Energetic performance of a series arrangement of irreversible power cycles

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In this paper we study the energetic performance of a series arrangement of totally irreversible heat engines (combined cycle (CC) type). The origin of the irreversibilities in these heat engines is external and internal. The external irreversibilities are associated with the heat flows towards and from the temperature reservoirs, while the internal irreversibilities are associated to all those dissipative processes that occur within the working fluids. On the other hand, our model includes information provided by different optimization criteria that are commonly used in studies of irreversible heat engine models. This model, allow us to analyze two commercial CC–power plants which use fossil fuel and compare them with two commercial nuclear power plants, from the energetic point of view. Under this first approach to the comparison, we found that, despite of the high efficiency reported by CC manufacturers, the large temperature gradients required by these plants result in irreversibility factors bigger than those for nuclear plants currently in operation. This so–called irreversibility factor is defined as the quotient between the entropy produced by internal processes and the thermal conductances involved in the heat fluxes. By means of this energetic approach we find that CC plants have some economical disadvantages with respect to the nuclear power plants.

Keywords: Non-equilibrium and irreversible thermodynamics; performance characteristics of energy conversion systems; figure of merit.

En este artículo estudiamos el desempeño energético de un arreglo en serie que emula un ciclo combinado (CC). Consideramos un modelo en donde las irreversibilidades provienen tanto de fuentes externas, como internas. Las fuentes externas son las resistencias térmicas que se oponen a los flujos de calor que se establecen entre la sustancia de trabajo y los almacenes de temperatura, mientras que las irreversibilidades internas están asociadas a todos aquellos procesos dissipativos que ocurren dentro de las sustancias de trabajo. Por otro lado, en nuestro modelo se incluye información proporcionada por los diferentes criterios de optimización que se utilizan comúnmente para estudiar el desempeño energético de modelos de plantas de generación de electricidad. En este trabajo, aplicamos nuestros resultados a la comparación entre dos plantas comerciales CC que utilizan hidrocarburos y dos nucleoeletricicas actualmente en operación. El resultado es que debido a las grandes diferencias de temperatura, requeridas para el funcionamiento de las plantas CC, su desempeño es de menor calidad que el de las nucleoeletricicas de acuerdo al factor de irreversibilidad de cada tipo de planta, definido como el cociente de la producción interna de entropía debida a los procesos internos y las conductancias térmicas asociadas a los flujos de calor. Este resultado significa, que las plantas CC, a pesar de las altas eficiencias reportadas por sus fabricantes, están en desventaja económica con respecto a las nucleoeletricicas.

Descriptors: Termodinámica irreversible y de no-equilibrio; desempeño de convertidores de energía; optimización.

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1. Introduction

One of the most important topics in Thermodynamics is that related with the formulation of approaches to study the operation of real thermal engines. When two or more thermal engines are connected by means of flows of heat and energy, and all the group is working between two main energy reservoirs at fixed temperatures, we have an arrangement of thermal engines [1]. In 1994 de Mey and de Vos [2] studied, the behavior of parallel and series arrays of endoreversible thermal engines operating with a linear heat transfer law between the temperatures $T_1$ and $T_2$ ($T_2 < T_1$), under the maximum power working regime. Later another authors studied the same arrangements in other working regime [3]. In the present study we analyze the energetic behavior of the thermal engines in a series arrangement as that depicted in Fig. 1.

In literature have been widely studied the real combined cycles, with reversible Carnot cycles [4, 5], non-adiabatic cycles [6], using several objective functions as the power density and the original ecological function [7] for its optimization. These engines have been analyzed using economic objectives also. For this study we use the following optimization criteria: Maximum Efficiency ($\eta_M$) [8], Maximum Power Output ($\text{MP}$) [9] and Maximum Ecological Function ($M\eta$) [12]. The $\text{MP}$ criterion has been broadly used to study thermal engines made by men [10, 11]. However, the dissipation of thermal engines working under this $\text{MP}$-regime is very high (sometimes comparable or bigger than its power output) and the efficiency is not to high [12, 13]. This issue has motivated to think about other working regimes, among them the $\eta_M$-regime and the generalized $M\eta$-regime. This last regime consisting in the maximization of the generalized ecological function [14–16].
Figure 1. Series arrangement of irreversible power cycles (combined power cycle).

