Entanglement on Ising model with Dzyalshinksy-Moriya interaction

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Recibido el 24 de junio de 2010; aceptado el 22 de marzo de 2011

We have studied the quantum phase transition of the Ising model with Dzyaloshinsky-Moriya (DM) interaction, by means of measures of entanglement as concurrence and von Neumann entropy. The study was performed using the Density Matrix Renormalization Group (DMRG). The parameter which drives the phase transition in this model is the D, which accounts for the strength of the DM interaction. The model has two phases, an antiferromagnetic phase and a chiral saturated phase, with critical point D = 1. By calculating the entanglement measures for both, open and periodic boundary conditions it was found that the critical point is signaled by an extreme behavior of the entanglement measure, maximum or minimum, it was also found these measures seem to tend to an asymptotic value when D $\rightarrow \infty$ representing the chiral saturated phase.

Keywords: Dzyaloshinsky-Moriya interaction; entanglement; quantum phase transitions.

Nosotros estudiamos la transición de fase cuántica del modelo de Ising con interacci´on Dzyaloshinsky-Moriya (DM) por medio de medidas del entreveramiento como la concurrencia y la entropía de von Neumann. El estudio fue realizado usando el método de grupo de renormalización de la matriz densidad. El parámetro que genera la transición de fase en este modelo es D el cual cuantifica la magnitud de la interacción DM. El modelo tiene dos fases, una antiferromagnetica y otra fase chiral saturada, con un punto crítico cuántico en D = 1. Calculando las medidas de entreveramiento para condiciones de frontera abiertas y periódicas, encontramos que el punto crítico cuántico es señalado por un comportamiento extremo en la medida del entreveramiento, un máximo o un mínimo, también se encontro de estas medidas tienden a un valor asintótico cuando D $\rightarrow \infty$ representando la fase chiral saturada.

Descriptores: Interacción Dzyaloshinsky-Moriya; entreveramiento; transiciones de fase cuánticas.

PACS: 03.67.Mn; 64.70.Tg; 75.10.Pq.La

1. Introduction

In the last years has been suggested a connection between Quantum Information Theory (QIT) and Condensed Matter Physics that have been usually associated with the fact that systems of condensed matter are useful to implement devices proposed in QIT, like was mentioned before. However it has been seen that this association can goes in the other sense too. Meaning that the tools that are employed in QIT, like entanglement, is being useful to study condensed matter systems and characterize them. Specifically it has been seen that entanglement could be a good indicator of quantum phase transitions in condensed matter systems, due to it gives account of quantum correlations.

Quantum phase transitions take place at zero temperature and are due to changes in some parameter order or coupling constants driven by quantum fluctuations [5, 6]. As classical phase transitions, quantum phase transitions are characterized by a non-analytical behavior of some physical quantity. This property can be the entanglement [2, 3, 5–8]. In some cases there have been found maximum of entanglement measures at the quantum critical points of the system [3, 9, 10]. Hence entanglement may be used as indicator of phase transitions. Nevertheless, this is a current field of study and there is not a unique criterion which specify which measure of entanglement is useful to point out a phase transition in a given system. Investigations have shown that for different systems, different entanglement measures indicates the critical point of the case studied.

In this paper, we want to study the quantum phase transition in the Ising model with Dzyaloshinsky-Moriya (DM) interaction, using the entanglement measures.

Dzyaloshinsky-Moriya interaction describes the phenomenon of weak ferromagnetism in some materials. Dzyaloshinsky [11] gave an explanation from symmetrical and hermodynamical considerations concluding that weak ferromagnetism is owing to the relativistic spin-lattice and magnetic dipolar interactions. Moriya [12,13] set up the analytical form of the interaction by means of perturbation theory and confirmed what Dzyaloshinsky previously concluded.

Materials in which Dzyaloshinsky-Moriya interaction is present has been widely studied in different fields such as condensed matter physics and quantum information theory. There has been found that DM interaction is important not only in weak ferromagnets but also in other classes of magnetic systems such as spin glasses, molecular magnets and magnetic surfaces among others and it also has effects in some systems to enhance the entanglement in some systems so they can be better quantum channel for teleportation [14–16]. There has also been mentioned that DM interaction is the simplest example of magnetic relativistic interactions. The fact that DM interaction vanishes in high symmetry crystal explains the importance of it on low symmetric cases such as molecules, clusters, surfaces and disordered systems [13].

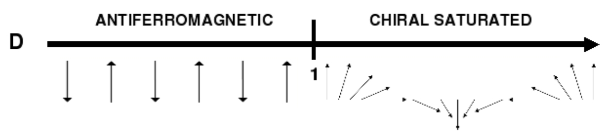

FIGURE 1. Phase diagram of the model.

