Current instabilities under HF electron gas heating in semiconductors with negative differential conductivity

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A nonlinear temperature dependence of the kinetic coefficients of semiconductor plasma can result in the appearance of regions of negative differential conductivity (NDC) in both the high-frequency (HF) and static current-voltage characteristics (CVC) [1, 2]. In the present paper the formation of the static NDC under simultaneous electron gas heating by HF and static electric field is studied. As is shown below, in this case the heating electromagnetic wave has a pronounced effect on the appearance of NDC caused by the overheating mechanisms [3] and the type of the static CVC as a whole.

Keywords: Current instability; electron gas heating; negative differential conductivity

Una dependencia no lineal de la temperatura de los coeficientes cinéticos del plasma del semiconductor puede llevar a la aparición de regiones con conductividad diferencial negativa (CDN) en las características corriente voltaje (CCV) de alta frecuencia (AF) y estática. En este artículo se estudia la formación de la CDN estática bajo la acción simultánea del calentamiento del gas de electrones por AF y el campo eléctrico estático. Como se muestra más adelante, en este caso la onda electromagnética que calienta a los electrones ejerce un fuerte efecto en la aparición de la CDN, que se obtiene por mecanismos de sobrecalentamiento [3], y en el tipo de CCV estática.

Descripciones: Inestabilidad de corriente; calefacción del gas de electrones; conductividad diferencial negativa

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1. Introduction

The electron temperature $\Theta$ in a sample can be found as a function of the coordinates and electric field by considering the equation of heat balance, which follows from the transport equation. Under steady-state conditions, the heat balance equation has the form [4]

$$
\frac{dV_z}{dz} + nT \nu_e(\Theta) \left( \frac{\Theta}{T} - 1 \right) = \bar{J} \mathbf{E},
$$

where

$$
\bar{W} = e J_{11} \mathbf{E}' - e J_{12} \nabla \ln \Theta
$$

is the thermal flux ($W_z$ is the $z$-component of this flux),

$$
\bar{J} = e^2 \mathbf{J}_{10} \mathbf{E}' - e J_{11} \nabla \ln \Theta
$$

is the electric current,

$$
\mathbf{E}' = \mathbf{E} + \frac{\mu}{e} \nabla \ln \Theta,
$$

$$
\mathbf{E} = -\nabla \phi;
$$

with $\phi$ the electrostatic potential, $\mu$ the chemical potential, and $\mathbf{E}$ the electric field.

$$
\mathbf{E}' = \mathbf{E} + \frac{\mu}{e} \nabla \ln \Theta,
$$

where $\nu(\Theta)$ is the electron concentration, and $m$ is the electron effective mass.

The values of $q$ and $r$ for different energy and momentum dissipation mechanisms are listed in Refs. 5 and 6.

The terms in Eq. (1) have simple physical meaning. The right side describes the Joule heat in the electron subsystem,
the first term on the left is the thermal flux in the electron subsystem, and the second one is the transfer of heat from the electron gas to the lattice. The heat balance equation [Eq. (1)] should be supplemented by boundary conditions describing the absorption of the carrier energy at the boundaries of the sample. If we assume that in the direction of the z-axis the planes bound the sample $z = \pm a$, we find that, in accordance with Ref. 7 they can be written in the form

$$W_s|_{z=\pm a} = \pm \eta_k f(\Theta)(\Theta - T)|_{z=\pm a},$$

where $\eta_k f(\Theta)$ are the functions associated with the inelastic scattering of electrons at the boundaries and $\eta_k > 0$ are the parameters representing this scattering ($\eta_k = 0$ correspond to the absence of the surface scattering mechanisms).

2. Classification of dissipation mechanisms and types of current-voltage characteristics

If the temperature of the electron subsystem is independent of the coordinates, this temperature is given by the equation [see Eqs. (1) and (3)]

$$n\nu_e(\Theta)(\Theta - T) = \sigma(\Theta)E^2,$$

where

$$\sigma(\Theta) = e^2 j_10$$

is the conductivity.

We must distinguish two possibilities, depending on the nature of the specific energy and momentum dissipation mechanisms:

1) The electron temperature defined by Eq. (7) is a single-valued function of the electric field.

2) The electron temperature is a many-valued function of the field.

This problem has been discussed in detail in Ref. 6. In particular, if the energy and momentum are dissipated by its own mechanism [see the expressions in Eq. (5)], we find that, subject to the inequality

$$r + q > 0,$$

the electron temperature is a single-valued function of the field (no overheating mechanisms). In this case, the current is also a single-valued function of the field. If $r - q > 0$, the field is also a single-valued function of the current. In the opposite case ($r - q < 0$), the current-voltage characteristics has an $N$-type negative resistance [6].

