(MD)* phenomena. An error in our model for the equilibrium distribution function is up to order $O(u^2)$. The magnetic field in our model is considered as a vector valued magnetic distribution function which follows a vector Boltzmann equation. We discuss the diffusion of magnetic field trough plasma in one or two dimensional configurations. Also we make a comparison between the analytical and simulation configurations finding a good agreement.

Keywords: Magnetic diffusion; lattice Boltzmann; magnetic simulation.

Se ha desarrollado un modelo de difusión magnético (MD), aplicando la técnica de lattice-Boltzmann (LBM). El error en nuestro modelo, asociado a la función de distribución de equilibrio, es de orden O(u2). El modelo de campo magnético se considera una función de distribución asociada a cada una de las componentes del campo, de tal forma, que se cumple una ecuación vectorial de Boltzmann. Se discute la difusión de plasma magnético en con_guraciones de una y dos dimensiones. De la misma forma, se realiza una comparación entre lo resultados analíticos y los dados por la simulación, encontrándose buena concordancia.

Descriptores: Difusión magnética; lattice Boltzmann; Simulación Magnética.

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1. Introduction

Lattice Boltzmann has been applied with success to many problems in Physics. The method is suitable to simulate hydrodynamic systems [1], multiphase fluids [2], charge distribution in electrolytes [3], chemical-reactive flows and the flow in porous media [4]. One of the first applications of the method comes from the pioneer work of Wolfram on lattice-gas automata [4-11].

There have been many attempts to build models of magneto-hydrodynamics (*MHD*) using Lattice-Boltzmann *(LB*). Ones of the first was a $2D$ model based on lattice-Gas automata [12]-[13], where a basic automata model (called *FHP*, Frisch, Hasslacher and Pomeau), is extended to include other degrees of freedom in order to give justification to the vector magnetic potential. Also a work done by Succi *et.al*, [14] shows that the lattice-Boltzmann scheme for the Navier-Stokes equations can be extended to include the effects of a two-dimensional magnetic field. Chen *et.al.*, [15],[16] and [17], in 1991, propose a *LB* equation model, for $2D$ and $3D$, that gives rise for moments as mass density, magnetic field, momentum for incompressible *MHD* simulations, and introduces a unique relaxation time that admits easy handling transport coeficients. Fogaccia, *et.al*, [18] in 1996, implement an algorithm using *LBE*, in the electrostatic limit studying $2D$ turbulence. Also, in ref. [19] a *BGK* (Bhatnagar-Gross-Krook) scheme models the collision term recovering the macroscopic dissipative MHD equations. In 2008, [20], Muñoz & Mendoza proposed a $3D$ lattice-Boltzmann model in a cubic lattice with 19 velocities ($D3Q19$), that recuperates *MHD* equations and reproduces the Hartmann flow and magnetic reconnection.

Based on the lattice-Boltzmann technique we find a novel solution to obtain the magneto-hydrodynamics equations. This paper aims to use the anzatz hypothesis [21], a solution to the magnetic diffusion equation using a definition of the tensor Π^0, in the Chapman-Enskog expansion [21]. In section (II), we begin presenting a short review of magnetic diffusion equations. In section (III), we present the basic set of equations of Boltzmann technique and the equilibrium functions for fluid and the magnetic field, based on the *2dq9* scheme [21]. After, we derivate the magnetic diffusion equations using the re-definition of Π^0. In section (IV), we obtain the equilibrium function which we use on the lattice for the implementation in the computational scheme. In section (V), we compare our results with the theoretical approach given by [22], in one and two dimensions for magnetic diffusion, and also we present the vorticity structure of the magnetic field starting from a random initial configurations. At the end of the section (V), we present conclusions.

