Effect of combined heat transfer on the thermoeconomic performance of an irreversible solar-driven heat engine at maximum ecological conditions

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In the present paper, we study the thermoeconomic optimization of an irreversible heat engine by using finite-time/finite-size thermodynamics theory. In this study we take into account losses due to heat transfer across finite time temperature differences, heat leakage and internal irreversibilities in terms of a parameter which comes from the Clausius inequality. In the considered heat engine model, the heat transfer from the hot reservoir to the working fluid is assumed to be simultaneous by radiation and conduction modes and the heat transfer to the cold reservoir is assumed of the conduction type. In this work, the optimum performance is analyzed under the effect of two design parameters when the heat engine operates under two objective functions: the power output per unit total cost and the ecological function per unit total cost. The effects of the technical and economical parameters on the thermoeconomic performance have been also discussed under the aforementioned two criteria.

Keywords: Thermoeconomic performance; irreversible heat engine; optimization.

En el presente trabajo estudiamos la optimización termo-económica de una máquina térmica irreversible dentro del contexto de la Termodinámica de Tiempo Finito/tamaño finito (TTF). En nuestro estudio se consideran pérdidas de calor a través de la transferencia directa de energía entre los almacenes térmicos, conocido como corto circuito (heat leak) e, irreversibilidades internas en términos de un parámetro que proviene de la desigualdad de Clausius. En el modelo de máquina térmica irreversible se considera que la transferencia de calor entre el almacén térmico a alta temperatura y la sustancia de trabajo, se da simultáneamente por radiación y conducción, mientras que, de la sustancia de trabajo al almacén térmico a baja temperatura, la transferencia de calor únicamente se da por conducción. Se determinan las condiciones de operación termo-económicamente óptimas del modelo de máquina térmica en términos de parámetros de diseño y construcción. Nuestra optimización se lleva a cabo bajo dos regímenes de operación: máxima potencia de salida y máxima función ecológica.

Descriptores: Optimización termo-económica; máquina irreversible; optimización.

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1. Introduction

In 2000, Sahin [1] studied the thermodynamics performance of an endoreversible solar-driven heat engine. In this study, he considered that the heat transfer from the hot reservoir to the working fluid is given by radiation, while the mode of heat transfer from the working fluid to the cold reservoir is given by a Newtonian heat transfer law. Sahin [1], calculated the optimum temperatures of the working fluid and the optimum efficiency of the engine operating at maximum power conditions. In 2008, Barranco-Jiménez et al. [2], studied the optimum operation conditions of an endoreversible heat engine with different heat transfer laws at the thermal couplings but operating under maximum ecological function conditions. The thermodynamic analysis based on the above works do not take account of the effect of economical aspects. The effect of the economical aspects was introduced by Sahin and Kodal [3] to study the termoeconomics of an endoreversible heat engine in terms of the maximization of a profit function defined as the quotient of the power output and the annual investment and the full consumption costs. Although this kind of thermo-economic analysis was early introduced by De Vos for study the thermoeconomic performance of a model of power plant of the Novikov type [4]. This thermoeconomic performance analysis, called the Finite-Time Thermo-economic optimization, was applied for Sahin et al. [5] to study the thermo-economic performance of an endoreversible solar driven-heat engine. Sahin et al [5] defined an objective function in terms of the ratio of power output per unit total investment cost. Recently, Barranco-Jiménez et al. [6] also studied the thermoeconomic optimum operation conditions of a solar-driven heat engine. In this study, Barranco-Jiménez et al. considered three regimes of performance: The Maximum Power Regimen (MPR) [7–9], the maximum efficient power [10] and the maximum ecological function regime (MER) [11, 12]. In 2007, Ust [13] studied the thermo-economic performance analysis performed by Sahin et al [5] for the endoreversible solar-driven heat engine model but including in his model internal irreversibilities and heat leakage effects. In all the aforementioned Finite-Time
Thermodynamics and Finite-Time Thermoeconomics studies we do not used a combined heat transfer law working at maximum ecological function. In the present work, we study the thermoeconomics of an irreversible heat engine with losses due to heat transfer across finite temperature differences, heat leakage between thermal reservoirs and internal irreversibilities in terms of a parameter which comes from the Clausius inequality. The heat transfer from the hot reservoir to the working fluid is assumed to be simultaneous conduction and radiation modes and the heat transfer to the cold reservoir is assumed of the conduction type [13]. In our study we use those two regimes of performance: The maximum power regime, and the so-called ecological function regime. The article is organized as follows: In Sec. 2, we present the irreversible heat engine model; in Sec. 3, the numerical results and discussion are presented, and finally, in Sec. 4, we present our conclusions.