If

$$r + q < 0,$$

the electron temperature is a double-valued function of $E$ (with the branches rising and falling with increasing $E$), i.e., overheating mechanisms are present. In this case, the condition

$$r - q > 0,$$

makes the current a double-valued (S-type) function of $E$ (the field $E$ is a single-valued function of the current). However, if

$$r - q < 0,$$

the dependence of the current on the field and of the field on the current are both many-valued (S-N-type).

The existence of drooping section at CVC $dj/dE < 0$, (negative differential conductivity) is widely used in the modern solid state functional electronics [8–13], mainly for generation of electromagnetic waves, and for high-velocity switches.

Nevertheless, the choice of materials with NDC is very limited, so the possibility of creation NDC with the help of combined external perturbation seems very perspective.

3. Current-voltage characteristics of a sample with S-type dependencies of the temperature on the field ($\eta_k = 0$)

If overheating mechanisms are active in a semiconductor at low temperatures and nonoverheating mechanisms predominate at high temperatures, it follows from Sec. 2 that Eq. (7) has three solutions in the range $E_1 < E < E_2$, i.e., the electron temperature and current are $S$-type functions of the field $E$ (if $d\sigma/d\Theta > 0$) [14].

Investigations of the various thermomagnetic and galvanomagnetic effects have been carried out first on thick samples and then on the thin one, whereas the $S$-type current-voltage characteristics first have been studied for the thin and then for the thick samples. The first investigations of this type characteristics have been concerned with Eq. (7), which corresponds to the thin samples (this feature was discussed later in Refs. 15–17). The many-valued current-voltage characteristics of thick samples have been studied relatively recently.

The limiting case of large transverse (in relation to the applied voltage) dimensions of a sample has been studied qualitatively [14, 18, 19]. However, as it is shown in Ref. 20, the true current-voltage characteristics can be obtained only after appropriate allowance for the boundary conditions, which have not been made consistently in the cited papers. Therefore, we shall follow Ref. 20.

It is convenient to rewrite Eqs. (1) and (6) with $\eta_k = 0$

$$\frac{d^2 w}{dz^2} + \frac{dU(w)}{dw} = 0,$$  \hspace{1cm} (13)

$$\left.\frac{dw}{dz}\right|_{z=0,a} = 0,$$  \hspace{1cm} (14)

where

$$U(w) = \int_U^w \{\sigma(w)E^2 - n\nu_e(w)[\Theta(w) - T]\} \, dw,$$

$$\int^T \chi(\Theta) \, d\Theta,$$ 

becomes infinite in fields \( E_1 \) and \( E_2 \), and passes through a minimum \( l_{c_{\min}} \) in some intermediate field.

There is a simple relationship between the characteristic length \( l_c \) and the cooling length \( L \) [21]:

\[
l_c = \frac{\pi \left[ \frac{\left| \frac{d\psi}{dE} \right| + \sigma}{2(\Theta - T) \frac{d\sigma}{d\Theta}} \right]^{1/2}}{L}.
\]

It follows from Eq. (16) that in the vicinity of the field \( E_{\min} \) in which \( l_c \approx l_{c_{\min}} \), we have \( l_c \approx L \) because in this region \( \left| \frac{d\psi}{dE} \right| \approx \sigma \). Closer to the field \( E_1 \) and \( E_2 \) where \( \left| \frac{d\psi}{dE} \right| \gg \sigma \), and \( l_c \gg L \).

Thus, we may conclude that if \( a < l_{c_{\min}} \) and \( R > (c/ab)|\frac{d\psi}{dE}|^{-1} \), a homogeneous temperature distribution is stable throughout the range \( E_1 < E < E_2 \). It is interesting to notice that in contrast to the thermal size effects the temperature (and current) distribution are homogeneous in thin samples, and if \( a < l_{c_{\min}} \) and \( b < l_{c_{\min}} \), the form of the current-voltage characteristics ceases to depend on the dimensions. If this restriction on the transverse dimensions is not obeyed, we have two regions adjoining \( E_1 \) and \( E_2 \) where the homogeneous temperature distribution is still stable. The boundaries of these regions \( E_{1}' \) and \( E_{2}' \) are found from the equation \( E_{c}(E) = a \). The range of fields \( E \) in which the homogeneous distribution is unstable \( (E_{1}' < E < E_{2}') \) increases in width with increasing thickness of the sample.

Now we shall consider all the possible inhomogeneous solutions within the range \( E_{1}' < E < E_{2}' \).