2. Magnetic diffusion equations

The equations governing the magneto-hydrodynamic phenomena are:

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\nabla\left(\vec{u}\right)\right)=-\nabla\left(p\right)+\vec{j}\times\vec{B}+\mu_f\nabla^2\vec{u}+\rho\vec{g} \qquad (1)$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\vec{u}\right)=0 \qquad (2)$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad (3)$$

$$\nabla\times\vec{B}=\mu_m\vec{j} \qquad (4)$$

$$\vec{j} = \sigma\left(\vec{E} + \vec{u} \times \vec{B}\right) \quad (5)$$

Where we have the momentum, the equation of mass continuity, Maxwell's equations and Ohm's law, respectively. The displacement current in Ampere's law, has not been taken into account reasons by it is valid for a non-relativistic approach for an inertial fluid.

Assumed in the above equations are the following relations:

$$\nabla \cdot \vec{B} = 0 \quad (6)$$

$$\nabla \cdot \vec{j} = 0 \quad (7)$$

If we take the curl of Ohm's law, we get:

$$\nabla \times \vec{j} = \nabla \times \left(\sigma \vec{E}\right) + \nabla \times \left(\vec{u} \times \vec{E}\right) \quad (8)$$

Using Ampere and faraday's laws and the identity:

$$\nabla \times \left(\nabla \times \vec{B}\right) = \nabla\left(\nabla \cdot \vec{B}\right) - \nabla^2 \vec{B} \quad (9)$$

We obtain

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{u} \times \vec{B}\right) + \frac{1}{\mu_m \sigma} \nabla^2 \vec{B} \quad (10)$$

Equation (10) is called the induction equation and describes temporal evolution of the magnetic field in terms of two effects, namely the advection of magnetic field with the plasma and its diffusion through the plasma.

Assuming the fluid in rest, eq. (11) it becomes a pure diffusion equation:

$$\frac{\partial \vec{B}}{\partial t} = D_m \nabla^2 \left(\vec{B}\right) \quad (11)$$

With $D_m = 1/\mu_m \sigma$. In general we can suppose that D_m is non-uniform, then we get:

$$\frac{\partial \vec{B}}{\partial t} = \nabla^2 \left(D_m \vec{B}\right) \quad (12)$$

Lattice velocities of the $D2Q9$ scheme.

3. The lattice Boltzmann model

The Boltzmann equation gives temporal evolution of a single particle probability distribution function $f(\vec{x}, t)$, which in the lattice-Boltzmann method is transform into a discrete function. In the model we use $f_i(\vec{x}, t)$ to justify the fluid and $g_i(\vec{x}, t)$ to the magnetic field. Both of them, at the site become as:

$$f_i(\vec{x} + \vec{e}_i, t+1) = f_i(\vec{x}, t) + \Omega_i(f_i(\vec{x}, t)) \quad (13)$$

$$g_{i,j}(\vec{x} + \vec{e}_i, t+1) = g_{i,j}(\vec{x}, t) + \Omega_{m,j}(g_{i,j}(\vec{x}, t)) \quad (14)$$

where i and m are local collision operators that give local interaction rules among particle collisions. Using *BGK* approximation [23], the collision operators could be approximated by a single time relaxation process that might happen for a given particle probability distribution at constant rate. These distributions are called $f_i^{eq}(\vec{x}, t)$ and $g_i^{eq}(\vec{x}, t)$ and they are given by:

$$\Omega_i = -\left(\frac{f_i(\vec{x}, t) - f_i^{eq}(\vec{x}, t)}{\tau_f}\right) \quad (15)$$

$$\Omega_m = -\left(\frac{g_{i,j}(\vec{x}, t) - g_{i,j}^{eq}(\vec{x}, t)}{\tau_m}\right) \quad (16)$$

Here τ_f and τ_m measure the approaching rate of the system to the statistical equilibrium. The physical conditions to this equilibrium are that the momentum and energy relations are conserved. Also, the analytical form of the equilibrium distribution has to ensure the isotropic and Galilean invariance.

