Method for solving the diffusion equation for a model of seismic dilatant zone

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In this paper we find the solution of the pressure diffusion equation \[PDE\] for a three-dimensional model of a seismic dilatant zone. We made a quantitative analysis of to the PDE solutions in order to see their applicability to explain the electric self-potential anomalies observed prior to earthquakes in different seismic regions around the world. The results here reported lead in a first approximation to reasonable comparisons with field measurements of electrotelluric activity in active seismic regions.

Keywords: Seismic precursors; diffusion equation; dilatant zone.

En este artículo encontramos la solución de la ecuación de difusión de la presión \[PDE\] para un modelo tridimensional de una zona sísmica con dilatancia. Presentamos un análisis cuantitativo de las soluciones de la PDE para ver su aplicabilidad en la explicación de anomalías de auto-potencial eléctrico observadas antes de la ocurrencia de sismos en diferentes regiones sísmicas del mundo. Los resultados aquí reportados conducen, en una primera aproximación, a comparaciones razonables con mediciones de campo de actividad electrotelúrica en regiones sísmicamente activas.

Descriptores: Precursos sísmicos; ecuación de difusión; zona de dilatancia.

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1. Introduction

Since several decades ago, different research groups around the world have registered fluctuations of the electric potential differences between electrodes buried into the ground in active seismic regions searching for anomalous patterns possibly associated with impending earthquakes [1-4]. The so-called electric self-potential difference $\Delta V$ is usually measured by means of shallow pairs of unpolarized electrodes located in $N-S$ and $E-W$ directions and with separations between 50 and 2500 m [5]. These electrodes and the complementary equipment form an electrotelluric station [4]. The location of these stations can be from very short distances (few meters) until several kilometers from the focal areas of the earthquakes [1-5]. Many authors, since the 1960’s until the present times, have reported self-potential patterns possibly linked with the preparation mechanism of earthquakes [6-10]. However, the tectonic origin of the precursory self-potential anomalies is not yet clearly established.

As Corwin and Morrison [2] assert, there exist many possible causes, some of which may not be related with tectonic activity, such as changes of the temperature and moisture content, also chemical composition of the soil at one electrode relative to another will produce changes in the potential, instrumental problems such as grounding of a connecting cable can be mentioned among other causes [2]. In the case of the cause of geoelectric anomalies be of tectonic origin, some proposed mechanisms are: the piezoelectric effect [11], the electrokinetic effect [12], point defects [13], the emission of electrons [14], and the motion of charged dislocations [15].

Many of the geoelectric anomalies reported possibly linked with impending earthquakes [2] show a similar pattern of rapid onset and slow decay, suggestive of a step-input diffusion process. As reported by Corwin and Morrison [2], one such process which might produce self-potential variations is based on the dilatancy hypothesis [16]. This model is based on laboratory studies which show that rock undergoes an inelastic volumetric increase (dilatancy) prior to failure. Dilatancy is produced by the formation and propagation of cracks within the rocks and begins to occur at stresses as low as half the breaking strength [17]. When a region of the earth becomes dilatant, the pore pressure in the region is reduced and groundwater diffuses into the dilatant zone [2]. As suggested by Mizutani et al. [12], this groundwater flow may generate an electric field at the surface of the earth by electrokinetic coupling, a process by which the flow of a fluid through a porous medium creates an electric potential gradient (the so-called streaming potential) across the medium. The so-called electrokinetic effect pertains to the well-known crossed phenomena of linear irreversible thermodynamics [12]. In this case a pressure or concentration gradient produces a mass flux, but if the molecules forming the fluid are electrically charged, then an electric potential gradient is also generated producing an electric current. Based in these kind of ideas, Corwin and Morrison [2] proposed the following simplified dilatant zone model, for a first estimate of streaming potentials generated by fluid flow into a dilatant zone. In this model, the pressure in the dilatant region $-a < x < a$ is zero and the pressure in the external region $|x| > a$ is $p_0$ at time $t = 0$.