The Eq. (13) is formally identical to the motion of a particle in a potential field where the function \( U(w) \) is the potential energy of the particle and the variables \( z \) and \( w \) replace the time and coordinate. It follows from the expression for \( U(w) \) [see Eq. (13)] that the potential energy depends on the field \( E \) as a parameter. In the fields \( E < E_1 \) and \( E > E_2 \) the potential energy \( U(w) \) is the function with one maximum, whereas in the fields \( E_1 < E < E_2 \) it has two maxima and one minimum, and finally in the field \( E = E_1 \) or \( E = E_2 \). The boundary conditions \( (\eta_\epsilon = 0) \) imposed on the temperature represent finite motion of the particle in the potential well and can be satisfied only if an integral number of half-oscillations can be fitted into a sample.

The minimum period of the motion of the particle corresponds to the minimum of the potential energy when the particle behaves as a linear oscillator with a period \( 2l_c(E) \). It is obvious that when \( a < l_c(E) \), there are no inhomogeneous solutions in the field being considered [22] and the criteria of the absence of inhomogeneous solutions is identical to the criteria of the stability of the homogeneous solution. Thus, if the homogeneous solution is stable, it is the only solution. Conversely, when inhomogeneous solutions exist \((a > l_c(E)) \), the homogeneous solution is unstable.

We note that two monotonic solutions exist throughout the range \( E_1 < E < E_2 \) (they also correspond to the "highest" energy level). An examination of the stability of these solutions shows that the only stable ones are two monotonic solutions \( w^{(1)}(z) \), and then only subject to the conditions

\[
R > \frac{c}{ab} \left| \frac{d\psi}{dE} \right|^{-1}
\]

where

\[
\frac{d\psi}{dE} < 0.
\]

It is clear that the value of \( \tilde{\psi} \) is independent of whichever solution \( [w^{(1)}] \) or \( [w^{(-1)}] \) is actually obtained.

Thus, if \( a > l_c(E) \), we find that there are fields \( E_1 < E < E_{1}' \) and \( E_2 < E < E_{2}' \) in which the homogeneous distribution of the temperature is stable and the only one.

4. Maxwell's equations and the boundary conditions

Maxwell's equation for the amplitude \( \tilde{E} = \tilde{E}_z - i\tilde{E}_y \) has the form [23]

\[
\frac{d^2\tilde{E}}{dz^2} + \kappa^2 \epsilon(\xi)\tilde{E} = 0,
\]

where

\[
\epsilon(\xi) = \epsilon_\infty + \frac{8\sqrt{2\pi} \left( mT \right)^{3/2} \omega_0^2}{3n\omega} \times \int_0^\infty \frac{x^{3/2}}{\omega + i\nu(x)} \frac{df_0(x,\xi)}{dx} dx.
\]
then \( \delta \varepsilon (\xi) = 4 \pi \sigma (\xi) / \omega \). Here \( \varepsilon (\xi) \) is the high-frequency dielectric constant, \( \varepsilon_0 \) is the static dielectric constant, \( f_0 (x, \xi) \) is the symmetric part of electron Fermi-Dirac distribution function with the electron temperature, \( x = \varepsilon / T, \xi = \Theta / T, \kappa = \omega / c, \omega \) is the frequency of HF electromagnetic field, and \( c \) is the light speed.

Maxwell's equation is nonlinear in virtue of the dependence of the dielectric constant on the electron temperature \( \xi \).

Now let us formulate the boundary conditions of the problem. Let the plasma (semiconductor or gas-discharge) occupy the half-space \( z > 0 \); a plane monochromatic wave coming from infinity \( (z = -\infty) \) falls normally at the surface \( z = 0 \). For simplicity we assume that the region of space \( z < 0 \) is filled by the linear nondissipative medium with an index of refraction \( n = 1 \). Then, for \( z < 0 \) the wave will have the form

\[
\vec{E} = \vec{E}_0 (e^{ikz} - Re^{-ikz}),
\]

where \( E_0 \) is the amplitude of the incident wave, \( R \) is the coefficient of reflection. Besides, we assume that the temperature of this medium coincides with the lattice temperature \( T \).

It is assumed below that the characteristic distance \( \lambda \) over which the field changes is much larger than the Debye radius \( d \approx \varepsilon / (4 \pi n e^2 n)^{1/2} \), where \( \varepsilon \) is the average energy. It is well known that under this assumption the plasma is quasineutral. For semiconductors containing carriers of a single sign this means that at any arbitrary point the charge density of the electrons (holes) is equal to the equilibrium density unless the processes like impact ionization, changing the recombination coefficient etc., are taken into account.

From here it follows that the carrier concentration in the doped semiconductors does not depend on the coordinates.

Above we have assumed that the wave in the plasma is circularly polarized. For this to occur \( n \), the polarization of the wave incident on the half-space must also be circular. We note that in contrast to the linear theory, where one can satisfy the boundary conditions for arbitrary polarization of the incident wave by the superposition of the normal waves, for the case of nonlinear propagation the superposition of the normal waves is not a solution of Maxwell's equation. Therefore, if the polarization of the incident wave does not coincide with the polarization of one of the normal waves in the plasma, the picture becomes more complicated [23]. We shall not dwell on this question here since the results obtained below remain qualitatively valid even for arbitrary polarization of the incident wave [23].