Expanding the distribution functions and the time and space derivatives, we obtain:

$$\begin{aligned} f_i &= f_i^{(0)} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} + \cdots \\ g_{i,\alpha} &= g_{i,\alpha}^{(0)} + \varepsilon g_{i,\alpha}^{(1)} + \varepsilon^2 g_{i,\alpha}^{(2)} + \cdots \\ \partial_t &= \varepsilon \partial_{t_1} + \varepsilon^2 \partial_{t_2} + \cdots \\ \nabla &= \varepsilon \nabla_1 + \cdots \end{aligned} \quad (17)$$

Where $f_i^{(0)} = f_i^{eq}$ and $g_{i,\alpha}^{(0)} = g_{i,\alpha}^{eq}$, and the parameter ε which is assumed small. Also, it is supossed that t_2 is smaller than the time scale t_1 and it is associated with diffusion phenomena.

Replacing equations (17) into eqs. (15-16) we obtain at first order:

$$\begin{aligned} \varepsilon : (\partial_{t_1} + \vec{e}_i \cdot \nabla_1) f_i^{(0)} &= -\frac{1}{\tau_f} f_i^{(1)} \\ (\partial_{t_1} + \vec{e}_i \cdot \nabla_1) g_{i,j}^{(0)} &= -\frac{1}{\tau_m} g_{i,j}^{(1)} \end{aligned} \quad (18)$$

And second order:

$$\begin{aligned} \varepsilon^2 : \partial_{t_2} f_i^{(0)} &+ \frac{1}{2} (\partial_{t_1} + \vec{e}_i \cdot \nabla_1)^2 f_i^{(0)} \\ &+ (\partial_{t_1} + \vec{e}_i \cdot \nabla_1) f_i^{(1)} = -\frac{1}{\tau_f} f_i^{(2)} \\ \partial_{t_2} g_{i,j}^{(0)} &+ \frac{1}{2} (\partial_{t_1} + \vec{e}_i \cdot \nabla_1)^2 g_{i,j}^{(0)} \\ &+ (\partial_{t_1} + \vec{e}_i \cdot \nabla_1) g_{i,j}^{(1)} = -\frac{1}{\tau_m} g_{i,j}^{(2)} \end{aligned} \quad (19)$$

Using some algebra, we have:

$$-\frac{1}{\tau_f} f_i^{(2)} = \partial_{t_2} f_i^{(0)} + \left(1 - \frac{1}{2\tau_f}\right)\left(\partial_{t_1} + \vec{e}_i \cdot \nabla_1\right) f_i^{(1)}$$
$$-\frac{1}{\tau_m} g_{i,j}^{(2)} = \partial_{t_2} g_{i,j}^{(0)} + \left(1 - \frac{1}{2\tau_m}\right)\left(\partial_{t_1} + \vec{e}_i \cdot \nabla_1\right) g_{i,j}^{(1)} \tag{20}$$

Proposing the next definitions:

$$\sum_i^N f_i = \rho$$
$$\sum_i^N g_{i,\alpha} = B_\alpha$$
$$\sum_i^N e_{i,\alpha} f_i = \mu_\alpha$$
$$\sum_i^N e_{i,\alpha} e_{i,\beta} f_i^0 = \Pi_{\alpha,\beta}^0$$
$$\left(1 - \frac{1}{2\tau_f}\right)\sum_i^N e_{i,\alpha} e_{i,\beta} f_i^1 = \Pi_{\alpha,\beta}^1 \tag{21}$$
$$\sum_i^N e_{i,\alpha} g_{i,\beta}^0 = \Gamma_{\alpha,\beta}^0$$
$$\left(1 - \frac{1}{2\tau_m}\right)\sum_i^N e_{i,\alpha} g_{i,\beta}^1 = \Gamma_{\alpha,\beta}^1$$
$$\sum_i^N e_{i,\alpha} e_{i,\beta} g_{i,\gamma}^0 = \Lambda_{\alpha,\beta,\gamma}^0$$
$$\left(1 - \frac{1}{2\tau_m}\right)\sum_i^N e_{i,\alpha} e_{i,\beta} g_{i,\gamma}^1 = \Lambda_{\alpha,\beta,\gamma}^1$$

