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Possible cosmological implications in electrodynamics due to variations of the fine structure constant

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Astronomical observations are suggesting that the fine structure constant varies cosmologically. We present an analysis on the consequences that these variations might induce on the electromagnetic field as a whole. We show that under these circumstances the electrodynamics in vacuum could be described by two fields, the "standard" Maxwell's field and a new scalar field. We provide a generalised Lorentz force which can be used to test our results experimentally.

Keywords: Classical electromagnetism; quantum electrodynamics; cosmology.

Observaciones astronómicas sugieren que la constante de estructura fina presenta variaciones cosmológicas. En este artículo hacemos un análisis sobre las consecuencias que estas variaciones posiblemente inducen en el campo electromagnético. Mostramos que bajo estas circunstancias la electrodinámica del vacío puede ser descrita por dos campos, el campo "estándar" de Maxwell y un nuevo campo escalar. Además, proponemos una fuerza de Lorentz generalizada que puede utilizarse para confirmar nuestros resultados de manera experimental.

Descriptores: Electromagnétismo clásico; electrodinámica cuántica; cosmología.

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1. Introduction

Since the first half of the 20th century different researchers [1–3] began to put forward the idea that the fine structure constant α could present cosmological variations. Today, recent observations of quasars have suggested [4–6] that the fine structure constant $\alpha \equiv e^2/\hbar c$ might present variations with respect to cosmic time. Here, e represent the electron charge, c the speed of light and $\hbar$ Planck's constant. These observations imply that the fluctuations $\Delta\alpha/\alpha$ of this fundamental constant are given by

$$\frac{\Delta\alpha}{\alpha} = -0.72 \pm 0.18 \times 10^{-5}, \qquad (1)$$

in the interval of redshifts z given by $0.5 < z < 3.5$.

Because the constants $\hbar$, e and c that define α might vary in different ways [7] so as to give the value given by Eq.(1), one may assume that the electromagnetic fields and the electric charges are coupled in different forms that depend on cosmic time. Since the electric charge could have variations of cosmological origin, possibly the continuity equation no longer holds and/or part of the electric charge is not generating electromagnetic field, or alternatively, it generates an extra electromagnetic field. In this letter we explore these possibilities and some of its immediate consequences on space–time.

Previous research has been conducted on this topic. Most notably the work by Bekenstein [8] and Chodos & Detweiler [9] had given in the past theoretical clues as to why the fine structure constant might vary in time or position as the universe expands. Bekenstein developed a complete analysis using the principle of least action. Chodos & Detweiler analysed α variations using a five dimensional (4+1) space–time based on ideas first proposed by Kaluza and Klein.

It is well known that one can decompose a vector field as the sum of one solenoidal component plus a non–rotational one (cf. Helmholtz decomposition theorem). A generalisation in terms of differential forms is given by the Hodge decomposition theorem for Riemannian manifolds. Also, an n–dimensional manifold can be foliated with submanifolds of smaller dimensions. For the electromagnetic case that we study in this letter, it is possible to foliate the space–time (which a 3+1 Lorentzian metric) with 2–dimensional and 0–dimensional manifolds that "emerge" from vector fields which represent the electric charge–current densities. It is then natural to use the formalism of differential forms in order to obtain a more general study of the problem through a Hodge–like decomposition of the differential form that represents the electromagnetic charge–current distributions.

2. Electrodynamics

Let us take the 1–form $\boldsymbol{J}_{\text{std}} = \rho_{\text{std}}\, \mathbf{d}x^0 + \left(j_k^{(\text{std})}/c \right) \mathbf{d}x^k$ representing the charge–current in the usual sense [10] with

$k = 1, 2, 3$. Here the signature of the metric is given by $(-, +, +, +)$, ρ_{std} is the charge density, $j_k^{(\text{std})}$ are the components of the current density, $\mathbf{d}x^\mu$ is a basis for the cotangent space with coordinates ($x^0 = ct$, x^1, x^2, x^3) and greek indices have values $0, 1, 2, 3$. $\boldsymbol{J}_{\text{std}}$ satisfies Maxwell's equations

$$\mathbf{d}\boldsymbol{F} = 0, \qquad \boldsymbol{\delta F} = 4\pi \boldsymbol{J}_{\text{std}}, \tag{2}$$

which imply naturally the continuity equation

$$\boldsymbol{\delta J}_{\text{std}} = 0. \tag{3}$$

In the previous equations $\boldsymbol{F}$ is a 2–form that builds up the standard electromagnetic field and is given [10] by

$$\boldsymbol{F} \equiv E_1 \mathbf{d}x^1 \wedge \mathbf{d}x^0 + E_2 \mathbf{d}x^2 \wedge \mathbf{d}x^0 + \ldots + B_3 \mathbf{d}x^1 \wedge \mathbf{d}x^2.$$

$\boldsymbol{E}$ and $\boldsymbol{B}$ represent the electric and magnetic components of the electromagnetic field. $\boldsymbol{\delta} \equiv^\star \mathbf{d}^\star$ is the co–differential operator and * is the Hodge star operator [10–13].