Following Nur [17], the diffusive one dimensional equation to solve is,

\begin{equation}
\frac{\partial^2 P}{\partial x^2} = \frac{1}{c} \frac{\partial P}{\partial t},
\end{equation}

where $P$ is the pressure, $x$ is the distance, $t$ is time and $c$ is hydraulic diffusivity. Taking as initial conditions,
we obtain
and special array of initial conditions, such types of solution
set of initial conditions. By using the superposition principle
the diffusion equation in 3 dimensions by using a particular
M
≥
anomalies possibly linked to the preparation mechanism of
eralization of Eq. (3) with its appropriate initial conditions
November 28, 1974.
Corwin and Morrison found that Eq. (3) fits very well a self-
earthquakes. (2)
In the present paper we show a three-dimensional gen-
the paper is organized as follows: in Sec. 2, we solve
the diffusion equation in 3 dimensions by using a particular
set of initial conditions. By using the superposition principle
and special array of initial conditions, such types of solution
are used in order to obtain a physical result in Sec. 3. Section
4 is advocated to briefly describe the electrokinetic ef-
fect. The connection between the diffusion equation and the
electrotelluric signals are analyzed in Sec. 5. Finally, some
concluding remarks are done in Sec. 6.
2. Diffusion equation with the auxiliary conditions
Clearly Eq. (1) is solved only in one dimension. Its gener-
alization to 3 dimensions is immediate and is given by the
following equation:
∇^2 P = \frac{1}{c} \frac{\partial P}{\partial t}. \tag{4}
This equation will be firstly solved by using a cube C of ra-
radius 2a whose center coincides with the origin such that at
time t = 0 the pressure is
P(\vec{x}, 0) = \begin{cases} \frac{1-a}{n} P_0 & \text{if } \vec{x} \in C \\ \frac{1}{n} P_0 & \text{if } \vec{x} \notin C \end{cases}, \tag{5}
being n a natural number. Even if we immediately notice that
the pressure is negative inside the cube C for n > 1, this set
of initial conditions will allow us to warranty that when a set
of cubes would be considered, the pressure inside of them
will vanish. By using Fourier transformation [18] in Eq. (4),
we obtain
-k^2 \tilde{P} = \frac{1}{c} \frac{\partial \tilde{P}}{\partial t}, \tag{6}
where the tilde, \tilde{f}, denotes the Fourier transformation of
the function f. The solution of this first order differential equation is
\[ \tilde{P}(\vec{k}, t) = \tilde{P}(\vec{k}, 0) \exp \left[ -ck^2 t \right]. \tag{7} \]
Applying the inverse Fourier transformation, we arrive to
\[ \tilde{P}(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int_{R^3} \tilde{P}(\vec{k}, t) \exp \left[ -ck^2 t \right] \exp \left[ -i \vec{x} \cdot \vec{k} \right] \, d^3k. \tag{8} \]
By using the convolution theorem,
\[ \int_{R^3} f(\vec{x}') g(\vec{x} - \vec{x}') \, d^3x' = \int_{R^3} \tilde{f}(\vec{k}) \tilde{g}(\vec{k}) \exp \left[ -i \vec{x} \cdot \vec{k} \right] \, d^3k, \tag{9} \]
by putting
f(\vec{k}) = \tilde{P}(\vec{k}, 0) \quad \text{and} \quad \tilde{g}(\vec{k}) = \exp \left[ -ck^2 t \right], \tag{10}
and
f(\vec{x}) = P(\vec{x}, 0) \quad \text{and} \quad g(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \exp \left[ -|x|^2 / 4ct \right], \tag{11}
we obtain
\[ \int_{R^3} P(\vec{k}, 0) \exp \left[ -ck^2 t \right] \exp \left[ -i \vec{x} \cdot \vec{k} \right] \, d^3k = \frac{1}{(2\pi)^{3/2}} \int_{R^3} P(\vec{x}', 0) \left[ \exp \left[ -|\vec{x}' - \vec{x}|^2 / 4ct \right] \exp \left[ -i \vec{x} \cdot \vec{k} \right] \right] \, d^3x'. \tag{12} \]
Substituting Eq. (12) in Eq. (8), we arrive to

\[ P(\vec{x}, t) = \frac{1}{(4\pi ct)^{3/2}} \int_{\mathbb{R}^3} P(\vec{x}, 0) \left[ \exp - \left( \left| \vec{x} - \vec{x}^* \right|^2 / 4ct \right) \right] d^3 x'. \]  

(13)

Then, we need to use the initial conditions described in Eq. (5). We have

\[ P(\vec{x}, t) = \frac{P_0}{(4\pi ct)^{3/2}} \left[ \frac{1}{n} \int_{\mathbb{R}^3} \left[ \exp - \left( \left| \vec{x} - \vec{x}^* \right|^2 / 4ct \right) \right] d^3 x' \right] . \]