Again, doing some algebra on eqs. (19) and eqs. (20), we obtain:

$$\partial_t \rho + \partial_\nu \mu_\nu = 0$$
$$\partial_t \mu_\nu + \partial_\mu \left(\Pi_{\mu,\nu}^0 + \varepsilon \Pi_{\mu,\nu}^1\right) = 0$$
$$\partial_t B_\nu + \partial_\mu \left(\Gamma_{\mu,\nu}^0 + \varepsilon \Gamma_{\mu,\nu}^1\right) = 0 \tag{22}$$
$$\partial_t \Gamma_{\mu,\nu}^0 + \partial_\alpha \left(\Lambda_{\alpha,\mu,\nu}^0 + \varepsilon \Lambda_{\alpha,\mu,\nu}^1\right) = 0$$

If we choose $\tau_m = 1/2$, then $\Gamma^1 = 0$ and $\Lambda^1 = 0$, we find:

$$\partial_t B_\nu + \partial_\mu \left(\Gamma_{\mu,\nu}^0\right) = 0 \tag{23}$$
$$\partial_t \Gamma_{\mu,\nu}^0 + \partial_\alpha \left(\Lambda_{\alpha,\mu,\nu}^0\right) = 0 \tag{24}$$

We will use the tensor Γ^0 as a diagonal matrix, where we define the diagonal components as the temporal derivative of the field B_ν, and the D is a factor that balances dimensions in the system:

$$\Lambda_{\alpha,\mu,\nu}^0 = D\delta_{\alpha,\mu}\partial_t B_\nu \rightarrow D \begin{pmatrix} \partial_t B_\nu & 0 \\ 0 & \partial_t B_\nu \end{pmatrix} \tag{25}$$

Replacing eq. (25) in equations (23) and (24), we obtain:

$$\partial_t \Gamma_{\mu,\nu}^0 + D\delta_{\alpha,\mu}\partial_\alpha \partial_t B_\nu = 0 \tag{26}$$

Applying the ∂_ν operator:

$$\partial_\nu \partial_t \Gamma_{\mu,\nu}^0 + D\partial_\nu \partial_\mu \partial_t B_\nu = 0 \tag{27}$$
$$\partial_t \delta_{\mu,\nu} \partial_\mu \Gamma_{\mu,\nu}^0 + D\delta_{\mu,\nu}\partial_t \partial_\mu \partial_\mu B_\nu = 0 \tag{28}$$

or

$$\partial_t \delta_{\mu,\nu}\left(\partial_\mu \Gamma_{\mu,\nu}^0 + D\partial_\mu \partial_\mu B_\nu\right) = 0 \tag{29}$$

Assuming

$$\partial_\mu \Gamma_{\mu,\nu}^0 + D\partial_\mu \partial_\mu B_\nu = 0 \tag{30}$$

Using eq. (23) in eq. (30)

$$-\partial_t B_\nu + D\partial_\mu \partial_\mu \left(B_\nu\right) = 0 \tag{31}$$

Which is the same as

$$\partial_t B_\nu = +D\nabla^2\left(B_\nu\right) = D_m \nabla^2\left(B_\nu\right) \tag{32}$$

4. The distribution function

We use the $d2q9$ scheme shown in Fig. 1, for directions e_i and weights w_i on each cell:

$$w_i = \begin{cases} \frac{4}{9} & if \quad i = 0 \\ \frac{1}{9} & if \quad i = 1,2,3,4 \\ \frac{1}{36} & if \quad i = 5,6,7,8 \end{cases} \tag{33}$$

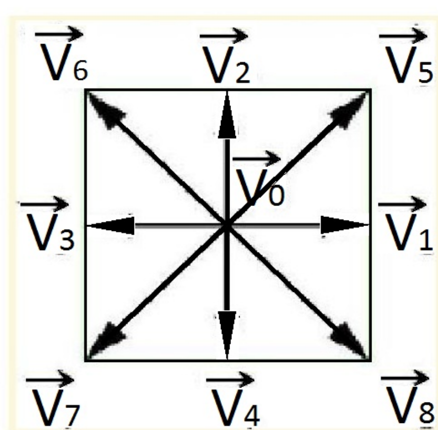

FIGURE 1. Lattice velocities of the $D2Q9$ scheme.