2. Theoretical model

The considered irreversible solar-driven heat engine operates between a heat source of temperature \( T_H \) (the collector) and a heat sink of temperature \( T_L \) (cooling water) see Fig. (1a). The temperatures of the working fluid exchanging heat with the reservoirs at \( T_H \) and \( T_L \) are \( T_X \) and \( T_Y \), respectively. A T-S diagram of the model including heat leakage, finite time heat transfer and internal irreversibilities is also shown in Fig. (1b). Heat transfer from the hot reservoir is assumed to be simultaneously conduction and radiation modes. The net heat flow rate \( \dot{Q}_H \) from the hot reservoir to the heat engine can be written as [13],

\[
\dot{Q}_H = \dot{Q}_{HC} + \dot{Q}_{HR} = U_{HC}A_H(T_H - T_X) + U_{HR}A_H(T_H - T_X),
\]

where \( U_{HC} \) and \( U_{HR} \) are the heat transfer coefficients for conduction and radiation heat transfer modes, respectively, and \( A_H \) is the heat transfer area of the hot-side heat exchanger. On the other hand, conduction heat transfer is assumed to be the main mode of the heat transfer to the low temperature reservoir and therefore the heat flow rate \( \dot{Q}_L \) from the heat engine to the cold reservoir can be written as,

\[
\dot{Q}_L = U_{LC}A_L(T_Y - T_L)
\]

where \( U_{LC} \) is the cold side heat transfer coefficient and \( A_L \) is the heat transfer area of the cold-side heat exchanger. The rate of heat leakage \( \dot{Q}_{LK} \) from the hot reservoir at temperature \( T_H \) to the cold reservoir at temperature \( T_L \) with thermal conductance \( \gamma \) is given by,

\[
\dot{Q}_{LK} = \gamma(T_H - T_L) = \xi U_H A_H(T_H - T_L),
\]

where \( \gamma \) is the internal conductance of the heat engine and \( \xi \) denotes the percentage of the internal conductance with respect to the hot-side conduction heat transfer coefficient and heat transfer area, that is, \( \xi = (\gamma/U_H A_H) \) [13]. Then the total heat rate \( \dot{Q}_{HT} \) transferred from the hot reservoir is,

\[
\dot{Q}_{HT} = \dot{Q}_H + \dot{Q}_{LK},
\]

and the total heat rate \( \dot{Q}_{LT} \) transferred to the cold reservoir is,

\[
\dot{Q}_{LT} = \dot{Q}_L + \dot{Q}_{LK}.
\]

Applying the first law of thermodynamics, the power output is given by,

\[
W = \dot{Q}_{HT} - \dot{Q}_{LT} = \dot{Q}_H - \dot{Q}_L.
\]

By using Eqs. (1), (2) and (6), we get a normalized expression for the power output \( \overline{W} = (W/U_{HC}A_H) \), given by,

\[
\overline{W} = (T_H - T_X) + \beta \left( T_H - \frac{T_H}{\overline{T}_H} \right) - \psi A_R(T_Y - T_L),
\]

where \( \beta = (U_{HR}/U_{HC})T_H^3 \), \( \psi = (U_{LC}/U_{HC}) \) and \( A_R = (A_L/A_H) \). Applying the second law of thermodynamics to the irreversible part of the model we get,