(14)

It is easy to see that

\[ P(\vec{x}, t) = \frac{P_0}{(4\pi ct)^{3/2}} \left[ \frac{1}{n} \int_{\mathbb{R}^3} \left[ \exp - \left( \left| \vec{x} - \vec{x}^* \right|^2 / 4ct \right) \right] d^3 x' \right] - \frac{1}{n} \int_{\mathbb{C}} \left[ \exp - \left( \left| \vec{x} - \vec{x}^* \right|^2 / 4ct \right) \right] d^3 x' . \]

(15)

We arrive to

\[ P(\vec{x}, t) = \frac{P_0}{(4\pi ct)^{3/2}} \left[ \frac{1}{n} \int_{\mathbb{R}^3} \left[ \exp - \left( \left| \vec{x} - \vec{x}^* \right|^2 / 4ct \right) \right] d^3 x' \right] - \frac{1}{c} \int_{\mathbb{C}} \left[ \exp - \left( \left| \vec{x} - \vec{x}^* \right|^2 / 4ct \right) \right] d^3 x' . \]

(16)

On the other hand, in order to solve Eq. (16), we need to know the following identities:

\[ \int_{-\infty}^{\infty} \left[ \exp - \left( \left| \vec{x} - \vec{x}' \right|^2 / 4ct \right) \right] d^3 x' = \int_{-\infty}^{\infty} \left[ \exp - \left( \left| x - x' \right|^2 / 4ct \right) \right] dx \times \int_{-\infty}^{\infty} \left[ \exp - \left( \left| y - y' \right|^2 / 4ct \right) \right] dy \int_{-\infty}^{\infty} \left[ \exp - \left( \left| z - z' \right|^2 / 4ct \right) \right] dz; \]

(17)

and

\[ \int_{\mathbb{C}} \left[ \exp - \left( \left| \vec{x} - \vec{x}' \right|^2 / 4ct \right) \right] d^3 x' = \int_{-a}^{a} \left[ \exp - \left( \left| x - x' \right|^2 / 4ct \right) \right] dx \times \int_{-a}^{a} \left[ \exp - \left( \left| y - y' \right|^2 / 4ct \right) \right] dy \int_{-a}^{a} \left[ \exp - \left( \left| z - z' \right|^2 / 4ct \right) \right] dz; \]

(18)

and making the change of variable, \( \left| x - x' \right| / 2\sqrt{ct} = u \), we have,

\[ \int_{-\infty}^{\infty} \left[ \exp - \left( \left| x - x' \right|^2 / 4ct \right) \right] dx' = \sqrt{4ct} \int_{-\infty}^{\infty} \exp - u^2 du = \sqrt{4\pi ct}; \]

(19)

\[ \int_{-a}^{a} \left[ \exp - \left( \left| x - x' \right|^2 / 4ct \right) \right] dx' = \int_{-a}^{0} \left[ \exp - \left( \left| x - x' \right|^2 / 4ct \right) \right] dx' + \int_{0}^{a} \left[ \exp - \left( \left| x - x' \right|^2 / 4ct \right) \right] dx' \]

\[ = \sqrt{4ct} \left[ \int_{-a}^{0} \exp \left[ -u^2 \right] du + \int_{0}^{a} \exp \left[ -u^2 \right] du \right] = \frac{\sqrt{4\pi ct}}{2} \left[ \text{erf} \left( \frac{a + x}{2\sqrt{ct}} \right) + \text{erf} \left( \frac{a - x}{2\sqrt{ct}} \right) \right]. \]
for each cube; that is, them. In such situation, the initial conditions for the pressure are chosen by adding the initial conditions described in Eq. (5) and Eq. (16) turns on

\[ \sum_{i=1}^{n} \left[ \exp \left( \frac{1}{4ct} \right) \right] d^3 x' = \left[ 4\pi ct \right]^{3/2}, \tag{21} \]

and Eq. (18) turns on

\[ \int_{C} \left[ \exp \left( \frac{1}{4ct} \right) \right] d^3 x' = \left[ 4\pi ct \right]^{3/2} \left\{ \text{erf} \left[ \frac{a+x}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-x}{2\sqrt{ct}} \right] \right\}, \tag{22} \]