Both, directions v_i and weights w_i, follow the next relations:

$$\sum_i w_i e_{i,\alpha} = 0 \tag{34}$$

$$\sum_i w_i e_{i,\alpha} e_{i,\beta} = \frac{1}{3}\delta_{\alpha,\beta} \tag{35}$$

$$\sum_i w_i e_{i,\alpha} e_{i,\beta} e_{i,\gamma} = 0 \tag{36}$$

The distribution function that we assume for $g_{i,\alpha}^{(eq)}$ is:

$$g_{i,\alpha}^{(eq)} = \begin{cases} w_i \left(A e_{i,\beta} u_\beta B_\alpha + C B_\alpha\right) & \text{if} \quad i > 0 \\ w_0 \left(E B_\alpha\right) & \text{otherwise} \quad i = 0 \end{cases} \tag{37}$$

Where A, C and E are quantities proportional to the magnetic field B_α an its temporal derivative $\delta_t B_\alpha$, and should be determined. Using eq. (25), the tensorial relations eqs. (34-34) and the definition of $g_{i,\alpha}^{(eq)}$, eq.(37), we obtain:

$$\begin{aligned} \Lambda^0_{\alpha,\mu,\nu} &= D_m \delta_{\alpha,\mu} \frac{\partial B_\nu}{\partial t} \\ \Lambda^0_{\alpha,\mu,\nu} &= \sum\nolimits_i e_{i,\alpha} e_{i,\beta} g^0_{i,\gamma} = C\frac{1}{3}\delta_{\alpha,\mu} B_\nu \\ D_m \delta_{\alpha,\mu} \frac{\partial B_\nu}{\partial t} &= C\frac{1}{3}\delta_{\alpha,\mu} B_\nu \\ 3D_m \frac{\partial B_\nu}{\partial t} &= C B_\nu \end{aligned} \tag{38}$$

Using

$$\begin{gathered} \sum\nolimits_i^N e_{i,\beta} g^0_{i,\gamma} = \Gamma^0_{\beta,\gamma} \\ \Gamma^0_{\beta,\gamma} = A \sum\nolimits_\alpha u_\alpha \tfrac{1}{3} \delta_{\alpha,\beta} \\ \Gamma^0_{\beta,\gamma} = \tfrac{A}{3} u_\beta B_\gamma \end{gathered} \tag{39}$$

Assuming

$$\Gamma^0_{\beta,\gamma} = u_\beta B\gamma \tag{40}$$

And equating eq.(39) and eq.(40), we have:

$$A = 3 \tag{41}$$

Finally

$$\sum\nolimits_i g_{i,\alpha} = B_\alpha \tag{42}$$

Using the tensorial relations eqs. (34-36) and the definition of $g_{i,\alpha}^{(eq)}$, eq.(37), we find:

$$\begin{aligned} \sum\nolimits_i g_{i,\gamma} &= C B_\gamma \frac{5}{9} + E B_\gamma \frac{4}{9} \\ B_\gamma &= C B_\alpha \frac{5}{9} + E B_\alpha \frac{4}{9} \end{aligned} \tag{43}$$

Replacing eq. (38) in eq. (43), we obtain:

$$\begin{aligned} B_\gamma &= \left(3D_m \frac{\partial B_\gamma}{\partial t}\right)\frac{5}{9} + E B_\gamma \frac{4}{9} \\ E B_\gamma &= \frac{9}{4} B_\gamma - \frac{15}{4} D_m \frac{\partial B_\gamma}{\partial t} \end{aligned} \tag{44}$$