\[ E(\eta) = P(\eta) - \varepsilon(\eta_C) T_2 \sigma(\eta) = P(\eta) - \varepsilon(\eta_C) \Phi(\eta), \]

where \( \eta_C \) is the Carnot efficiency of the CC,

\[ \varepsilon(\eta_C) = \eta_{MP}(\eta_C) / [\eta_C - \eta_{MP}(\eta_C)] \]

and \( \Phi(\eta) = T_2 \sigma(\eta) \) is the Function of Dissipation given by Tribus [18]. Equation 1, represents a measure of the commitment between the power output of a thermal cycle and their dissipation, in such a way that a thermal cycle working in the generalized \( ME \)-regime represents a good trade-off among these two characteristic functions, which is simultaneously desirable from the energetic and ecological points of view. In fact, for a single thermal cycle, the power output under the generalized \( ME \)-regime reaches around 75% of the power output in the \( MP \)-regime, but the dissipation considerably decreases (approximately 25% of the dissipation in the \( MP \)-regime) [13–15]. Here we should mention that the values taken by the generalized ecological function, in the physically allowed range of values of the high reduced temperature, can be negative, it happens because the dissipation of the CC is larger than its power output. For these reasons, the ecological function presents a very important characteristic: the efficiency of a thermal cycle working in this operation mode is bigger than the \( MP \)-regime efficiency (for \( E > 0 \)).

On the other hand, during the last years, the electric generation plants vendors have promoted the installation of CC plants or the transformation of plants of single thermal cycle to CC plants. The main motivation of the users to acquire these plants is the high efficiency that the vendors report. In these cycles, it is common to have a high temperature approximately of 1500 K for the first working fluid. Then the working fluid goes through the first turbine where descends their temperature until 550 K. In this stage we obtain water vapor, which goes across a second turbine, for finally to condense at 300 K [1]. From the above description, we can model a combined cycle by means of a series arrangement of two thermal cycles. In this work we carry out the study of a model of this type which includes both external and internal irreversibilities; the external irreversibilities are associated to the flows of heat between reservoirs and the working fluids, without including a direct flow of heat between reservoirs, called short-circuit [17]. These heat transfers are modeled by means of a Newton’s cooling law. The internal irreversibilities, are not modeled and they are only considered by means of a lumped quantity that is defined. In this way we take into account, with the help of the information that provides the different operation modes, all and each one of the contributions to the entropy production of the irreversible processes that happen within the working fluids. This article is organized as follows: In Sec. 2 we will build the CC model, we will establish their constitutive equations also (first law, second law and heat transfer laws). We will introduce the characteristic functions or process variables and their linear combinations, these functions will serve like objective functions for the thermodynamic optimization of the arrangement. The most important hypothesis used in the construction of the model, is that the working substances works in irreversible cycles, and therefore the entropy change of each working substance is null. We find that, contrary to other arrangement models [3], the characteristic loop shaped curves of power output versus efficiency are obtained similar to those observed in real thermal engines [21] and without using the thermal “short circuit”. In these loop shaped curves, it is possible to locate the points corresponding to different optimization criteria: Maximum Efficiency, Maximum Power Output and Maximum Ecological Function (see Fig. 2).

In Sec. 3, we compare the energetic performance of this series arrangement, under the mentioned working regimes,
for three characteristic functions: the irreversible efficiency of conversion, the irreversible power output and the dissipation of the process. In Sec. 3.1, starting from the experimental reported data, we will compare the energetic performance of two commercial CC plants [19, 20], with some nuclear modern plants of single thermal cycle [16], and we will show that the CC plants have a bigger irreversibility parameter than the nuclear ones. It means that CC plants are in economic disadvantage with respect the nuclear plants. This fact is not clearly observed if we only compare their reported efficiencies. Finally, we will sketch some conclusions on this type of models and their utility to qualitatively describe the energetic behavior of the real CC plants.