In this paper, we study the one-dimensional Ising model Dzyaloshinsky-Moriya interaction at zero tempera-ture using White?s density matrix renormalization group (DMRG). In section 2 we discuss the model and the results; and the conclusions are presented in section 3

2. Model and results

If the interaction is set only about the z axis the Hamiltonian of the Ising model with Dzyaloshinsky-Moriya interaction is given by

$$H = J\sum \left(S_i^z S_{i+1}^z + \frac{i}{2} D \left(S_i^+ S_{i+1}^- - S_i^- S_{i+1}^+ \right) \right) \quad (1)$$

Where, S_i^+, $S_i^{-?}$ and S_i^z are the spin operators, J is the exchange parameter and D represents the intensity of the interaction in the z axis:

In the absence of the interacccion term (D = 0) we obtain the model well-known Ising model, which ground state is antiferromagnetic and is shown in the Fig. 1. But, when the Dzyaloshinsky-Moriya interaction is present (D 6= 0) this causes an ordering of the spins in the XY plane. When D is increased the AF order in z direction begins to disappear, appearing instead a chiral order which increases until it is saturated [2]. The phase diagram of this model is represented in Fig. 1.

At D = 1 there is a phase transiton between an antiferromagnetic state to a chiral one. Jafari et al. show that the staggered magnetization

$$S_M = \frac{1}{N} \sum_i (-1)^i \left(\sigma_i^z \right) \quad (2)$$

is the orden parameter of this transition, *i.e.* when $D < 1$ the staggered magnetization is nonzero, while for $D > 1$ this vanish [2].

Using the quantum renormalization group (QRG) and numerical exact diagonalization methods Jafari et al. calculate the local von Neumann entropy and found that this quantity is zero for $D < 1$ and one for $D > 1$, with a discontinuity at D = 1. They concluded that the local von Neumann entropy is a suitable quantity to identify the critical point at this model.

With Jafari's results on mind, we wondered if other en tanglement measures and boundary conditions gave account of the phase transition of the Ising model with DM interaction. In order to answer this question, we study long chains using the density matrix renormalization group (DMRG).

We focus our attention in the bipartite entanglement measures of von Neumann entropy and concurrence. Von Neumann entropy is defined as

$$E(\rho) = -\sum_i \alpha_i^2 log\left(\alpha_i^2\right) \quad (3)$$

where ρ is the reduced density matrix of the system and α_i^2 the eigenvalues of it.

On the other hand, concurrence has been widely used to investigate several typical localized spin models [3,10]. It was first used as an auxiliar quantity to calculate the entanglement of formation, however it can be used as an entanglement measure by itself [17]. The concurrence for a bipartite two-level system, of a mixed state is defined using the following matrix.

$$S = \rho\left(\sigma_y \otimes \sigma_y\right) \rho^* \left(\sigma_y \otimes \sigma_y\right) \quad (4)$$

where σ_y is the second Pauli matrix of dimension 2. Diagonalizing S we obtain the eigenvalues λ_i^2 , with them we can calculate the concurrence defined as

$$C(\rho) = \max\left(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4.0\right) (5)$$

We justify to use these entanglement measures since for a quantum many-body system, the reduced density matrix, which is constructed by keeping some local variables intact and integrating out all the rest degrees of freedom, contains information not only on the subset itself, but also on the correlation between this part and the rest of the system. This observation is dictated by the superposition principle of quantum mechanics. Therefore, one is allowed to investigate some global properties of the system by a local measurement [9].

By using the DMRG method, there were calculated the entanglement measures block von Neumann entropy and concurrence for a chain of L = 60 sites when varying the D parameter, with periodic and open boundary conditions.

In Fig. 1a we show the entanglement measure of concurrence as a function of the interaction parameter D, using periodic boundary conditions. As it is expected, concurrence is zero at D = 0, the state is not entangled at this point. The state

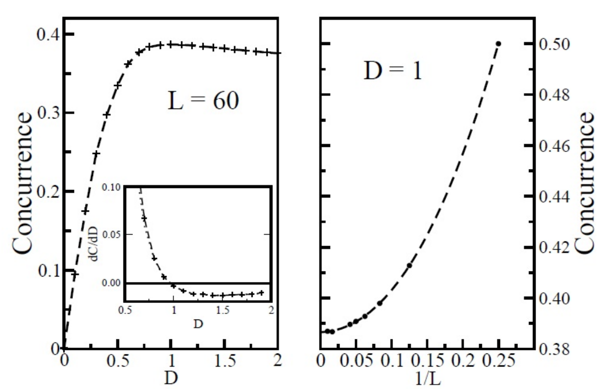

FIGURE 1. Left side: Concurrence as a function of D for a chain of L = 60 sites. Inside the first derivative od the concurrence. Right side: Concurrence at D = 1 as a function of 1=L. Here we consider periodic boundary conditions.

can be described as the direct product between states of spaces of one site since it is completely antiferromagnetic. When $D > 0$ the state of the system starts to become entangled, this is reflected in the rapid growth of the concurrence. The reason of this behavior is attributed to the Dzyaloshinsky-Moriya interaction. The state of the whole system can not be expressed anymore as a product of individual states. At D = 1 the concurrence reaches a maximum, which is ratified by the zero of its derivative (Inside right side Fig. 1a). At this point, the phase transition takes place and the system changes from a antiferromagnetic order to a saturated chiral one. When D>1 the concurrence diminishes in a small quantity remaining almost constant, *i.e.* the Dzyaloshinsky-Moriya interaction do not entangle more the states and leave them almost in the same way.