Collecting the results given in eq. (38), eq. (41) and eq. (44) and replacing them in eq. (37), we obtain:

$$g_{i,\alpha}^{(eq)} = \begin{cases} w_i \left(3 e_{i,\beta} u_\beta B_\alpha + 3 D_m \dfrac{\partial B_\alpha}{\partial t}\right) & \text{if} \quad i>0 \\ w_0 \left(\dfrac{9}{4} B_\alpha - \dfrac{15}{4} D_m \dfrac{\partial B_\alpha}{\partial t}\right) & \text{otherwise} \quad i=0 \end{cases} \tag{45}$$

For the distribution function, $f_i^{(eq)}$ we assume the expression given in ref. [24], which is:

$$f_i^{(eq)} = \begin{cases} \rho w_i \left(1 + 3\vec{e}_i \cdot \vec{u} + \dfrac{9}{2}(\vec{e}_i \cdot \vec{u})^2 - \dfrac{3}{2}\vec{u}^2\right) & \\ -\rho w_i \left(\dfrac{1}{2}\left(\vec{e}_i \cdot \vec{B}\right)^2 - \dfrac{3}{2}\vec{B}^2\right) & \text{if} \quad i > 0 \\ \rho w_i \left(1 - \dfrac{3}{2}\vec{u}^2\right) & \text{otherwise} \quad i = 0 \end{cases} \tag{46}$$

5. Results

In the work of *Wilmot*, [22], the diffusion of a magnetic field has been considered. There have been obtained results for configurations in one, two and three dimensions. In order to compare and validate our simulation results in one and two dimensions we use the analytical results given in *Wilmot*, [22].

5.1. Results for uniform diffusivity

Taking the one-dimensional magnetic diffusion equation

$$\frac{\partial B}{\partial t} = D \frac{\partial^2 B}{\partial x^2} \tag{47}$$

and the magnetic field fixed at two points ($\pm$l), then we get:

$$B(l,t) = -B(-l,t) = B_0 \tag{48}$$

D is assumed constant, eqs. (12) and (32), and the magnetic initial profile is:

$$B(x,0) = \begin{cases} +B_0, & \text{if} \quad x > 0 \\ -B_0, & \text{if} \quad x < 0 \end{cases} \tag{49}$$

The solution is

$$B(x,t) = B_0 \frac{x}{l} + \frac{2B_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left(\frac{-n^2\pi^2 Dt}{l^2}\right) \sin\left(\frac{n\pi x}{l}\right) \tag{50}$$

Figure 2 shows the analytical results, equation (50), for different values of τ_h. In order to compare the analytical result given by equation (50), with the result provided by the Lattice-Boltzmann simulation, sections (III) and (IV), we purport the two curves in Fig. 3. The one-dimensional simulation result was obtained by projecting the result of a two-dimensional Lattice-Boltzmann simulation, Fig. 4, on the $B-x$ plane. The two curves coincide quite well, obtaining a good agreement between the theory and simulation.

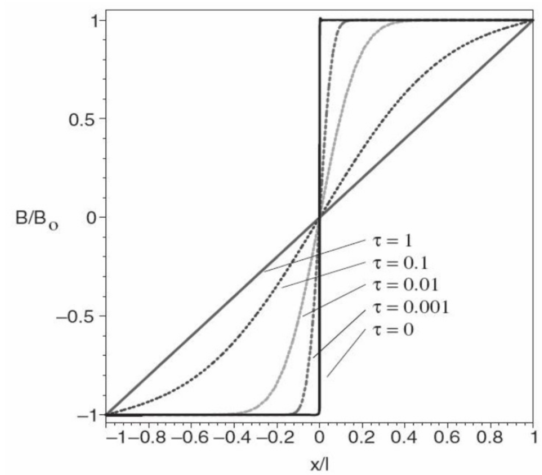

FIGURE 2. Analytic Solution, ref. [22], with different τ_{th}, which is defined as τ_{th}=*Dt/l*^{2}.