2. Characteristic functions of the Arrangement

The total entropy production of the Irreversible Thermal Cycles Arrangement (ITCA) without “short circuit”, is shown in Fig. 1., and given by,

$$\sigma_T = -\frac{Q_1}{T_1} + \frac{Q_1}{T_{1w}} - \frac{Q_2}{T_{2w}} + \frac{Q_2}{T_{3w}} - \frac{Q_3}{T_{4w}} + \sigma_{i1} + \sigma_{i2} > 0,$$

where each term is given by,

$$\sigma_s = \left[ \frac{Q_3}{T_2} - \frac{Q_1}{T_1} \right]$$

and

$$\sigma_{st} = \left[ \frac{Q_1}{T_{1w}} - \frac{Q_2}{T_{2w}} + \sigma_{i1} \right] + \left[ \frac{Q_2}{T_{3w}} - \frac{Q_3}{T_{4w}} + \sigma_{i2} \right],$$

because we don’t have direct flux of heat between the reservoirs. The first term in 3, given by 4, corresponds to the entropy change of the energy reservoirs, while the second term given by 5, corresponds to entropy change of the working substances 1 and 2. The total entropy production (Eq. 2) includes the supposition that into thermal cycle, the flows of heat can stay finite and the working substance returns to its initial state, that is, the reservoirs entropy production is positive, while that of the working substances are null. Since the working fluids operates in cycles, their changes of internal energy are,

$$\Delta U_{st1} = \Delta U_{st2} = 0.$$

As the entropy is a state function also, then the entropy production of both working fluids satisfy,

$$\Delta S_{st1} = \frac{Q_1}{T_{1w}} - \frac{Q_2}{T_{2w}} + \sigma_{i1} = 0,$$

and

$$\Delta S_{st2} = \frac{Q_2}{T_{3w}} - \frac{Q_3}{T_{4w}} + \sigma_{i2} = 0.$$

On the other hand, we can write the irreversible power output of each engine as:

$$P_1 = Q_1 - Q_2 \quad \text{and} \quad P_2 = Q_2 - Q_3.$$

After the above equations the total power output is:

$$P = P_1 + P_2 = Q_1 - Q_3.$$

Finally, the thermal efficiency is given by

$$\eta = \frac{P}{Q_1} = 1 - \frac{Q_3}{Q_1}.$$
therefore, $a_h$, $a_i$, $a_c$ and $\eta_C \in [0, 1]$. Using these definitions and Eqs. 7 and 8 we can rewrite the constitutive equations. The efficiency is given by,

$$\eta = 1 - \frac{(1 - \eta_C) \left( \frac{22}{\gamma_1} + \frac{21 \gamma_2 \sigma_2}{\alpha} - \gamma_2 \right) \{ \alpha (1 - a_i) + \gamma_1 (1 - \sigma_1) \}}{\gamma_1 \gamma_2 \left( \alpha - \gamma_1 \sigma_1 - \alpha a_i \right) \left( 1 + \gamma_2 - \frac{21 \gamma_2 \sigma_2}{\alpha} - \frac{22}{\gamma_1} \right)}, \quad (16)$$

with $\gamma_1 = \alpha / g$ and $\gamma_2 = \beta / g$.

The total power output is,

$$P = \alpha T_1 \left( 1 - \frac{\alpha \gamma_1}{\alpha (1 - a_i) + \gamma_1 (1 - \sigma_1)} + \frac{1 - \eta_C}{\gamma_1 \gamma_2} \frac{\gamma_2 \left( 1 - \frac{1}{\alpha} - \frac{21 \gamma_2 \sigma_2}{\alpha} \right)}{1 + \gamma_2 \left( 1 - \frac{1}{\alpha} - \frac{21 \gamma_2 \sigma_2}{\alpha} \right)} \right) \quad (17)$$

The parametric graph of the power output versus efficiency, shows the loop shaped curve behavior that characterizes the irreversible conversion of energy [21] (see Fig. 2). In this graph all the points on the loop are physically accessible. We mark only four points which correspond to those working regimes that are of practical interest: the $MP$-regime, generalized $ME$-regime, the $M\eta$-regime, and the minimum dissipation regime ($\Phi$) (which is the origin of the graph where simultaneously the power and the efficiency are zero). Finally, the dissipation function can be written as,

$$\Phi = T_2 \sigma_T = \alpha T_1 (1 - \eta_C) \left[ 1 - \frac{\alpha \gamma_1}{\alpha + \alpha \gamma_1 - \gamma_1 \sigma_1 - \alpha a_i} + \frac{1}{\gamma_1 \gamma_2} \frac{\gamma_2 - \frac{21 \gamma_2 \sigma_2}{\alpha} - \frac{22}{\gamma_1}}{1 + \gamma_2 - \frac{21 \gamma_2 \sigma_2}{\alpha} - \frac{22}{\gamma_1}} \right] \quad (18)$$

Substituting 17 and 18 into 1, gives us the generalized ecological function, in terms of the reduced temperatures and the other physical parameters,

$$E = E [\alpha, T_1, \gamma_1, \gamma_2, a_i, a_h, a_c, \sigma_1, \sigma_2, \eta_C] \quad (19)$$

this expression is not shown in explicit form because is very extensive.

