Computer simulations were used to create perfectly ordered binary virtual samples, formed by magnetic (56.25%) and nonmagnetic (43.75%) atoms in a BCC phase and with DO$_3$ order. Subsequently, the samples were disordered and then atomic migration between the lattice sites was simulated; this migration was governed by a migration probability $P_m$ (specified by us) and we achieve the phase A2 for $P_m = 1.0$. An exponential relation was obtained between the long range order parameter (measurable experimental parameter $S$) and the migration probability (microscopic parameter $P_m$). Magnetization, susceptibility, the Edward Anderson parameter, and the mean energy as a function of the dimensionless temperature $\Theta$ and long range order parameter were calculated using the Monte Carlo method with Metropolis dynamic; they allow us to determine the critical temperature for samples with phase transition. The saturation magnetization results showed that using the 1/2 Ising model, the disorder induces a para-ferro phase transition due to the substitutional disorder. The highly ordered samples showed a paramagnetic behavior in the entire temperature range, while those samples with migration probability $P_m > 0.06$ have a ferromagnetic phase transition at low temperatures. Additionally, it was found that the critical order parameter is 0.882.

Keywords: Magnetism; binary alloy; long range order parameter; migration probability; Monte Carlo.

This concentration (56.25%) was chosen because it has the DO$_3$ order when it is perfectly ordered and is easily constructed as a series of supercells of two lattice parameter ($l = 2a$) with clusters of nine magnetic atoms surrounded by planes of diluter atoms (Fig. 1). Direct interactions between the spins are restricted to clusters of nine atoms and the system will exhibit a global paramagnetic behavior.

2. Theoretical model

The long-range order parameter $S$ (Eq. (1)) determines the regularity and uniformity with which the components are distributed in the crystal structure, with reference to a predetermined atomic configuration [6].

$$S = \frac{r_A - F_A}{1 - F_A} = \frac{r_B - F_B}{1 - F_B}$$

(1)

With:

$$r_\alpha = \frac{N_\text{mag}}{N_\alpha}; \quad F_\alpha = \frac{N_\text{nonmag}}{N}$$

(2)
Figure 1. Two supercells (l = 2a) of a binary alloy with a concentration of magnetic atoms of 56.25% (black spheres) and 43.75% of diluter atoms (white spheres) with a perfect configuration order (DO₃). a is the lattice parameter. Note the isolated clusters of nine magnetic atoms.

where \( r_A (r_B) \) is the fraction of atoms A (B) occupying right A (B) lattice sites, and \( F_A (F_B) \) is the fraction of atoms A (B) in the solution (\( F_A = 0.5625 \) and \( F_B = 0.4375 \)). When the long-range order is perfect the fraction of atoms that are well placed is equal to unity (\( r_A = 1 \)) and therefore \( S = 1 \).

To study the magnetic response of the system the spin-\( \frac{1}{2} \) Ising model is considered. In this model the magnetic atoms (A) have net spin \( \sigma (\sigma = \pm 1) \) and diluter atoms (B) have no net spin (\( \sigma = 0 \)).

The Hamiltonian of the system, without external magnetic field, is expressed as:

\[
H = -J \sum_{\langle ik \rangle} \sigma_i \sigma_k
\]  

(3)

where the symbol \( \langle ik \rangle \) means that the sum is performed on the nearest neighbor (first neighbors) and \( J \) is the exchange energy between \( i \)-th and \( j \)-th spins.

3. Results and discussion

To analyze the effect of the order parameter in A_{56.25}B_{43.75} binary system it was initially built a DO₃ ordered sample (Fig. 1) in a bcc lattice with sized \( L \times L \times L \) (\( L = 10 \) lattice parameters). To disorder the sample it was used the Aguirre’s algorithm [7], which uses by they called the Migration Probability (\( P_m \)). \( P_m \) provides the probability that an atom in a lattice site exchanges position with another atom in another lattice site.

In this study, samples were disordered and the order parameter was calculated by Eq. (1), taking as reference the initial sample DO₃.

The exponential decay was excellently fitted by:

\[
S = 1 - A \left[ 1 - \exp \left( -\frac{P_m P}{P_r} \right) \right]
\]  

(4)

where \( A = 1.2461 \approx [1 - \exp(-P_{-1}^{-1})]^{-1} \) and \( P_r = 0.6121 \) are constants.

After constructing the virtual sample, Monte Carlo’s thermal scans were performed (with Metropolis dynamic) for a dimensionless temperature (\( \Theta = k_B T/J \)) varying from 8.00000 to 0.10102 (\( \pm 5 \times 10^{-5} \)). For each temperature the properties were analyzed with \( 8 \times 10^4 \) Monte Carlo steps and \( 5 \times 10^3 \) MC thermalization steps. ZFC process was emulated by changing the temperature according to the relationship \( \Theta_k = 0.99 \Theta_{k+1} \), where \( k \) and \( k+1 \), are the present and back state, respectively.

Figure 3 shows the behavior of: (a) the susceptibility, and (b) the Edward Anderson factor \( Q_{EA} \) (fourth cumulant of the magnetization) as a function of dimensionless temperature (\( \Theta \)), for different order parameters (\( S \)). Lambda peaks are ob-

Figure 2. Shows that the migration probability \( P_m \) (statistical effect of microscopic thermal actions, mechanical or otherwise) produces an exponential decay in the order parameter \( S \) (parameter experimentally measurable by X-ray diffraction [5]).
served in susceptibility curves that match with abrupt jumps in $Q_{EA}$. This means that there is a phase transition from a ferromagnetic to a paramagnetic state at the peak temperature. The perfectly ordered sample ($P_m = 0.0$ and $S = 1.0$) has not phase transition, when $P_m > 0.06$ phase transition occur at increasingly higher temperatures. The critical temperature behavior is presented in Fig. 4. It shows the phase diagram of the system $A_{56.25}B_{43.75}$ as a function of the order parameter.

Energy and magnetization curves were also simulated. Figure 5 shows the fundamental energy ($E_0$) and the obtained magnetization by spin ($\Theta \rightarrow 0$). We see that the fundamental energy per spin presents a linear behavior with $S^2$, the constant term $(2.26 \pm 0.1)$ is close to the average number of expected bonds, assuming a binomial distribution of magnetic atoms in the BCC structure $(zp/2 = 8(0.5625)/2 = 2.25)$. The magnetization curve shows saturation close to $S^2 \approx 0.56$ and a fall around $S^2 \approx 0.8 \approx (0.89)^2$, consistent with the phase diagram of Fig. 4.

In Eq. (3) (Ising Hamiltonian), the value of the fundamental energy decreases as increases the number of terms of the sum; Fig. 5, and the above, confirms that the decrease of the order parameter (disorder of the sample) increases the number of magnetic bonds. Then, it is reasonable that the critical temperature must increase as increases the disorder of the sample (decrease of S in Fig. 4).

**4. Conclusions**

Under spin $1/2$ Ising model disorder induces the appearance of a phase transition from a paramagnetic state (highly ordered samples, $S \sim 1$) to a ferromagnetic state (substitutional disordered samples). Moreover, if the long-range order parameter is less than 0.882 then the phase transition happens. This transition is the result of the increasing of percolation paths as the sample is lest ordered.

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