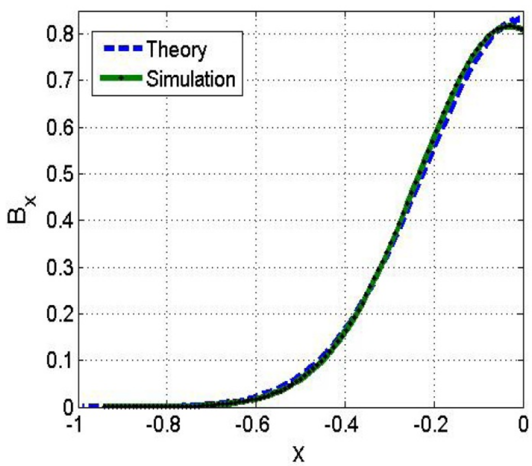

FIGURE 3. Analytic one-dimensional solution, dash line, superimposed over the simulation result, continuous line, with τ_{th}=0,20295.

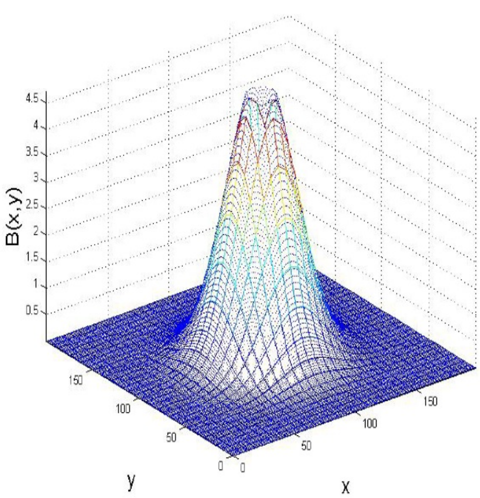

FIGURE 4. The two-dimensional Lattice-Boltzmann simulation result, for a system size $L = 200 \times 200$, with $\tau_m = 1/2$.

5.2. Results for a magnetic field with circular field lines

Taking the radial 2D-dimensional magnetic diffusion equation

$$\frac{\partial B}{\partial t} = D\left(\frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r} - \frac{B}{r^2}\right) \tag{51}$$

For an initial magnetic diffusion flux of radius a, we have:

$$B\left(r,0\right) = F_0 \delta\left(r-a\right) \tag{52}$$

The solution, [22], is:

$$B\left(r,t\right) = -r\frac{F_0}{4D^2t^2}\left(\int_a^\infty s\exp\left(\frac{-r^2+s^2}{4Dt}\right) I_0\left(\frac{rs}{2Dt}\right) ds\right) + \frac{F_0}{4D^2t^2}\left(\int_a^\infty s^2\exp\left(\frac{-r^2+s^2}{4Dt}\right) I_1\left(\frac{rs}{2Dt}\right) ds\right) \tag{53}$$

Where I_0 and I_1 are the hyperbolic Bessel function of zero and first order, respectively.

In the same way we compare the results for the radial analytical solution, eq.(53), which is shown in Fig. 5, with that obtained from the Lattice-Boltzmann simulation, Fig. 4. The two results have the same overall behavior.

5.3. Results for a random initial configuration

We performed simulations on a 200×200 spatial grid with periodic boundary conditions. We start from an initial random configuration, for velocities and magnetic fields. Figures 6, 7 and 8 show the magnetic field configuration for different time steps, such as $100, 400$ and 4000, respectively. Clearly, we can see the vorticity structure for magnetic field.

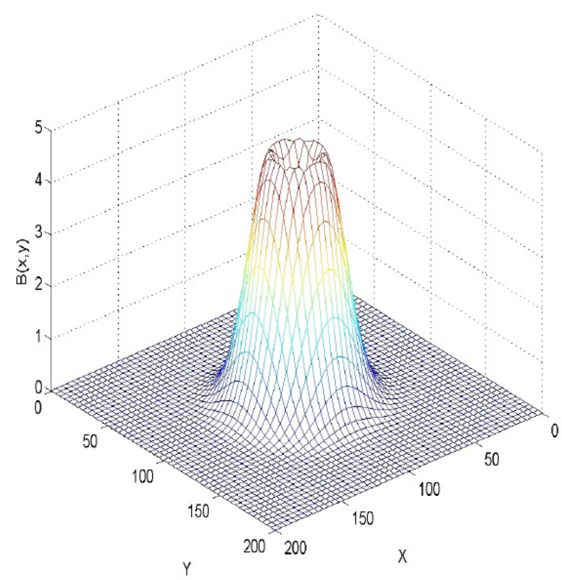

FIGURE 5. The two-dimensional analytic result, Eq. (53) for a system size $L = 200 \times 200$.