### 2.1. $M\eta$, $ME$ and $MP$ working regimes for the series arrangement

To establish each one of the optimal operation modes of the series arrangement, it is necessary to calculate the maxima of the power output, efficiency and ecological function, respect to the intermediate reduced temperature $a_i$.

#### 2.1.1. Reduced temperature and characteristic functions in the $M\eta$-regime

The intermediate reduced temperature $a_i$ of the $M\eta$-regime is given, in closed form, by the solution of the equation,

$$\left( \frac{\partial \eta}{\partial a_i} \right)_{a_i M\eta} = 0, \quad (20)$$

that is,

$$a_i M\eta = a_i M\eta(\alpha, T_1, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \eta_C), \quad (21)$$

again this expression isn’t shown in explicit form because is very extensive, but note that this solution doesn’t depends of the other two reduced temperatures. Substituting the above closed forms of the intermediate reduced temperature in Eqs. (16, 17, 18) we obtain the analitycal characteristic functions in the $M\eta$-regime:

$$\eta_{M\eta} = \eta [\alpha, T_1, \gamma_1, \gamma_2], \quad a_i M\eta(\alpha, T_1, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \eta_C), \quad (22)$$

Their behavior versus $\eta_C$ are showing in Fig. 3. These behaviors reduces to the endoreversible limits if $\sigma_1$, $\sigma_2$ tends to zero.

#### 2.1.2. Reduced temperature and characteristic functions in the $ME$-regime

Following the same procedure as in the $M\eta$-regime we will obtain a closed solution for $a_i ME$:

$$\left( \frac{\partial E}{\partial a_i} \right)_{a_i ME} = 0, \quad (25)$$

with the $ME$-reduced temperature

$$a_i ME = a_i ME(\alpha, T_1, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \eta_C), \quad (26)$$

Finally in the MP–regime we have
\[
\left( \frac{\partial P}{\partial a_i} \right)_{a_{i,MP}} = 0
\]
and the reduced temperature of this regime is
\[
a_{i,MP} = a_{i,MP}(\alpha, T_1, \gamma_1, \gamma_2),
\]
Using this closed solution in the characteristic functions of Sec. 2, we obtain,
\[
\eta_{MP} = \eta[\alpha, T_1, \gamma_1, \gamma_2,
\]
\[
a_{i,MP}(\alpha, T_1, \gamma_1, \gamma_2, \sigma_{i1}, \sigma_{i2}, \eta_C), \sigma_{i1}, \sigma_{i2}, \eta_C],
\]
\[
P_{MP} = P[\alpha, T_1, \gamma_1, \gamma_2,
\]
\[
a_{i,MP}(\alpha, T_1, \gamma_1, \gamma_2, \sigma_{i1}, \sigma_{i2}, \eta_C), \sigma_{i1}, \sigma_{i2}, \eta_C],
\]
\[
\Phi_{MP} = \Phi[\alpha, T_1, \gamma_1, \gamma_2,
\]
\[
a_{i,MP}(\alpha, T_1, \gamma_1, \gamma_2, \sigma_{i1}, \sigma_{i2}, \eta_C), \sigma_{i1}, \sigma_{i2}, \eta_C].
\]

When we evaluate the characteristic functions in each one of the three optimal and analytical reduced temperatures, we obtain the complete energetics of the CC in each working regime. In next section we will use the reported data for a commercial unit by TOSHIBA (109FA-MS9001FA) with the purpose to illustrate the use of all closed expressions obtained in this Section: \(\alpha = 3.9 \times 10^6\) W/K, \(T_1 = 1573\) K, \(\gamma_1 = 1/2, \gamma_2 = 2, \sigma_{i1} = 5\sigma_{i2}/4, \sigma_{i2} = 0.155 \times 10^6\) W/K and \(\eta_C \in [0.6, 1]\).