Substituting this last equations, Eqs. (21) and (22), into Eq. (14), we obtain

\[ \frac{P(\vec{x},t)}{P_0} = \frac{1}{n} - \frac{1}{8} \left\{ \text{erf} \left[ \frac{a+x-x_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-x+x_i}{2\sqrt{ct}} \right] \right\} \times \left\{ \text{erf} \left[ \frac{a+y+y_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-y+y_i}{2\sqrt{ct}} \right] \right\} \times \left\{ \text{erf} \left[ \frac{a+z+z_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-z+z_i}{2\sqrt{ct}} \right] \right\}. \tag{23} \]

It has been noticed that if \( n = 1 \), for \( \vec{x} \in C \), \( P(\vec{x},0) = 0 \) and for \( \vec{x} \notin C \), \( P(\vec{x},0) = P_0 \). Therefore this case, \( n = 1 \), could be used to describe a physical situation where the pressure vanishes inside the dilatant zone and it is constant outside at \( t = 0 \). For \( n \neq 1 \), we will use \( n \) cubes in order to describe initial conditions where inside each cube the pressure will also vanish and outside it will be constant.

3. \( n \) cubes and the superposition method

The idea now consists of using the solution of the last section in order to describe a set of \( n \) cubes with initial conditions such as we mentioned before; that is to say that inside the cubes the pressure vanishes and outside is constant. This is permitted because the diffusion equation is a linear one.

In the last section, the cube is centered at the origin. But, if we want to describe a cube not centered in the origin we just need to give their coordinates; that is, the center of the cube is \( \vec{c}_i = \{x_i, y_i, z_i\} \). Indeed, in order to calculate the pressure, for a cube not centered at the origin, the position vector \( \vec{x} \) must be substituted by \( \vec{x} - \vec{c}_i \) in Eq. (23). Therefore,

\[ \frac{P(\vec{x},t)}{P_0} = \frac{1}{n} \left( \text{erf} \left[ \frac{a+x-x_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-x+x_i}{2\sqrt{ct}} \right] \right) \times \left( \text{erf} \left[ \frac{a+y+y_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-y+y_i}{2\sqrt{ct}} \right] \right) \times \left( \text{erf} \left[ \frac{a+z+z_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-z+z_i}{2\sqrt{ct}} \right] \right). \tag{24} \]

Let us consider \( n \) cubes of side \( 2a \) centered each one in \( \vec{c}_i \), with \( i = 1, 2, \ldots, n \), such that there is no intersection among any of them. In such situation, the initial conditions for the pressure are chosen by adding the initial conditions described in Eq. (5) for each cube; that is,

\[ P(\vec{x},0) = \left\{ \begin{array}{ll} 0 & \text{if } \vec{x} \in C_1 \cup C_2 \cup \ldots \cup C_n \\ P_0 & \text{if } \vec{x} \notin C_1 \cup C_2 \cup \ldots \cup C_n \end{array} \right\}, \tag{25} \]

where \( C_1 \cup C_2 \cup \ldots \cup C_n \) represents the union of all the cubes \( C_i \). The sum of such cubes could always represent, as a good approximation, an earthquake focal zone whose pressure initially vanishes. Therefore, adding the contribution of each cube for calculating the pressure, we arrive to

\[ \frac{P(\vec{x},t)}{P_0} = \sum_{i=1}^{n} \frac{P_i(\vec{x},t)}{P_0} = \frac{1}{n} \sum_{i=1}^{n} \left( \text{erf} \left[ \frac{a+x-x_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-x+x_i}{2\sqrt{ct}} \right] \right) \times \left( \text{erf} \left[ \frac{a+y-y_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-y+y_i}{2\sqrt{ct}} \right] \right) \times \left( \text{erf} \left[ \frac{a+z-z_i}{2\sqrt{ct}} \right] + \text{erf} \left[ \frac{a-z+z_i}{2\sqrt{ct}} \right] \right). \tag{26} \]

4. Electrokinetic effect

By using \( n \) cubes, Eq. (26) represents a good approximation for describing the pressure of the dilatant zone, which will be the location of the future earthquake focus as function of the position (outside the dilatant zone) and the time. Nevertheless, the question now is how to link it with a physical measurable observable. The answer is given by noticing that the pressure and the electric potential can be related due to the electrokinetic effect [12]. Following Prigogine [19], for a system consisting of two
vessels I and II, which communicate by means of a porous wall or a capillary, where the temperature and the concentrations are supposed to be uniform throughout the entire system and both phases differ only with respect to pressure and electrical potentials, the entropy production due to the transfer of the constituents from vessel I to vessel II, is given by

\[
\frac{dS}{dt} = J \frac{\Delta P}{T} + I \frac{\Delta \phi}{T},
\]

where \(I\) is the electrical current due to a transfer of charges from I to II and \(J\) the resultant flow of matter. The phenomenological equations are given by

\[
I = L_{11} \frac{\Delta \phi}{T} + L_{12} \frac{\Delta P}{P},
\]

\[
J = L_{21} \frac{\Delta \phi}{T} + L_{22} \frac{\Delta P}{P},
\]

and with Onsager relation,

\[
L_{12} = L_{21}.
\]