In a universe with varying α, the continuity equation is not necessarily valid. This can be interpreted as if a universal "total charge–current" 1–form $\boldsymbol{J}_e$ given by

$$\boldsymbol{J}_e \equiv \boldsymbol{J}_{\text{std}} + \boldsymbol{J}_{\text{n}} + \boldsymbol{J}_{\text{h}}, \tag{4}$$

is associated to the "global electrodynamics" of the universe at all cosmological times. In Eq. (4), the 1–form $\boldsymbol{J}_e$ is such that it accepts a Hodge–like decomposition (cf. Hodge decomposition theorem in [12,13]). With this assumption, the differential 1–forms $\boldsymbol{J}_{\text{std}}$, $\boldsymbol{J}_{\text{n}}$ and $\boldsymbol{J}_{\text{h}}$ are coexact, exact and harmonic 1–forms respectively.

Equation (4) is a natural generalisation of the result expressed by Eq. (1). Indeed, Eq. (1) means that $\alpha \approx \left(1 - 0.72 \times 10^{-5}\right) \alpha_{\text{today}}$. If the total charge–current $\boldsymbol{J}_e$ obeys a similar relation, that is

$$\boldsymbol{J}_e = (1 + \eta) \boldsymbol{J}_{\text{std}}, \tag{5}$$

where η is a scalar 0–form, then it follows that $\boldsymbol{J}_{\text{n}} + \boldsymbol{J}_{\text{h}} = \eta \boldsymbol{J}_{\text{std}}$. To simplify things it is possible to assume that η can be decomposed in to two additive terms, η_{n} and η_{h} such that $\eta = \eta_{\text{n}} + \eta_{\text{h}}$. These terms satisfy

$$\boldsymbol{J}_{\text{n}} = \eta_{\text{n}} \boldsymbol{J}_{\text{std}}, \qquad \text{and} \qquad \boldsymbol{J}_{\text{h}} = \eta_{\text{h}} \boldsymbol{J}_{\text{std}}. \tag{6}$$

From the previous considerations it follows that the 1–form $\boldsymbol{J}_e$ does not satisfy a continuity–like equation when $\eta_n \neq 0$.

3. Mathematical relations between fields

In order to analyse the electrodynamics imposed by the conditions of the previous section, let us multiply Eq. (4) by 4π and substitute Eq. (2) and (6) on this to obtain

$$4\pi \boldsymbol{J}_e = \boldsymbol{\delta F} + \mathbf{d}M + 4\pi \eta_{\text{h}} \boldsymbol{J}_{\text{std}}, \tag{7}$$

in which the scalar 0–form M is such that

$$\mathbf{d}M = 4\pi \eta_{\text{n}} \boldsymbol{J}_{\text{std}}, \qquad \text{and} \qquad \boldsymbol{\delta} M = 0. \tag{8}$$

Note that Eq. (7) reduces to the standard Maxwell's equations when there is no cosmological variation of $\boldsymbol{J}_e$. That is, when $\eta_{\text{n}} = \eta_{\text{h}} = 0$ and so $\boldsymbol{J}_e = \boldsymbol{J}_{\text{std}}$. In the general case, when this condition is not valid, the electromagnetic field is such that it is represented by two mathematical objects, the Maxwell 2–form $\boldsymbol{F}$ and the 0–form M. $\boldsymbol{F}$ satisfies Maxwell's equations, Eq. (2), and M satisfies a set of Maxwell's-like equations given by Eq. (8). In other words, the cosmic time variations of $\boldsymbol{J}_e$ imply that the electrodynamics of space–time are given by two fields. One field turns out to be the standard Maxwell 2–form $\boldsymbol{F}$. The other is a scalar field M introduced by the cosmological variations of $\boldsymbol{J}_e$.

According to Eq.(7), the 0–form M satisfies the following "Poisson's" equation

$$\Delta M \equiv (\boldsymbol{\delta} + d)^2 M =^\star \{ \mathbf{d}\eta_{\text{n}} \wedge \mathbf{d}^\star \boldsymbol{F} \}. \tag{9}$$

In other words, the scalar field M is produced by the changes in the 2–form field $\boldsymbol{F}$ and the scalar η_{n}.

