Neutrino mixings and right-handed currents in $\tau_{M2}$ decays

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Abstract. The $\tau \rightarrow M\nu_\tau$ decays are revisited in the framework of an effective weak interaction Hamiltonian with neutrino mixings and right-handed currents. Hierarchical and Kobayashi-Maskawa neutrino mixings are considered in the evaluation of the ratio $R = \frac{w(\tau \rightarrow M\nu_\tau)}{w(\tau \rightarrow M\nu_\tau)}$, and manifest left-right symmetry is assumed in our calculations.

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1. Introduction

With the advent of new $\tau$-lepton factories, the decays and properties of this particle will be studied to a great extent, leading to a deep insight in the nature of the weak currents involved in these processes. In particular, the $\tau_{M2}$ decays, where $M$ is a meson, are the simplest of the $\tau$-decays to look for massive neutrinos and right-handed currents, since we are in a two-body final state process [1]. Furthermore, some of these decays have high branching ratios, allowing high statistics studies. [For instance if $M$ is a vector (pseudoscalar) particle like the $\rho(\pi)$, we have branching ratio of about 22% (10%)]. The $\tau_{M2}$ decays are a subset of all the exclusive decay modes of the tau. This decays have been extensively studied, within an effective $V-A$ theory and massless neutrinos by some authors. We refer the reader to Ref. [2] for a review.

Since, up to now, there is no fundamental principle requiring neutrino zero mass, massive neutrinos are to be considered in any theory of weak interactions. In particular models of weak interactions with right-handed currents must involve a finite neutrino mass. Within the framework of this class of models, the $\tau \rightarrow M\nu_\tau$ decays must be considered as an incoherent sum of decay modes $\tau \rightarrow M\nu_i$, where $\nu_i$ denotes a neutrino mass eigenstate of mass $m_i$ distinct from the weak eigenstate $\nu_\tau$ which is a sum of the $\nu_i$ times a neutrino mass-mixing matrix factor $U_{\tau i}$. The index $i$ runs from one to three for a 3-generation model. The masses of the neutrino mass eigenstates are commonly supposed to be in ascending order of values, i.e. $m_1 < m_2 < m_3$. This is the situation in the nondegenerated case when $\nu_1$, $\nu_2$, and $\nu_3$ are, respectively, $\nu_e$, $\nu_\mu$, and $\nu_\tau$ [3].
In this paper we devote ourselves to the study of these decays, looking for effects due to neutrino mixings and right-handed currents. In Sect. 2 we give the amplitude for the four Feynman diagrams and some details of the model under consideration. In Sect. 3 we give the results for the ratio \( R = \sum_i w(\tau \rightarrow M\nu_i)/w(\tau \rightarrow M\nu_\tau) \), with \( M \) a pseudoscalar \((\pi, K)\) or vector meson \((\rho, K^*)\), for the Hierarchical and the Kobayashi-Maskawa mixings, and for manifest left-right symmetry. In Sect. 4 we present our conclusions.

2. Amplitude for \( \tau \rightarrow M\nu_i \)

In considering neutrino mixing and right-handed currents we have four diagrams contributing to the amplitude for \( \tau \rightarrow M\nu_\tau \). These are

\[
M_{(a)} = \frac{G}{2\sqrt{2}} \bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma_5) u_\tau \langle M | J_{WL}^\mu (0) | 0 \rangle
\]

mediated by \( W_L \),

\[
M_{(b)} = \frac{G}{2\sqrt{2}} \eta \bar{u}_{\nu_\tau} \gamma_\mu (1 + \gamma_5) u_\tau \langle M | J_{WL}^\mu (0) | 0 \rangle
\]

mediated by \( W_L - W_R \) mixing,

\[
M_{(c)} = \frac{G}{2\sqrt{2}} K \bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma_5) u_\tau \langle M | J_{WR}^\mu (0) | 0 \rangle
\]

mediated by \( W_L - W_R \) mixing, and

\[
M_{(d)} = \frac{G}{2\sqrt{2}} \lambda \bar{u}_{\nu_\tau} \gamma_\mu (1 + \gamma_5) u_\tau \langle M | J_{WR}^\mu (0) | 0 \rangle
\]

mediated by \( W_R \).