That is, we have two irreversible effects, the flows \(I\) and \(J\) with a cross effect given by the coefficients \(L_{12} = L_{21}\), which is due to the interference of the two irreversible processes.

One of the electrokinetic effects is characterized by the streaming potential defined as the potential difference per unit pressure difference in the state with zero electrical current, that is,

\[
\left( \frac{\Delta \phi}{\Delta P} \right)_{t=0} = - \frac{L_{12}}{L_{21}} = K. \tag{30}
\]

For the dilatancy model, the difference of pressure between two points \(\overrightarrow{x}_a\) and \(\overrightarrow{x}_b\),

\[
\frac{\Delta P(\overrightarrow{x}, t)}{P_0} = \frac{\Delta P(\overrightarrow{x}_a, t)}{P_0} - \frac{\Delta P(\overrightarrow{x}_b, t)}{P_0},
\]

is then proportional to the difference of electric potential \(\Delta \phi\) being \(\overrightarrow{x}\) any position between \(\overrightarrow{x}_a\) and \(\overrightarrow{x}_b\) since \(\Delta \overrightarrow{x} = \overrightarrow{x}_a - \overrightarrow{x}_b \ll \overrightarrow{x}\); that is:

\[
\Delta \phi = K \frac{\Delta P(\overrightarrow{x}, t)}{P_0}, \tag{32}
\]

where \(K\) given by Eq. (30) \([19]\) represents the electrokinetic coefficient related to streaming potential,

\[
\frac{\Delta P(\overrightarrow{x}, t)}{P_0}
\]

is given by Eq. (26) and \(P_0\) is the pressure within the dilatancy zone. Eq. (32) can be written as

\[
\frac{\Delta P(\overrightarrow{x}, t)}{P_0} = K^{-1} \frac{\Delta \phi}{P_0}. \tag{33}
\]

5. Plausibility of the diffusion equation solution to explain the electrotelluric signals

According to Bernard \([20]\) most of self-potential electric anomalies reported in Russia, Greece, China, Japan and USA last between a few minutes and a few hours, but some last up to several days. They are usually up to 100 mV, significantly above the electrode or background noise (usually about 1 to 10 mV). Bernard \([20]\) also asserts that the most quantitative model for explaining the anomalous electrical currents uses the so-called streaming potential (or electrokinetic effect, see Sec. 4) produced by fluid percolation in the crust, driven by pore pressure gradients related to precursory deformation. Laboratory experiments have investigated the streaming potential properties of common minerals (see \([20]\) and references therein) showing that the values of self-potential anomalies could indeed be produced by streaming potential effects near the electrodes, considering standard streaming potential coefficients and pressure gradients of several MPa, which could be expected in the vicinity of an active fault \([20]\). Regarding pore pressure differences, Bernard \([20]\) asserts that \(P = 1\) MPa could be available in the vicinity of an active fault. In fact, at 6 km in depth, \(P\) may reach 100 MPa, and at 15 km, \(P\) may reach 250 MPa \([20]\). On the other hand, Mizutani \([12]\) estimated that the electrokinetic coefficient \(K\) is of the order of,

\[
K = 10^2 \sim 10^3 \text{ mV} / \text{kb} \quad \frac{1b}{10^9 \text{ Pa} \text{1 MPa} } = 10^3 \sim 10^4 \text{ mV MPa}. \tag{34}
\]

By using Eqs. (33) and (34), we have for the mentioned limits of pore pressure

\[
\frac{100 \text{ mV}}{250 \text{ MPa} (10^4 \text{ mV MPa}^{-1})} < \frac{\Delta P}{P_0} < \frac{100 \text{ mV}}{1 \text{ MPa} (10^3 \text{ mV MPa}^{-1})}; \tag{35}
\]

that is:

\[
4 \times 10^{-5} \lesssim \frac{\Delta P}{P_0} < 10^{-1}. \tag{36}
\]