Since the plasma is semi-bounded, it is necessary to add to Eq. (19) the boundary conditions at the planes \( z = 0 \) and \( z \to \infty \).

The boundary conditions for the field have the usual form

\[
\vec{E}(-0) = \vec{E}(+0), \quad \frac{\partial \vec{E}(-0)}{\partial t} = \frac{\partial \vec{E}(+0)}{\partial t}.
\]

In the presence of attenuation

\[
\lim_{z \to \infty} \vec{E}(z) \to 0,
\]

i.e., the heating of the electron gas is absent at infinity; therefore

\[
\lim_{z \to \infty} \vartheta(z) \to 1,
\]

5. Propagation of weakly damped electromagnetic waves

As it has already been indicated above, two mechanisms exist for the removal of energy from the electrons: the thermal conductivity which is described by the first term on the left-hand side of Eq. (1), and the transfer energy to the lattice, which corresponds to the second term of the left-hand side of this Equation. The ratio of these terms is of the order of \( L_e^2 / L_\Theta^2 \), where \( L_\Theta \) is the characteristic length over which the temperature changes, and

\[
L_e \approx \frac{\bar{v}}{\sqrt{\nu v_e}}
\]

is the energy mean free path, characterizing the transfer energy to the lattice, where \( \bar{v} \) is the electron heat velocity.

One can neglect the thermal conductivity provided \( L_e^2 / L_\Theta^2 \ll 1 \). In this connection, as follows from Eq. (1) the relation between the temperature and the field is local and consequently \( L_\Theta \approx \lambda \), where \( \lambda \) is the damping depth of the electromagnetic wave.

If the inequality

\[
\frac{L_e^2}{L_\Theta^2} \approx \frac{L_e^2}{\lambda^2} \ll 1
\]

is satisfied, then we shall talk about the normal skin effect.

By the anomalous skin effect we mean that situation when the inequality

\[
l \ll \lambda \ll l_e
\]

is satisfied (\( l = \bar{v} / \nu \) denotes the mean free path connected with momentum transfer), because the case \( \lambda \leq l \) is actually not realized in semiconductors.

6. Current instabilities

Let us consider a semiconductor plate oriented so that a constant electric field is applied along the OY-axis, and the incident plane HF electromagnetic wave \( \vec{E}_0^0 \) travels in the positive OZ-axis direction. We consider a one-dimensional energy balance equation

\[
-\frac{dW}{dz} + \sigma |\vec{E}|^2 + \sigma E^2 = n \nu_e (\Theta - T),
\]

where \( \vec{E} \) and \( E \) are the intensities of the HF and static fields, respectively; \( \sigma (\Theta) = \sigma_0 (\Theta/T)^{-\gamma} \) is the static...
CURRENT INSTABILITIES UNDER HF ELECTRON GAS HEATING IN SEMICONDUCTORS WITH NEGATIVE...
Figure 1. Effect of the HF heating on the CVC with S-like NDN \((q = -1.5 \text{ and } r = 0.5)\). a) Electron temperature, b) direct currents as functions of the static electric field for different values of HF field, \((1. \bar{E} = 0, 2. \bar{E} = 5, \text{and } 3. \bar{E} = 10)\), c) electron temperature as a function of the HF field with the static field as a parameter, \((1. E = 0, 2. E = 0.3, \text{and } 3. E = 0.4)\).

Since the right-hand side of the Eq. (34) is positive and monotonic in comparison with the electron temperature for 
\(\xi > \xi^* \equiv \Theta^* / T\), the switching field decreases monotonically with an increase of the HF wave amplitude, and the associated temperature increases (Fig. 1a). As a result, the NDC region of the CVC shrinks with an increase of the HF wave amplitude and it moves to the region of the strong currents (Fig. 1b).

Note that the Eq. (34) also predicts the extreme of the function \(\bar{E}(\xi)\). When the HF field amplitude exceeds a certain critical value \(\bar{E}_{cr}\), then the electron temperature starts to increase until another mechanism of the energy relaxation turns on. Therefore, due to the HF field the electron gas goes to the state described by the low-resistance branch of the static CVC (Fig. 1c).

7. Conclusions
Formation of current-voltage characteristics with regions of static negative differential conductivity under simultaneous additional heating by the static and high frequency electric fields has been studied. It is shown that when the static CVC is \(S\)-like, heating by HF field results in the narrowing of the NDC region and in its shift toward high currents. The effect of transformation of \(N\)-like CVC into a mutually multivalued one is predicted.

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