\[
\int \frac{dQ}{T} = \frac{\dot{Q}_H}{T_X} - \frac{\dot{Q}_L}{T_Y} < 0.
\]
One can rewrite the inequality in Eq. (8) as,
\[
\frac{\dot{Q}_H}{T_X} = \frac{\dot{Q}_L}{T_Y},
\tag{9}
\]
where \( R \) is the so-called nonendoreversibility parameter \([14–17]\). This parameter, which in principle is within the interval \( 0 < R \leq 1 \) (\( R = 1 \) for the endoreversible case), can be seen as a measure of the departure from the endoreversible regime \([14–17]\). Substituting Eqs. (1) and (2) into Eq. (9), a relationship between \( T_Y \) and \( T_X \) is obtained as,
\[
\frac{T_Y}{T_L} = \frac{R\psi A_R - (1-\eta) - \beta (1-\eta^4)}{R\psi A_R},
\tag{10}
\]
where \( \theta = (T_X/T_H) \). On the other hand, the thermal efficiency of the irreversible heat engine is,
\[
\eta = 1 - \frac{Q_L}{Q_{HT}} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H},
\tag{11}
\]
In thermoeconomic analysis of power plant models, an objective function is defined in terms of a characteristic function (power output \([4, 5, 18, 19]\), ecological function \([2, 6, 11, 15, 20]\) and the cost involved in the performance of the power plant. In his early paper on this issue, De Vos \([4]\) studied the thermoeconomics of a Novikov power plant model in terms of the maximization of an objective function defined as the quotient of the power output and the performing costs of the plant. In that paper \([4]\), De Vos considered a function of costs with two contributions: the cost of the investment which is assumed as proportional to the size of the plant and the cost of the fuel consumption which is assumed to be proportional to the quantity of heat input in the Novikov model. Analogously, Sahin and Kodal \([3]\) made a thermoeconomic analysis of a Curzon and Ahlborn \([8]\) model in terms of an objective function which they defined as power output per unit total cost taking into account both the investment and fuel costs \([3]\), but assuming that the size of the plant can be taken as proportional to the total heat transfer area, instead of the maximum heat input previously considered by De Vos \([4]\). Following the Sahin et al. procedure \([5]\), the objective function has been defined as the power output per unit investment cost, due to a solar driven heat engine does not consume fossil fuels. In order to optimize power output per unit total cost, the objective function is given by \([5, 13]\),
\[
F_P = \frac{W}{C_i},
\tag{12}
\]
where \( C_i \) refers to annual investment cost. The investment cost of the plant is assumed to be proportional to the size of the plant. The size of the plant can be proportional to the total heat transfer area. Thus, the annual investment cost of the system can be written as \([5]\),
\[
C_i = aA_H + bA_L,
\tag{13}
\]
where the investment cost proportionality coefficients for the hot and cold sides \( a \) and \( b \) respectively are equal to the capital recovery factor times investment cost per unit heat transfer area, and their dimensions are \( \text{ncu/(year} \cdot \text{m}^2) \), \( \text{ncu} \) being the national current unity. In analogous way to Eq. (12), we define another objective function in terms of the ecological function and the unit total cost \([6, 15, 21]\),
\[
F_E = \frac{E}{C_i} = \frac{W - T_L \Sigma}{C_i},
\tag{14}
\]
where \( \Sigma \) is the total entropy production of the irreversible heat engine model. If we apply the second law of thermodynamics to the model of Fig. 1, the total entropy production \( \Sigma \) can be expressed as,
\[
\Sigma = -\frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_H}{T_X} - \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_L}{T_H} + \frac{\dot{Q}_L}{T_L}.
\tag{15}
\]
Using Eqs. (1) - (3), (7), (12) and (13), we get a normalized expression for the objective function \( (\bar{F}_P = \left(aF_P/UHTC_T\right)) \) associated to the power output given by,
\[
\bar{F}_P = \frac{\tau \left(1 - \theta \right) + \beta \left(1 - \theta^4\right)}{\psi A_R \left(T_X/T_H - 1\right)} + 1
\tag{16}
\]
\text{Figure 2.} Variation of the thermo-economic objective functions respect to \( \theta = T_X/T_H \), for several values of the parameter \( R \), under a) power output conditions and under b) ecological function conditions.
FIGURE 3. Variation of the dimensionless thermo-economic objective function $\overline{F_P}$ with respect to thermal efficiency for several a) $R$, b) $\beta$, c) $A_R$ and d) $f$ values, respectively. ($\xi = 0.02$).