We observe that the curve of the concurrence as a function of D preserves same form for any lattice size, *i.e.* the concurrence increases from zero reaches a maximum at D=1 and decreases slightly; but the maximum of the concurrence decreases with the lattice size, as can be seen in Fig. 1b. A cubic regression was done and the value of the concurrence for D = 1 within the thermodynamic limit obtained was 0.38753. The actual value for L = 60 differs in 0.2% of the thermodynamic limit value.

The results of the concurrence with open boundary conditions are shown in 2. We observe that the concurrence at D = 0 is zero, indicating that the the state can be described as the direct product between states of spaces of one site, i.e the system is not entangled. When the interaction parameter D increases the concurrence grows up to reach a maximum value at D = 1, which is higher than the periodic boundary conditions value. For $D > 1$ we observe that the concurrence decrease more faster than the periodic boundary case. This behavior of the concurrence holds for all lattices sizes. However the maximum value of the concurrence increases with the lattice size and tends to 0.623 within the thermodynamic limit.

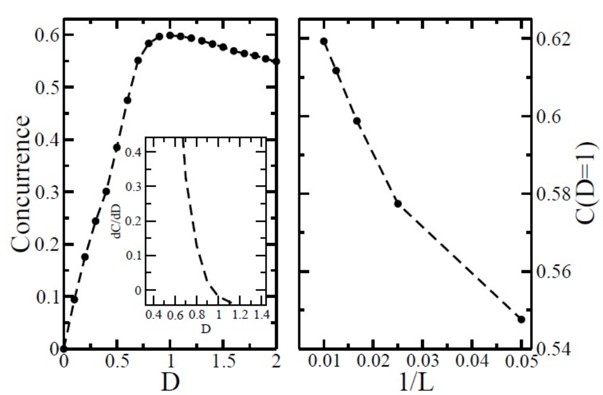

FIGURE 2. Left side: Concurrence as a function of D for a chain of L = 60 sites. Inside the first derivative od the concurrence. Right side: Concurrence at D = 1 as a function of 1=L. Here we consider open boundary conditions.

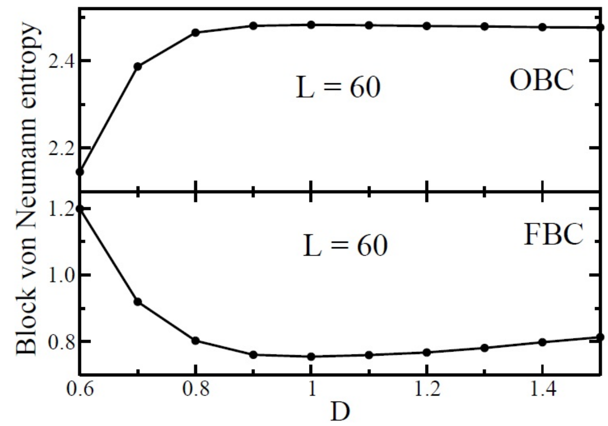

FIGURE 3. Block von Neumann entropy as a function of D for a chain of L = 60 sites. Top: periodic boundary conditions. Botton: open boundary conditions.

Thus, we observe that the concurrence is a suitable quantity to indentify the critical point of this model, which is at D = 1 but the boundary conditions afects the relative value at this point.

In Fig. 3 we show the block von Neumann entropy as a function of the D parameter for both periodic and open boundary conditions. We note that for periodic boundary conditions the block von Neumann entropy increases and reaches a maximum at D = 1 pointing the critical point of the model (Fig. 3 top). For open boundary conditions, we observe that the block von Neumann entropy decreases with the D parameter and reaches a minimum at D = 1, after that this increases (Fig. 3 bottom).

As we saw and have been pointed out by other authors [2–4, 7, 9, 10, 18, 19] the research on quantum phase transitions and its relation with entanglement is a vastly field of study. There is no yet a unique criterion which stablishes what specific entanglement measure one must use to find the phase transitions of a model. Hence the way to follow is to study different measures based on previous results which may give a clue of what measure could be useful and serves as an indicator of the quantum critical points.points.

3. Conclusions

We calculate the concurrence and the block von Neumann entropy for the Ising model with Dzyalshinksy-Moriya interaction using the density matrix renormalization group (DMRG). We found that these measures of entanglement presents extreme values at the critical point of this model for both open and periodic boundary conditions. Our results agree and complement the existing results for this model.

Acknowledgements

This work was financed by Universidad Nacional de Colombia (DIB-8003357).

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