In Eqs. (1) to (4), \( \langle M | J_{WL(R)}^\mu (0) | 0 \rangle \) represents the hadronic matrix element of the left \((L)\) or right \((R)\) handed current associated to the \( W_{L(R)} \). The parameters \( \eta, \kappa \) and \( \lambda \) measure the magnitude of the left-right mixing and right-handed currents. The weak eigenstates neutrino \( \nu_{\tau_L}, \nu_{\tau_R} \) are assumed to be superpositions of mass-eigenstate neutrinos \( N_j \) with mass \( m_j \) [4]

\[
\bar{u}_{\nu_{\tau_L}} = \sum_j U_{\tau j} N_{jL},
\]

\[
\bar{u}_{\nu_{\tau_R}} = \sum_j V_{\tau j} N_{jR}.
\]
An appropriate choice of the matrices \( U \) and \( V \) leads us to the Dirac and Majorana neutrino cases. (No mixing means \( U_{rj} = V_{rj} = \delta_{rj} \)). The hadronic matrix element is given by

\[
\langle M | J_{WL}^\mu (0) | 0 \rangle = \begin{cases} 
  f_p U_{KMP}^* \mu \\
  f_v U_{KM}^* \epsilon^\mu (p) 
\end{cases}
\]  

(7)

and

\[
\langle M | J_{WR}^\mu (0) | 0 \rangle = \begin{cases} 
  f_p U_{KMP}^* \mu \\
  f_v U_{KM}^* \epsilon^\mu (p) 
\end{cases}
\]  

(8)

where \( f_p (f'_p) \) and \( f_v (f'_v) \) are the decay form factors for the case where \( M \) is a pseudoscalar meson of 4-momentum \( p^\mu \), and a vector meson of polarization four-vector \( \epsilon^\mu (p) \). \( U_{KM} \) and \( U_{KM}' \) are Kobayashi-Maskawa mixing matrices for the left and right handed hadronic currents, respectively.

Adding Eqs. (1)-(4) and substituting Eqs. (5)-(8) we obtain for the decay amplitude

\[
M_j = \frac{G}{2\sqrt{2}} U_{KM}^* \left\{ \begin{array}{c} f_p \\ f_v \end{array} \right\} \left[ F_j \tilde{N}_L \gamma^\mu (1 - \gamma_5) u_\tau + F'_j \tilde{N}_R \gamma^\mu (1 + \gamma_5) u_\tau \right] \left\{ \begin{array}{c} p^\mu \\ \epsilon^\mu (p) \end{array} \right\}
\]  

(9)

where

\[
F_j = \left( 1 + K \left\{ \begin{array}{c} f_p \\ f'_p \end{array} \right\} \right) U_{rj},
\]

(10)

\[
F'_j = \left( \eta + \lambda \left\{ \begin{array}{c} f_p \\ f'_p \end{array} \right\} \right) V_{rj},
\]

(11)

and

\[
f = \frac{f_p U_{KMP}}{f_p U_{KM}'}, \quad f' = \frac{f'_p U_{KMP}'}{f_v U_{KM}'},
\]

(12)

3. Total decay rate

To compute the total decay rate \( \omega (\tau \rightarrow M \nu_\tau) \), we proceed as usual: we sum over final spins (or polarization) and average over the initial one, and integrate over final
phase space. The result is

$$w(\tau \to M\nu_\tau) = \left(\frac{G}{\sqrt{2}}\right)^2 \sum_j \frac{1}{4\pi} p(j) \left\{ \frac{M_{f_p}}{f_v} \right\}^2 \times \left[ (|F_j|^2 + |F_j'|^2) \sum_j \left\{ \frac{(1 + \delta_j)(1 - \delta + \delta_j) - 4\delta_j}{\delta} \right\} + 2(1 - \delta + \delta_j) \right]$$

where $\delta = m^2/M^2$, $\delta_i = m_i^2/M^2$, with $m$ the meson mass and $M$ the $\tau$ mass; $p(j)$ is given by

$$p(j) = \frac{M}{2} \lambda^{1/2}(1, \delta, \delta_j)$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

In Eqs. (13) we have absorbed the $|U_{KM}|^2$ factor into the decay constants $f_p$ and $f_v$, and the upper (lower) line corresponds to the pseudoscalar (vector) meson case. The sum is over the incoherent neutrino mass eigenstates only.

For left-handed currents only and no neutrino mixings we obtain the result [5–6]

$$w^0(\tau \to M\nu_\tau) = \left(\frac{G}{\sqrt{2}}\right)^2 \frac{p}{4\pi} \left\{ \frac{M_{f_p}}{f_v} \right\}^2 \left\{ \frac{(1 - \delta)}{(1/\delta + 2)(1 - \delta)} \right\}$$

where $p = \frac{M}{2}(1 - \delta)$.