FIGURE 4. Variation of the dimensionless thermo-economic objective function $\overline{F_E}$ with respect to thermal efficiency for several a) $R$, b) $\beta$, c) $A_R$ and d) $f$ values, respectively. ($\xi = 0.02$).
where \( \tau = (T_H/T_L) \) and the parameter \( f \), is the relative investment cost of the hot size heat exchanger and is defined as [5],

\[
\begin{align*}
f = \frac{a}{a + b}.
\end{align*}
\]

In Fig. 2a, we depict the objective function given by Eq. (16) versus \( \theta \), for several values of the parameter \( R \).

Analogously to Eq. (16), by using Eqs. (1) - (3) and (13) - (15), we can obtain a normalized objective function \( \overline{F}_E = (aF_E/U_HC_T_L) \) associated to the ecological function, this objective function is given by,

\[
\begin{align*}
\overline{F}_E & = \Lambda \left( (1 - \theta) + \beta \left( 1 - \theta^4 \right) \right) - \xi \tau \left( 1 - \frac{1}{\tau} \right)^2 \frac{A_R}{\left( 1 - \frac{1}{\tau} \right)^2 + 1},
\end{align*}
\]

where

\[
\begin{align*}
\Lambda & = \tau + 1 - \frac{1}{\theta} + \frac{1}{R} \left( \frac{T_L}{T_Y} - 2 \right).
\end{align*}
\]

On the other hand, by using Eqs. (1), (2), (3) and (11) the thermal efficiency, \( \eta_{th} \), of the irreversible heat engine can be expressed by,

\[
\begin{align*}
\eta_{th} = \frac{1}{1 + \frac{\psi A_R \theta - \beta (1 - \theta^4)}{\xi (1 - \frac{1}{\theta^4}) + \beta (1 - \theta^4)}}.
\end{align*}
\]

In Fig. (2b), we depict the objective function given by Eq. (18) versus \( \theta \), for several values of the parameter \( R \). The dimensionless thermo-economic objective functions (Eqs. (16) and (18)), can be plotted with respect to the thermal efficiency (Eq. (19)) for given values of \( f \), \( \beta \), \( \psi \) and \( A_R \) as shown in Figs. 3(a)-3(d) and Figs. 4(a)-4(d) for the cases of the maximum power output and maximum ecological function conditions respectively. In all cases we use \( \tau = 4 \), as in Ref. [5], where \( T_L \approx 300K \) and therefore \( T_H \approx 1200 \text{ K} \), this value of \( \tau \) is for comparison with [5], however a more realistic value of \( T_H \) could be of the order of 431K [9], which is the effective sky temperature stemming from the dilution of solar energy. As it could be seen from Figs. (2a) and (2b), there is a value of \( \theta \) that maximizes the objective functions for given \( f \), \( \psi \) and \( \tau \) values. Since the two objective functions and thermal efficiency depend on the working fluid temperatures \( (T_X, T_Y) \), the objective functions given by Eqs. (16) and (18) can be maximized with respect to \( T_X \) or \( T_Y \), that is, we calculate \( dF / d\theta \theta = \theta^* = 0 \), for Eqs. (16) and (18), the \( \theta^* \) values obtained give us the maximum values for \( F_P \) and \( F_E \) functions, respectively. This optimization procedure has been numerically carried out in the next section.

3. Numerical results and discussion

We can observe from Figs. (2a) and (2b), that the maximum thermo-economic objective functions \( F_P \) and \( F_E \) diminish while the corresponding optimum hot working fluid temper-
Figures 7 and 8. Variation of the maximum thermoeconomic objective functions \( \bar{F} \) respect to \( A_R \), for several values of the parameter \( \tau \) under a) maximum power conditions and b) maximum ecological function conditions.

Figures 7 and 8. Variation of the maximum thermoeconomic objective functions \( \bar{F} \) respect to \( A_R \), for several values of the parameter \( R \) under a) maximum power conditions and b) maximum ecological function conditions.