We can also give an expression for Dirac's equation. From Eq.(7), using again a Hodge–like decomposition, it follows that we can introduce a 1–form $\boldsymbol{A}$ that represents the electromagnetic potential given by

$$\boldsymbol{A} = \boldsymbol{A}_{\text{std}} + \boldsymbol{A}_M + \boldsymbol{A}_{\text{h}}, \tag{10}$$

where $\boldsymbol{A}_{\text{std}}$, $\boldsymbol{A}_M$, and $\boldsymbol{A}_{\text{h}}$ are co–exact, exact and harmonic 1–forms respectively. In Eq. (10) we have added the 1–form $\boldsymbol{A}_{\text{h}}$ for mathematical completeness, despite the fact that it is usually discarded in standard physics. With this, and because $e = (1 + \eta) e_{\text{std}}$, where e_{std} is the standard charge of an electron, then Dirac's equation takes the form

$$\left(i \not{d} - \frac{\alpha}{(1 + \eta)\, e_{\text{std}}} \not{A} \right) \Psi = \frac{mc}{\hbar} \mathbf{1} \Psi. \tag{11}$$

Here $i^2 = -1$, $\not{d} = \gamma^\mu \partial_\mu$, m is the electron's rest mass, $\not{A} = \gamma^\mu A_\mu$, Ψ is Dirac's spinor and $\mathbf{1}$ is the identity element of the algebra generated by Dirac's matrices γ^μ that satisfy the following equation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} \mathbf{1},$$

where $g^{\mu\nu}$ are the metric components assigned to space–time.

4. Discussion

The previous analysis was made under the assumption that the variations of the 1–form $\boldsymbol{J}_e$ are time dependent. However, all the presentation is still valid if the variations are not only functions that depend on time, but also functions that could vary on space. That is, the variations can equally occur on space and/or time and the coupling of the two fields $\boldsymbol{F}$ and M

will still occur in the same form. More generally, the results obtained in the previous section are also valid if space–time variations on the "fundamental" constants $\hbar$, e and c, or even [14] m occur.

It is intriguing that our daily experiments do not show any evidence of the physical properties that the field M might induce on space–time. However, there has been a report [15,16] in which such a field produces longitudinal electrodynamic waves. One can also think that the reason for a non–observable field M happens because it vanishes at our present epoch. This is the same as saying that we live in a very peculiar place or time in the universe, something that is difficult to believe. On the other hand, one can think that we have constructed our standard Maxwell electrodynamics in such a way that the properties of the field M do not affect any of our experiments. This is also difficult to believe. Another possible way in which the field M might had been missed by our experiments is if its strength is tiny. For example, since Eq. (1) suggests that η is a small quantity, then it follows that the field M is weak. Indeed, when $\eta = 0$ then $\eta_{\mathrm{h}} = -\eta_{\mathrm{n}}$. This result together with Eq. (6) and combined with the properties of $\boldsymbol{J}_{\mathrm{n}}$ and $\boldsymbol{J}_{\mathrm{h}}$ imply that $\eta_{\mathrm{h}} = \eta_{\mathrm{n}} = 0$. Thus, the trivial solution of Eq. (8) occurs when $\eta = 0$ and gives $M = 0$ because M is not harmonic. When η is a small quantity, one has to proceed slightly differently. The Lorentz force can be naturally generalised as $\mathrm{d}\boldsymbol{P}/\mathrm{d}\tau = {}^*\boldsymbol{F} \cdot {}^*\boldsymbol{J}_e + M\boldsymbol{J}_e = (1+\eta)\,({}^*\boldsymbol{F} \cdot {}^*\boldsymbol{J}_{\mathrm{std}} + M\boldsymbol{J}_{\mathrm{std}})$, where τ is the proper time and $\boldsymbol{P}$ is the 1–form momentum. So, if η is small and M is not negligible then we would had already observed the properties of the field M in our laboratories. However, this Lorentz force can be used in experiments to test the validity of our reasoning.

On the other hand, when $\eta_{\mathrm{n}} = 0$, then $\boldsymbol{M} = 0$, and the Lorentz force is given by

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}\tau} = (1+\eta_{\mathrm{h}})\,{}^*\boldsymbol{F} \cdot {}^*\boldsymbol{J}_{\mathrm{std}}. \tag{12}$$

This means that the standard Lorentz force is changed by a factor $(1+\eta_{\mathrm{h}})$ because the variations of η_{h} produce deviations in the intensities of the electromagnetic interactions.

However, Eq. (12) can be written as

$$\frac{1}{(1+\eta_{\mathrm{h}})}\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}\tau} = {}^*\boldsymbol{F} \cdot {}^*\boldsymbol{J}_{\mathrm{std}}. \tag{13}$$

This equation means that the electromagnetic forces are producing deviations from the standard dynamics, since $\eta_{\mathrm{h}} \neq 0$ associates changes on the momentum which are not Newtonian.

The duality presented in Eqs.(12)-(13) is similar to that presented by some researchers [17–20] for the gravitational forces in order to explain the rotation curves of galaxies, and other astronomical observations. These theories, the so called Modified Newtonian Dynamics (MOND) theories, suggest that our standard ideas of dynamics should be changed. For the electromagnetic case considered in the present article, this modification occurs naturally.

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