3. Energetic performance and irreversibility factor

In the study of the energetic performance of the models of irreversible thermal cycles, the process variables: efficiency, power output and dissipation are the quantities that give us the estimated time recovery of the investment in the power plant construction.

In Fig. 3a the efficiencies of each proposed operating mode are shown. As it was expected, the \(M\eta\)-efficiency is the biggest one, however, the efficiency that corresponds to the ME-regime is very near to the \(M\eta\)-efficiency but with a power output larger than the \(M\eta\)-power output. On the other hand, the \(MP\)-efficiency always goes under the efficiencies of the other two working regimes. Respect to the power output, when we observe Fig. 3b we see that the power output corresponding to the ME-regime always is above of the \(M\eta\)-power output and it is reasonably near to the power output of the \(MP\)-regime. Finally, in Fig. 3c we show the plots corresponding to the dissipation of the arrangement in serie, here we observe that the dissipation in the \(MP\)-regime is bigger than both the \(ME\)-dissipation and the \(M\eta\)-dissipation within the typical operation temperatures interval of actual CC plants (\(\eta_C \in [0.6, 1]\)). On the contrary, the dissipations of these last two working regimes are very near to each other and they are smaller than the \(MP\)-dissipation in this interval. In summary, these results show that for a series arrangement of two completely irreversible power cycles the ecological
approach of optimization reaches a good trade-off between the power output and the efficiency of the array.

3.1. Combined cycle power plants versus nuclear power plants

Now, we will use the model developed here to make a comparison between two commercial CC power plants and two modern nuclear power plants. This comparison is carried out by using the factor,

\[
\sigma_{12} = \left( \frac{\sigma_{12}}{\alpha} \right)_{\text{op}},
\]

which we call “irreversibility factor”; with \(\alpha\) the thermal conductance associated with the input heat flux, \(\sigma_{12}\) the entropy production in the vapor cycle of the CC and \(\text{op} = M_{\eta}, ME, MP\), i.e., one of the working regimes studied above. This irreversibility factor measures the “negative” balance between the Clausius uncompensated heat \(\sigma_{12}\) and the absorbed heat \(\alpha\) associated with each working regime.

Using the data of the observed power output \(P_O\) and observed efficiency \(\eta_O\), reported by salesmen, for the CC power plants, and supposing that these power plants operate in some of the working regimes studied in Sec. 2, one can invert and solve Eqs. 16 and 17 for each working regime (Eqs. 22-33) to obtain \(\sigma_{12}\) and \(\alpha\) the two quantities involved in 34, i.e., starting from the experimental data and the information provided by the optimization process, one can quantitatively estimate \(f_{\text{op}}\) for each power plant and working regime. The results obtained by means of this procedure in the \(M_{\eta}\)-regime are shown in Table I. This working regime is recommend by the manufacturers for the the operation of the CC plants, The results for the and \(ME\) and \(MP\)-regimes are shown in the same Table.

These results show us that in spite of the high efficiencies of the CC-power plants, the relationship between their global conductance and the internal irreversibilities (in this case those associated to the vapor turbine) are two orders of magnitude bigger than the irreversibility factors of the nuclear power plants, in two working regimes \((M_{\eta} \text{ and } ME)\). This fact means, that the CC-power plants could have high operation costs rising the price of the kilowatt-hour. On the other hand, for three of the four power plants the maximum power output regime con not be reach due a second law restrictions (see Fig. 2).

4. Concluding Remarks

In this work we have shown that the series arrangement of completely irreversible power cycles, have the qualitative properties (loop shaped curves and operating modes) of the real energy converters [8]. In particular it was shown that it is possible to operate the CC–power plants in a working regime representing a good trade–off between a high power output and low dissipation. Later on, with this model it was possible to make a comparison among power plants with different types of cycle. This result reissues one of the most appreciated characteristics of thermodynamic theory [22]: with equivalent process variables (power output and efficiency) it is possible to study the energetic performance of two irreversible energy converters with different internal details. This previous result would justify an exhaustive revision of data of CC plants actually in operation, to calculate their irreversibility factors and to estimate more accurately the real costs of operation and maintenance. However, we must take into account that our study has an important limitation, that is, our thermal engine models are very simplified versions of the real ones. Nevertheless, our models can be used to provide general guides to evaluate the thermodynamic performance of actual engines.

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