The optimal temperature \( T_X \) and \( T_Y \) shift towards \( T_H \) when the internal irreversibility parameter \( R \) decreases. On the other hand, the thermoeconomic objective function at maximum ecological regime (MER) is lesser than the thermoeconomic objective function at maximum power regime (MPR). In Figs. 3 and 4, for both MPR and MER cases, the variation of the dimensionless thermoeconomic objective functions with respect to thermal efficiency for several values of \( R \), \( \beta \), \( \psi \), and \( f \) are presented. From Figs. 3(a)-(d) and 4(a)-(d), we see that the loop curves become smaller as \( f \), \( \beta \) and \( \psi \) decrease. We can also see that the maximum thermal efficiency is independent of \( f \) values, while the maximum \( \bar{F}_P \) (or \( \bar{F}_E \)), decreases for decreasing \( f \) values (see Figs. 3a) and 4a)). In Figs. 5 and 6, we show the variations of optimal temperatures \( T_X \) and \( T_Y \) with respect to \( \beta \) for different values of \( \psi \) and for both maximum power and maximum ecological function conditions, respectively. We observe in Figs. (5) and (6), that for smaller values of \( \psi \) and higher values of \( \beta \), the ratio of the optimum temperatures \( T_X \) and \( T_Y \) to the source temperature \( T_H \) and \( T_L \) is generally higher. We can also see when the radiation \( \beta \) increases, \( T_X^* / T_H \) increases and approaches to 1. This means that the optimal temperature gets closer to the source temperature \( T_H \) as radiation increases. We can also observe in Figs. 5 and 6 that, the effect of \( \beta \) on \( T_X^* \) and \( T_Y^* \) is more important in the interval \( 0 < \beta < 2 \), under both MPR and MER conditions. In Figs. 7 and 8, we show the variation of the maximum dimensionless thermo-economic objective functions (\( \bar{F}_{max} \), for both MPR and MER cases) with respect to the ratio \( A_R = A_L / A_H \) for different values of the parameter \( R \) (see Figs. (7a) and (8a)) and for several values of the temperature ratio \( \tau = T_H / T_L \) (see Figs. (7b) and (8b)). We can observe in Figs. 7 and 8, for both MPR and MER, as \( A_R \) increases, \( \bar{F}_{max} \) increases to its peak value, an then decreases smoothly. We can also observe that \( \bar{F}_{max} \) increases and the optimal \( A_R \) value decreases considerably for increasing \( R \) and \( \tau \) values.

Finally, by using Eq. (19), we can calculated the optimal thermal efficiencies. In Fig. 9, we show the optimal thermal efficiencies for both MPR and MER cases. In this figure, we can observe that the optimal thermal efficiencies under MER (for all values of the parameter \( R \), see Fig. 9a) and for several values of the parameter \( \tau \), see Fig. 9b) are bigger than the optimal thermal efficiencies at MPR. Besides these optimal efficiencies satisfy the following inequality:

\[
\eta_{CA} < \eta_{opt}^{MPR} < \eta_{opt}^{MER} < \eta_{CA},
\]

where the subscripts \( C \) and \( CA \) refer to Carnot and Curzon-Ahlborn respectively. The previous inequality was obtained.

Figure 9. Optimal thermal efficiencies vs. $A_R$ for the two regimes. a) For three values of $R$ with $\tau = 4$ and b) For three values of $\tau$ with $R = 0.8$. In both cases, $\beta = 1$, $f = 0.7$ and $\xi = 0.02$

by Barranco-Jiménez et al. for the case of an endoreversible heat engine using different heat transfer laws in the thermal coupling [2] and was recently obtained, for a thermoeconomic optimization model of a solar driven-heat engine using a Dulong-Petit heat transfer law in the thermal coupling [21].

4. Conclusions

In this work, we studied an irreversible solar-driven heat model where we consider simultaneously conduction and radiation modes at the superior thermal coupling as in Ref. #. Following the Ust procedure, a thermoeconomic performance analysis using finite time/finite size thermodynamics has been carried out in terms of the maximization of two objective functions. The objective functions have been defined as the quotient between power output and the ecological function per unit total investment cost, respectively. By means of the maximization of these objective functions, the optimum thermoeconomic performance and the corresponding best design parameters of the solar-driven heat engine were determined. In this context, the effects of the economic parameter, $f$, and the ratio of heat transfer areas, $A_R$, on the optimal thermoeconomic performance have been investigated. Besides, we show how the efficiency under maximum ecological function is greater than the efficiency under maximum power conditions. This result has systematically been observe in all kinds of thermo economical heat engine models studied in the literature.

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