Figure 2 describes \(\Delta P/P_0\) versus \(t\) (see Eq. (26)) for a cubic dilatant zone of size 100 km. An approximate relation between the surface of the rupture zone \(A\) and the magnitude \(M\) is \([21]\)

\[
\log A \simeq M - 4, \tag{37}
\]

with \(A\) in km² and \(\log\) is the base 10 logarithm. For example, for a rupture zone of \(100 \times 10 \text{ km} = 1 \text{ 000 km}^2\), \(M = 7\). If the cubic dilatant zone is centered at the origin and the electrodes are at the positions 55, 65, 75, 85 and 95 km respectively (that is, at 5, 15, 25, 35 and 45 km from the border of the dilatant zone, respectively, (see Fig. 2), and the separation between the electrodes is of 100 m \([3]\), then it can be noticed that while the electrodes are farther the quantity \(\Delta P/P_0\) is
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Figure 2. Behavior of $\Delta P/P_0$ vs $t$ (days) for different distance in the dilatant zone.

Figure 3. Behavior of $\Delta P/P_0$ vs $t$ (days) for different sizes of the dilatant zones.

smaller. Figure 3 describes $\Delta P/P_0$ versus $t$ for cubic dilatant zones of different sizes: 20, 40, 60, 80 and 100 km with the electrodes at 20 km of the border of the dilatant zone. In this case, it can be observed that the greater the volume of the dilatancy, the greater the measured signal. Finally, Figs. 4 and 5 show $\Delta P/P_0$ for a dilatant zone with a size of 40 km in the $x$ axis and 2 km in the $y$ axis (corresponding approximately to an earthquake of $M = 6$) with one electrode in the position $x_a = (10, 10, 0)$ km and the other electrode at $x_b = (10.1, 10, 10)$ km. Equations (26) and (31) are first obtained for cubes of size 0.4 km (Fig. 4) and then for cubes of size 1 km (Fig. 5). It has to be noticed that there is no practically difference between both figures. That is, the result is independent of the size of the unitary cubes. In all the mentioned cases the signal $\Delta P/P_0$ is converted in an electric self-potential signal by using Eq. (33); that is, $\Delta \phi$ between the points $x_a$ and $x_b$ at time $t$ is found by multiplying $\Delta P/P_0$ by the assumed value of $P_0$ and by the streaming potential coefficient. For example, if $P_0 \approx 10^6$ Pa [20] and $K \approx 10^3$ V/kb [20], and taking $\Delta P/P_0 \approx 10^{-3}$ (see Eq. 36), we have $\Delta \phi = 10^{-3}P_0K \approx 10^{-2}$ V = 10 mV, or $\Delta \phi \approx 10^{-1} \approx 100$ mV for $\Delta P/P_0 \approx 10^{-4}$, $P_0 = 10^8$ Pa and $K = 10$ V/kb. That is, by means of reported values for $P_0$ and $K$ and the $\Delta P/P_0$ values obtained in our calculations, we obtain $\Delta \phi \approx 10 - 100$ mV, which are typical values of self-potential reported data [20].

6. Concluding Remarks

Although there is no consensus in the geophysical literature about the model which explains the so-called electrotelluric anomalies, it is widely accepted that the phenomenon is related with the diffusion of the electric charges in the fractured region of the future focal zone. A model suitable for the description of this phenomenon is the so-called dilatancy.
model. This model was used by Corwin and Morrison [2] to explain some precursory electric anomalies prior to earthquakes in California, USA. Such a model was developed in one dimension. In this work, we have extended this model to a three-dimensional version by incorporating a structure composed by unitary cubes of different sizes. The curves depicted in Figs. 2, 3 and 4 reasonably reproduce electric anomalies reported by means of field measurements. Our model predicts that the greater the dilatant zone, the greater the maximum of the $\Delta P/P_0$ vs $t$ curves. This is quantitatively consistent with other models [2,12]. We have applied the model to a hypothetical dilatant zone corresponding to an earthquake of $M = 7$, with electrode positions at 95 km of the epicenter. Our calculations for this case are in reasonable agreement with self-potential anomalies reported by Bernard [20]. Finally, we want to emphasize that nevertheless the relative simplicity of our extended model, it permits the obtention of very reasonable results in good agreement with self-potential anomalies reported by means of field measurements.

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