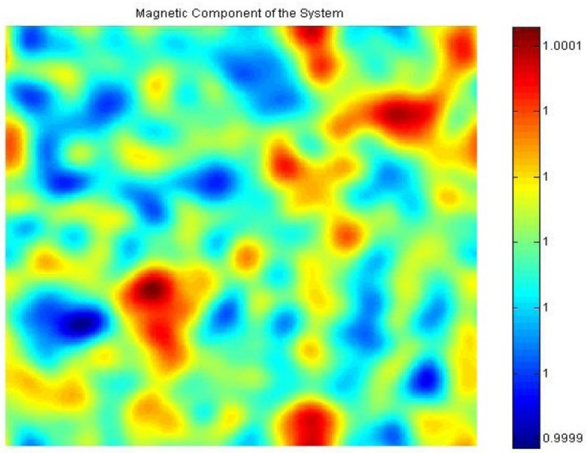

FIGURE 6. Magnetic simulation result for a system of spatial grid of 200×200, for a simulation time $t = 100$.

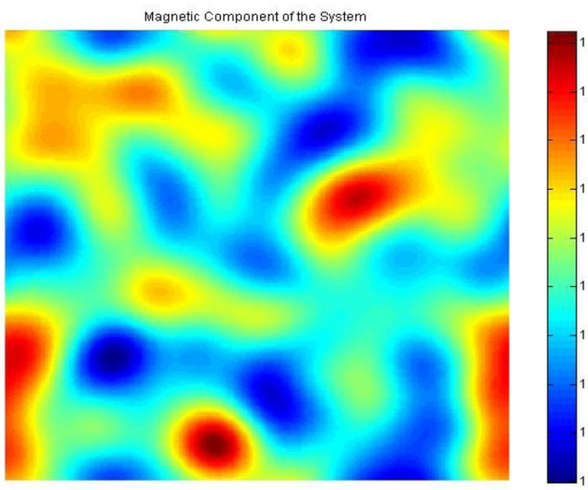

FIGURE 7. Magnetic simulation result for a system of spatial grid of 200×200, for a simulation time t = 400.

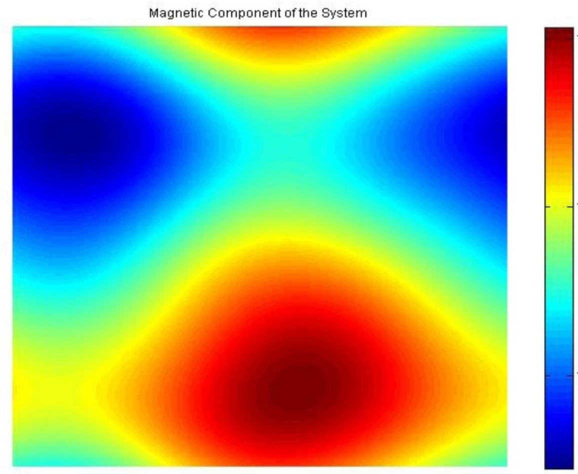

FIGURE 8. Magnetic simulation result for a system of spatial grid of 200×200, for a simulation time $t = 4000$.

6. Conclusions

We have implemented a new strategy using the lattice-Boltzmann technique to obtain the magneto-hydrodynamic equations and in particular the magnetic diffusion equation using the anzatz hypothesis [21], using a defintion of the tensor Π^0, extracted from the Chapman-Enskog expansion. The comparison between the theory and the lattice-Boltzmann simulation agrees quite well for the one and two-dimensional case. As a future work we will extend the method to reproduce equation (10) and to study magnetic reconnection phenomena.

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