The rate for the mode $\tau \to M\nu_\tau$, Eq. (13), relative to that for the conventional decay $\tau \to M\nu_\tau$, with $m_{\nu_\tau} = 0$ and no right-handed currents, is given by

$$R_p = \sum_j \frac{p(j)}{p} \frac{1}{1 - \delta} \left[ (|F_j|^2 + |F_j'|^2) ((1 + \delta_j)(1 - \delta + \delta_j) - 4\delta_j) \right]$$

$$+ 2 \text{Re}(F_j F_j'^*) \delta \sqrt{\delta_j}$$

for the case when $M$ is a pseudoscalar meson, and

$$R_v = \sum_j \frac{p(j)}{p} \frac{1}{(1 - \delta)(2 + 1/\delta)} \left[ (|F_j|^2 + |F_j'|^2) \right]$$

$$\left[ \frac{(1 + \delta_j)(1 - \delta + \delta_j) - 4\delta_j}{\delta} + 2(1 - \delta + \delta_j) \right] - 6 \text{Re}(F_j F_j'^*) \sqrt{\delta_j}$$

(17)
for the case when $M$ is a vector meson. We proceed to the study of these results as follows. For manifest left-right symmetry [6] we have $\eta = K$, $V_{\tau j} = U_{\tau j}$ and $f = f' = 1$. Then $F_j = (1 + K)U_{\tau j}$ and $F_\tau = (K + \lambda)U_{\tau j}$. The parameters $K$ and $\lambda$ are expressed by the gauge coupling constant, the masses of the gauge bosons and the mixing angle between the light and heavy gauge bosons [7]: $K \simeq -1.44 \times 10^{-3}$, $\lambda = 0.4028$ (for $M_{\text{WR}} = 400 \text{ GeV}$). In Fig. 1 we plot $R_\tau$ vs $m_3$, for hierarchical mixing (H) and Kobayashi-Maskawa mixing (KM) in the neutrino sector. We use, for Hierarchical mixing, the values of $U_{\tau j}$ given in [8]

$$|U_{13}|^2 = 0.0003, \quad |U_{23}|^2 = 0.059 \quad \text{and} \quad |U_{33}|^2 = 0.94.$$ 

For KM mixing, we use, as an example, solution ($C$) from Ref. [9]

$$|U_{13}|^2 = 0.1681, \quad |U_{23}|^2 = 0.0004 \quad \text{and} \quad |U_{33}|^2 = 0.8281.$$ 

For these values of $U_{\tau j}$ we observe that the dominant contribution comes, as expected, from $m_3$ only. In Fig. 2 we do the same for $R_\rho$. We note for $R_\tau$ that H-mixing is greater that KM-mixing in about 1% for the full range of $m_3$. For $R_\rho$ we note the same behaviour up to $m_3$ around 90 GeV. Above this $m_3$ value KM-mixing is greater that H mixing by 0.1%. Eq. (13) do not incorporate radiative corrections, which depend on $m_3$ and meson structure functions. For the $\tau \rightarrow \pi \nu$ decay, with no neutrino mixing and no right-handed currents, in an effective $V - A$ theory, radiative corrections give a contribution $-5.4\%$ to $-4.4\%$ for $0 \leq m_{\nu \tau} \leq 100$ (MeV) [5]. For the $\tau \rightarrow \rho \nu_\tau$ decay the contribution of the radiative corrections is in the range $-0.77\%$ to $0.66\%$ for $0 \leq m_\tau \leq 80$ (MeV) [6]. Then, radiative corrections in $\tau \rightarrow \pi \nu_\tau$ are much greater than the contributions arising from right-handed currents. But for
Neutrino mixings and right-handed currents in $\tau_{M_2}$ decays

$\tau \to \rho \nu_{\tau}$ the contributions coming from right-handed currents amount to around 3%, for both H and KM mixings. We conclude that the $\tau \to \rho \nu_{\tau}$ decay mode is a suitable one to look for right-handed currents, without taking into account radiative corrections. Eqs. (16) and (17) are insensitive to radiative corrections, except for radiative corrections coming from diagrams mediated by the heavy right-handed weak boson, which are small for $M_{WR} \geq 400$ GeV.

4. Conclusion

We have calculated the $\tau \to M \nu_{\tau}$ decays in the framework of a model of weak interactions with neutrino mixing and right-handed currents. Our results show that, for manifest left-right symmetry and for $M_{WR} = 400$ GeV, these decays are 1.6% ($\tau_{\tau \nu}$) and 3.4% ($\tau_{\rho \nu}$) greater than the corresponding one in the absence of right handed currents and neutrino mixing. For both cases the experimental result do not exclude the kind of contribution studied here.

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References


**Resumen.** En el contexto de un hamiltoniano de interacciones débiles efectivo con mezcla de neutrinos y corrientes derechas se revisan los decaimientos $\tau \rightarrow M\nu_r$. En la evaluación de la razón $R = \sum_i w(\tau \rightarrow M\nu_i)/w(\tau \rightarrow M\nu_r)$ se consideran la mezcla de neutrinos tipo jerárquica y la tipo Kobayashi-Maskawa. Suponemos simetría manifiesta izquierda-derecha en nuestros cálculos.